

# Direct N-body simulations of distant halo globular clusters

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# Outline

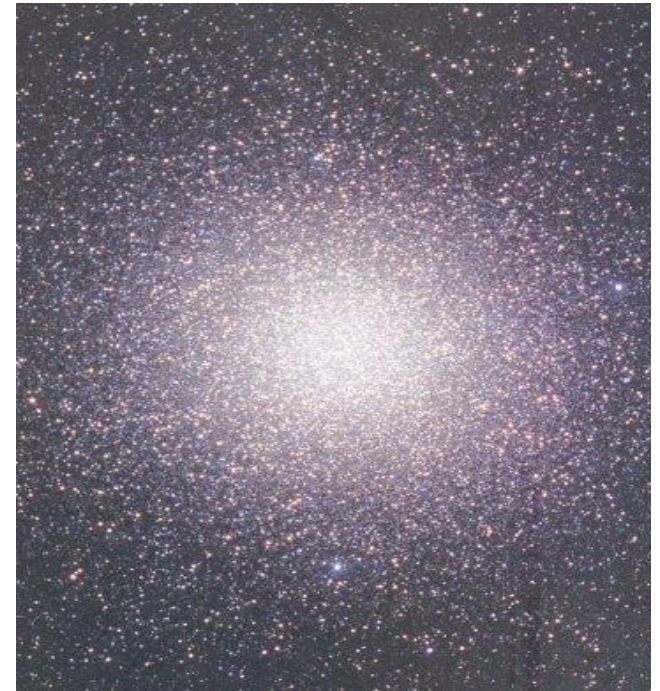
- Globular clusters (GCs)
- Dynamical evolutionary modeling of GCs
- The importance of the **Distant Diffused** star clusters
  
- Palomar 14 & Palomar 4

# Introduction: Globular clusters (GCs)

Median mass  $\sim 3 \times 10^5 M_{\odot}$

Median Size:  $\sim 10$  pc

- The GCs of the Milky Way extends out to more than 100 kpc.
- They contain virtually no gas or dust
- They contain coeval stars
- Almost all have ages  $\sim 12$ Gyr



GCs are **collisional** systems v.s. Galaxies that are **collisionless**.

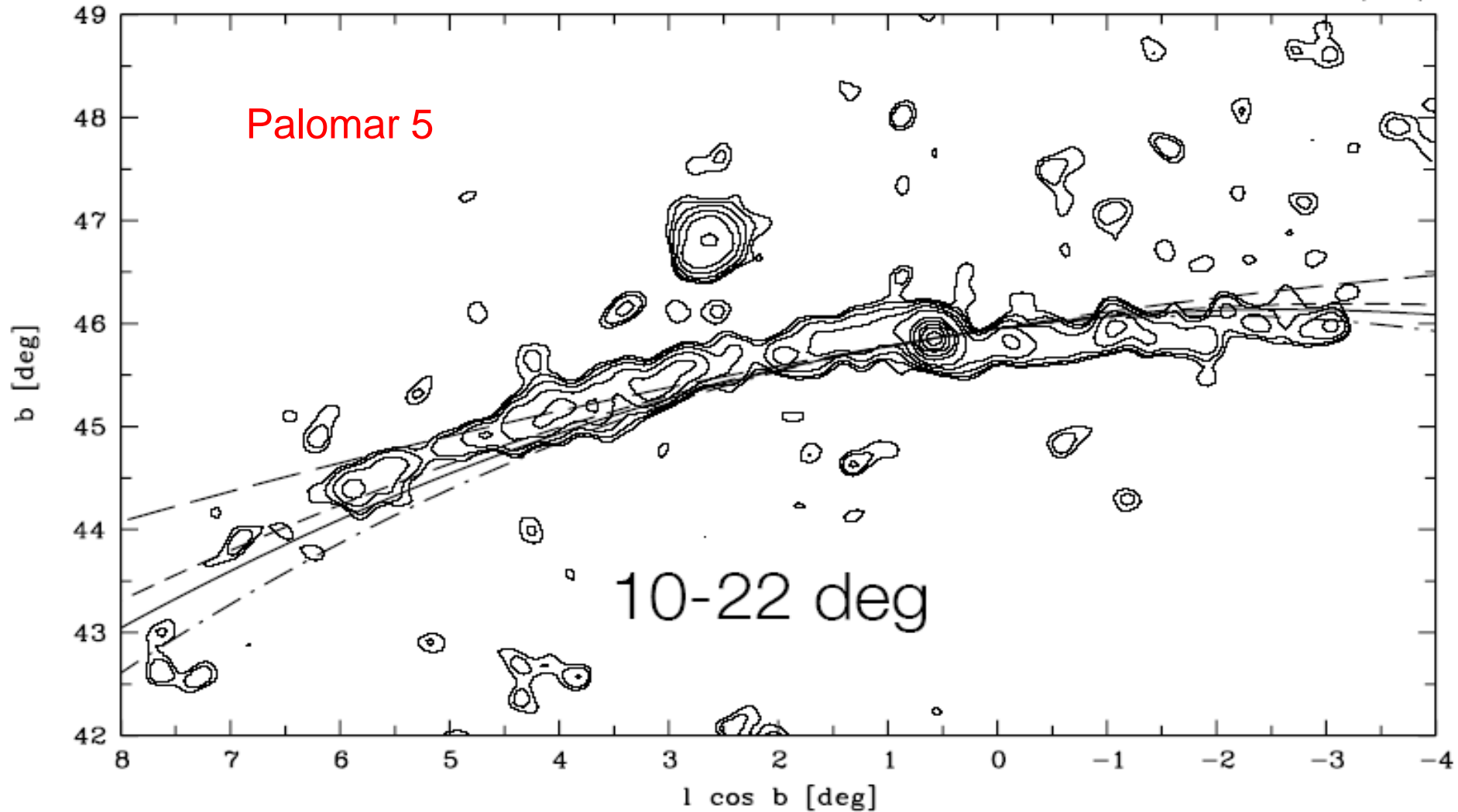
2-body interactions of stars are important in driving the dynamical evolution: **bumpy** systems

stars are mainly moving in the collective gravitational field

A perfect laboratory to explore the effects of **2-body encounters** on dynamical evolution.

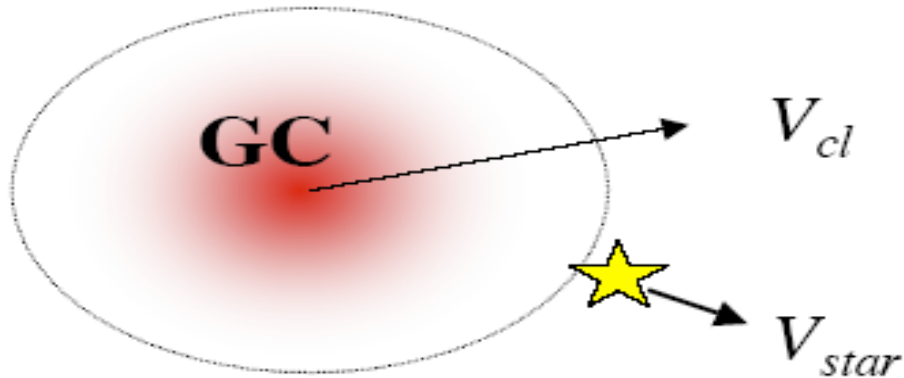
# Dynamic Evolutionary modeling of GCs

Odenkirchen et al. (2003)



# Dynamic Evolutionary modelling of GCs

Tidal boundary



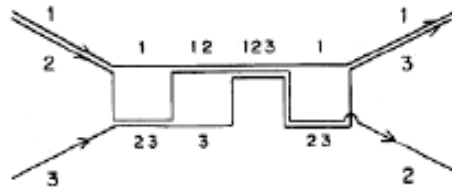
## Galaxy

- Star clusters:  $10^7 - 10^9$  yrs
- Galaxies:  $10^{16}$  yrs

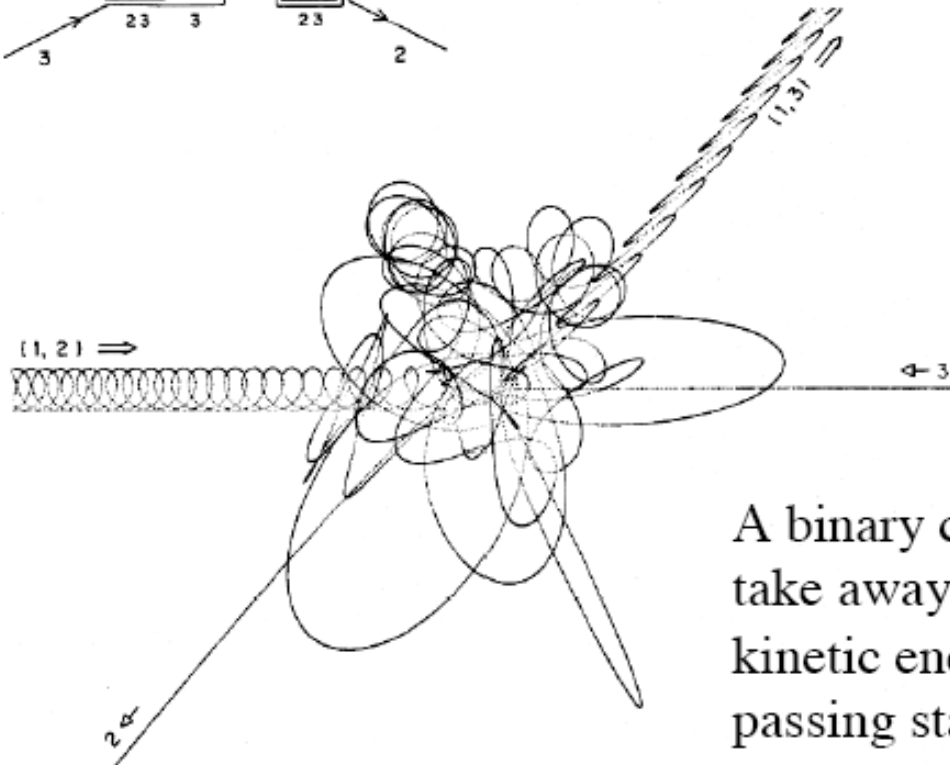
# Dynamical evolution of globular clusters: N-Body simulations



## Examples of 3-Body Interactions



*(Numerical simulation by  
P. Hut and J. Bahcall)*



A binary can either  
take away or give  
kinetic energy to a  
passing star

The binary acts as an energy source in the core thus halting core collapse.

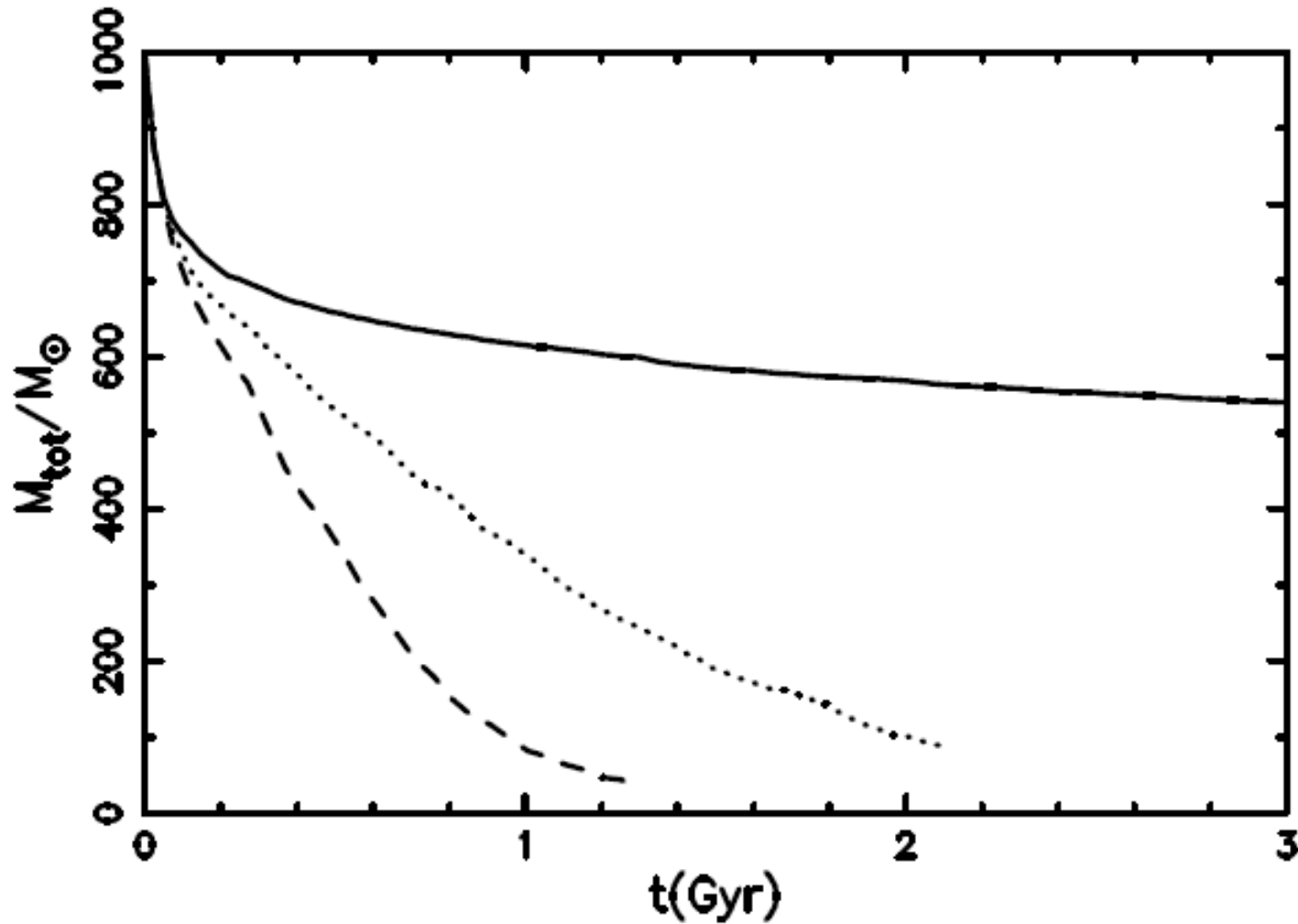
# Dynamic Evolutionary modeling of GCs

- A detailed understanding of the effects of **evolutionary** processes is essential to be able to disentangle the properties which result from **dynamical evolution** from those **imprinted at the time of cluster formation**.



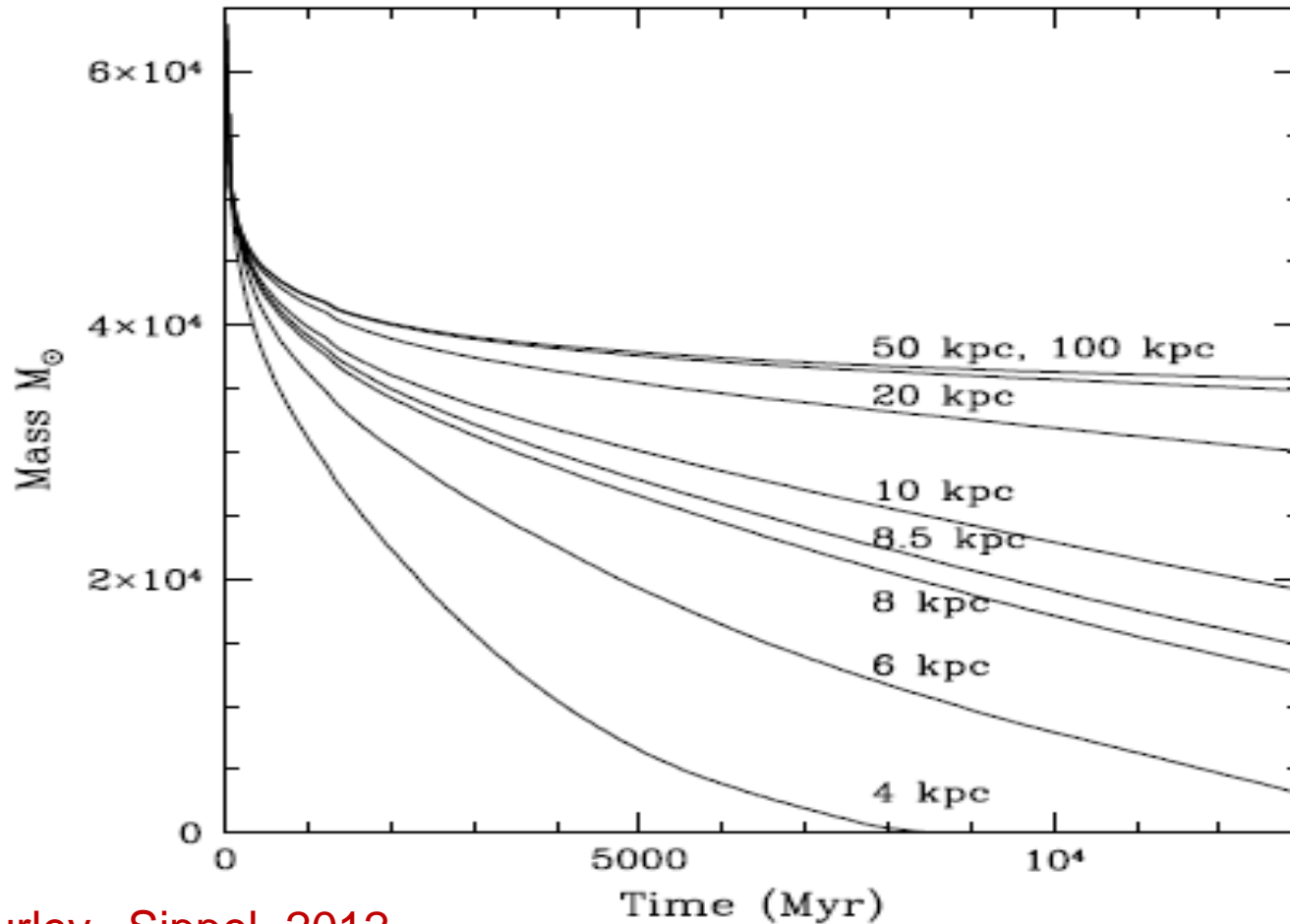
# The effect of Dynamical Evolution on the Basic properties of clusters

## Mass



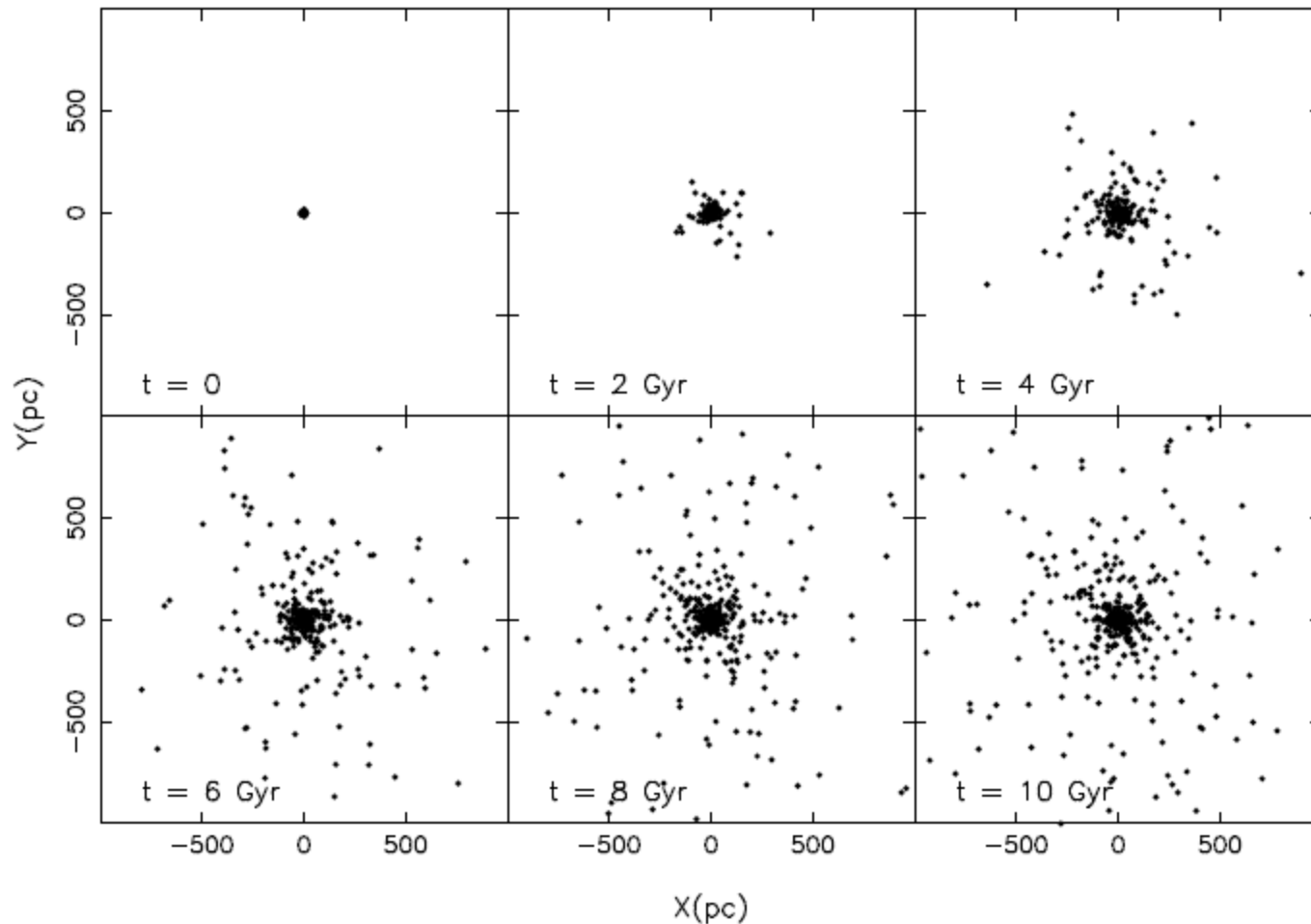
# The effect of Dynamical Evolution on the Basic properties of clusters

## Mass



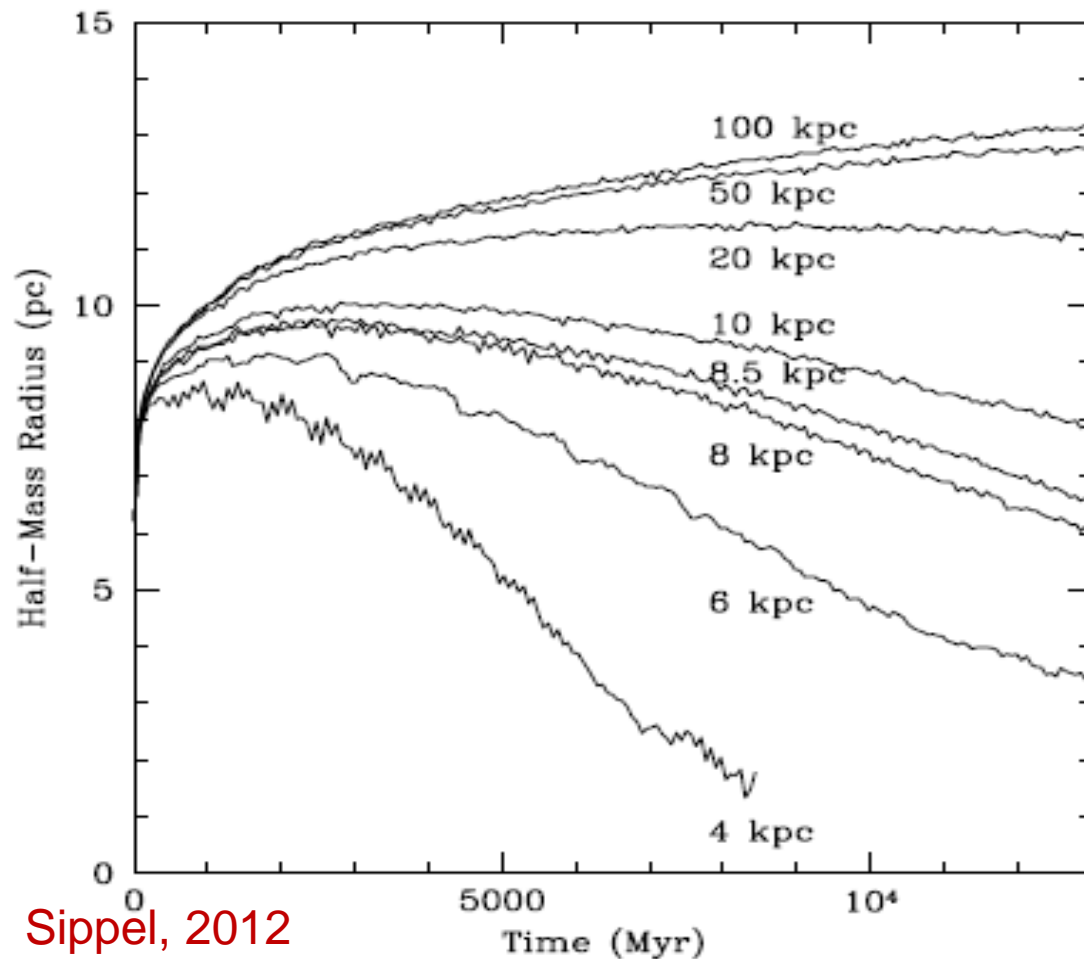
# The effect of Dynamical Evolution on the Basic properties of clusters

## Size scale



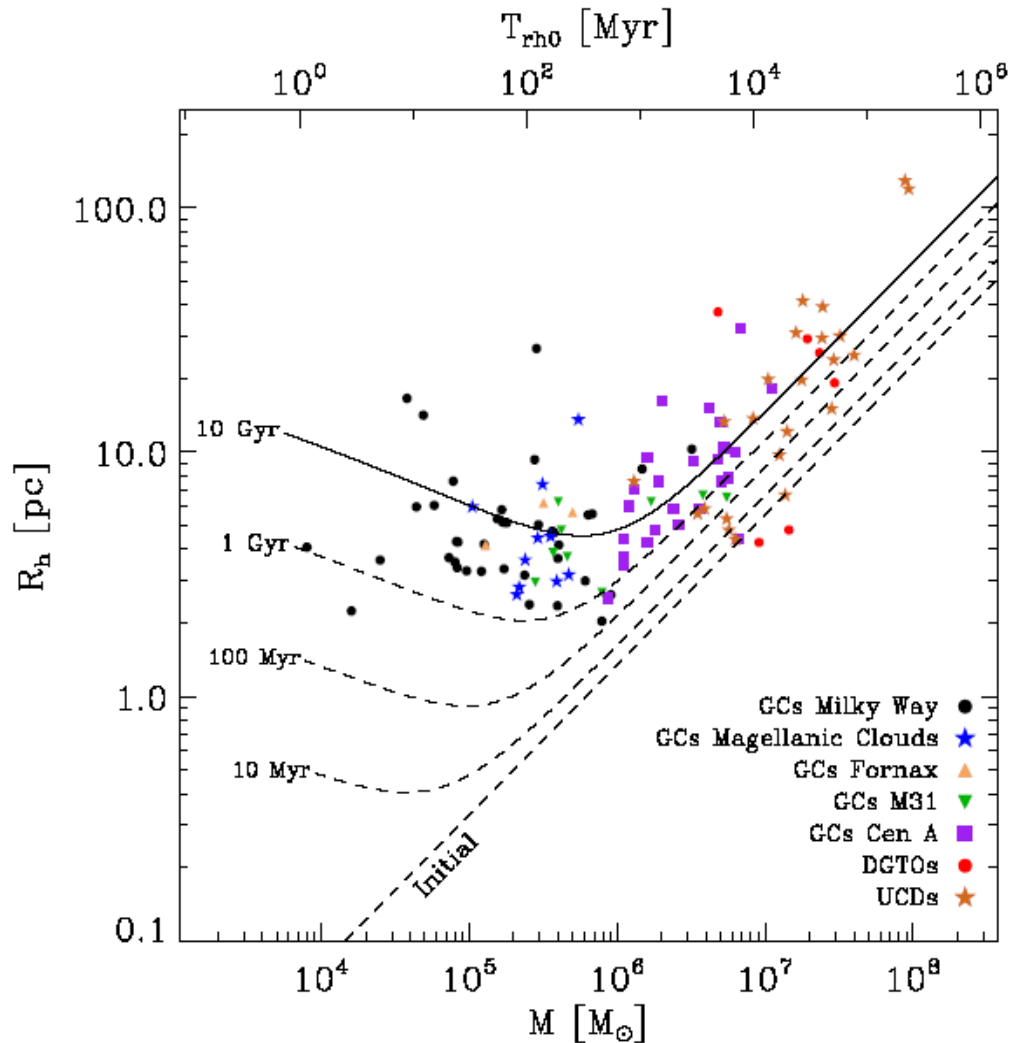
# The effect of Dynamical Evolution on the Basic properties of clusters

## Size scale



# Mass-radius relation

Gieles et al. (2010)



Globular clusters: no apparent correlation between mass and radius.

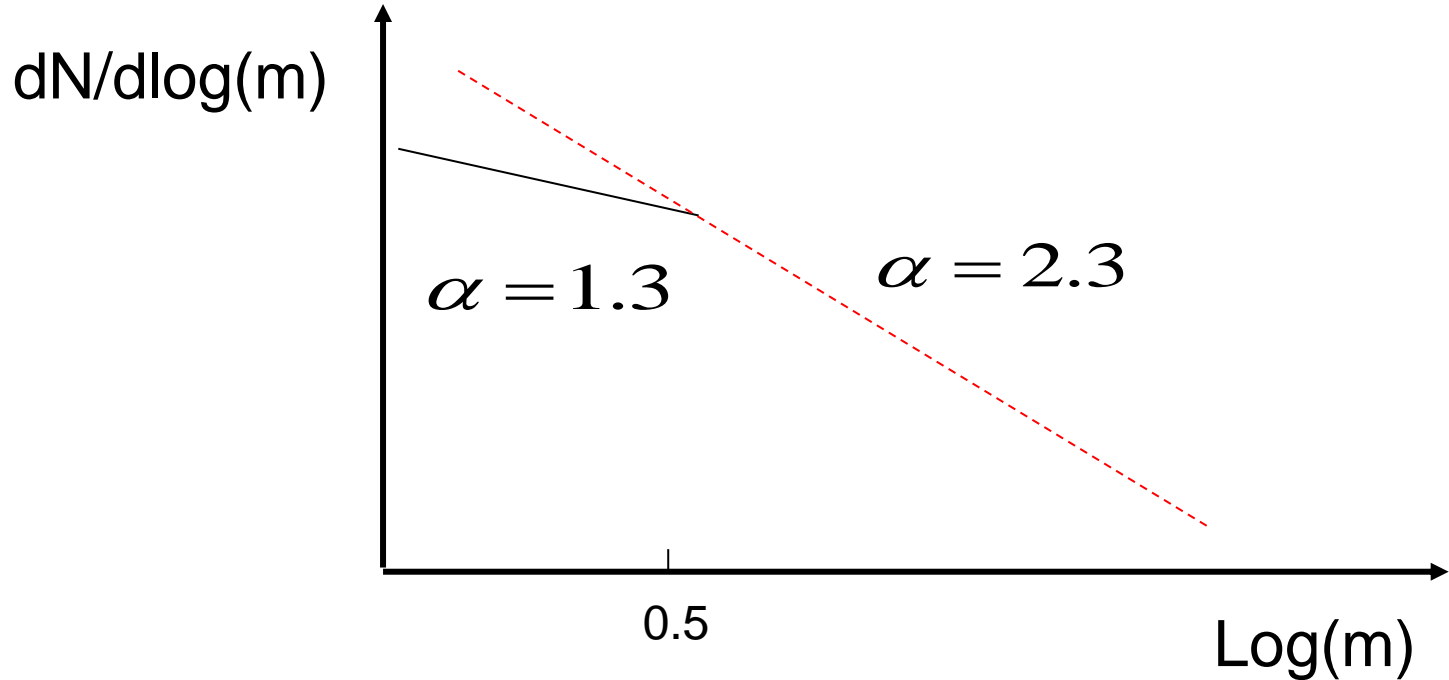
Elliptical galaxies: a strong positive correlation between mass and radius.

Globular clusters, have undergone expansion driven by stellar evolution and hard binaries.

# Initial mass function (IMF)

**IMF:** The initial mass distribution of stars

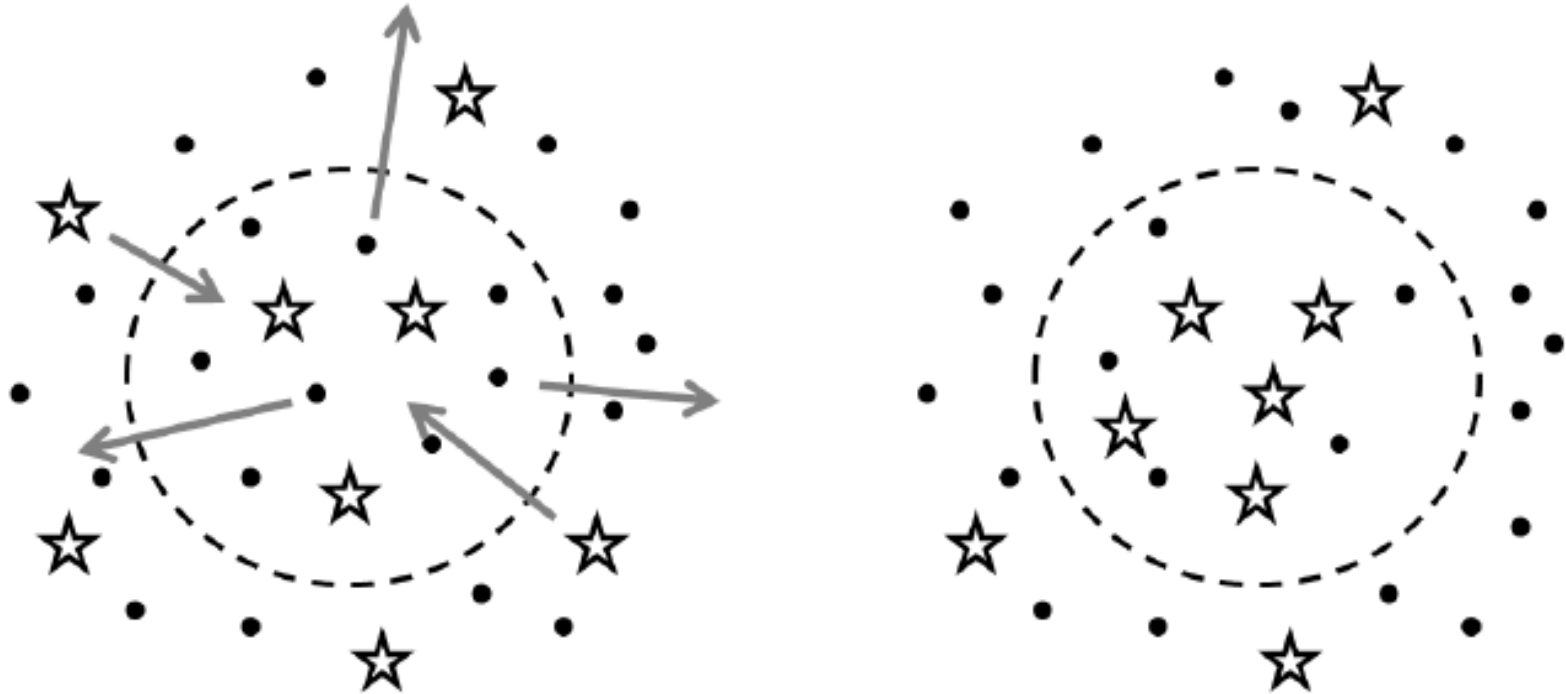
(Salpeter 1955, Kroupa 2001, 2012)



The mass function of stars in clusters evolves through **stellar and dynamical evolution**. Therefore it is hard to extract the IMF from the observed mass function. (Baumgardt & Makino 2003).

# Mass Segregation

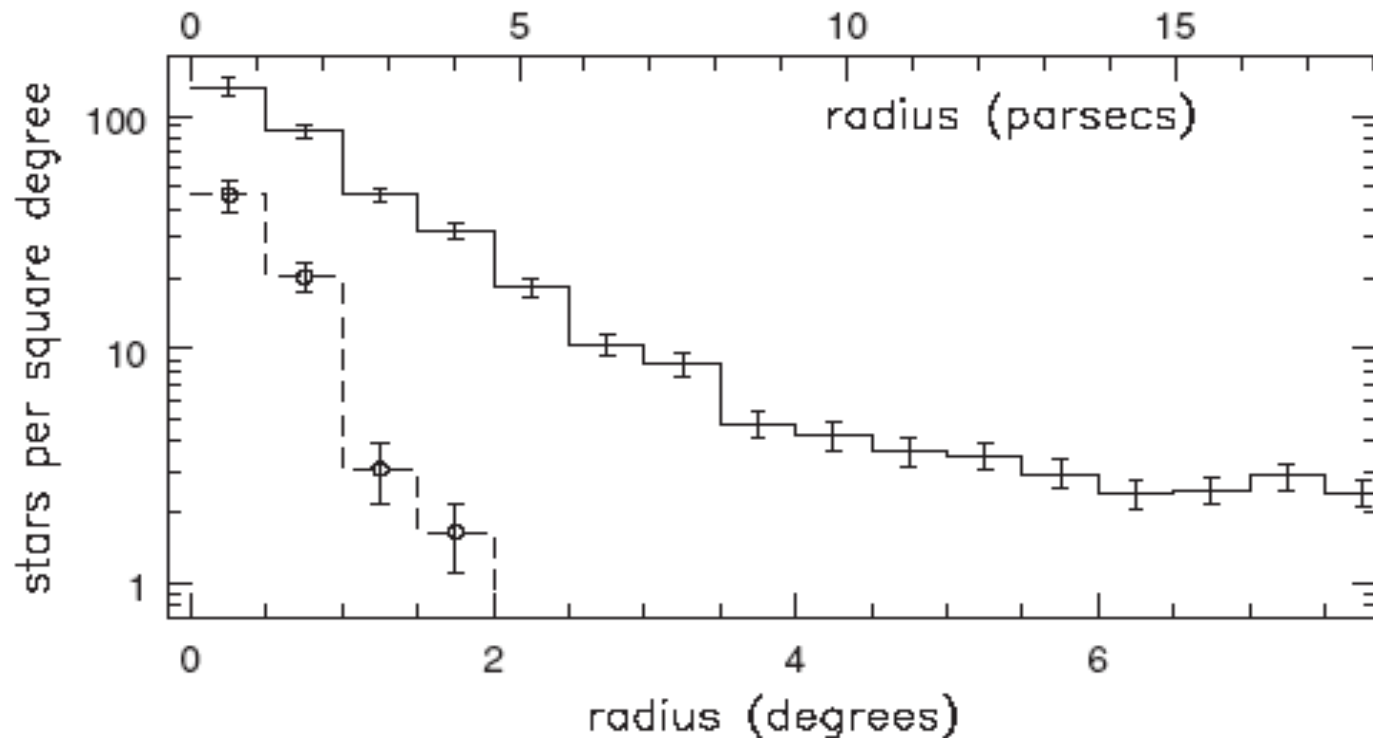
a result of energy equipartition



**t = 0**

The prediction of energy equipartition is confirmed in **globular cluster 47 Tucanae**: Of the 15,000 stars, **only 23 stars** were in the **targeted blue group**. The average squared speed to be **72 (km/s)<sup>2</sup>**, which is half the value we found for **the red stars**, **144 (km/s)<sup>2</sup>**. (Meylan 2007)

# Mass Segregation

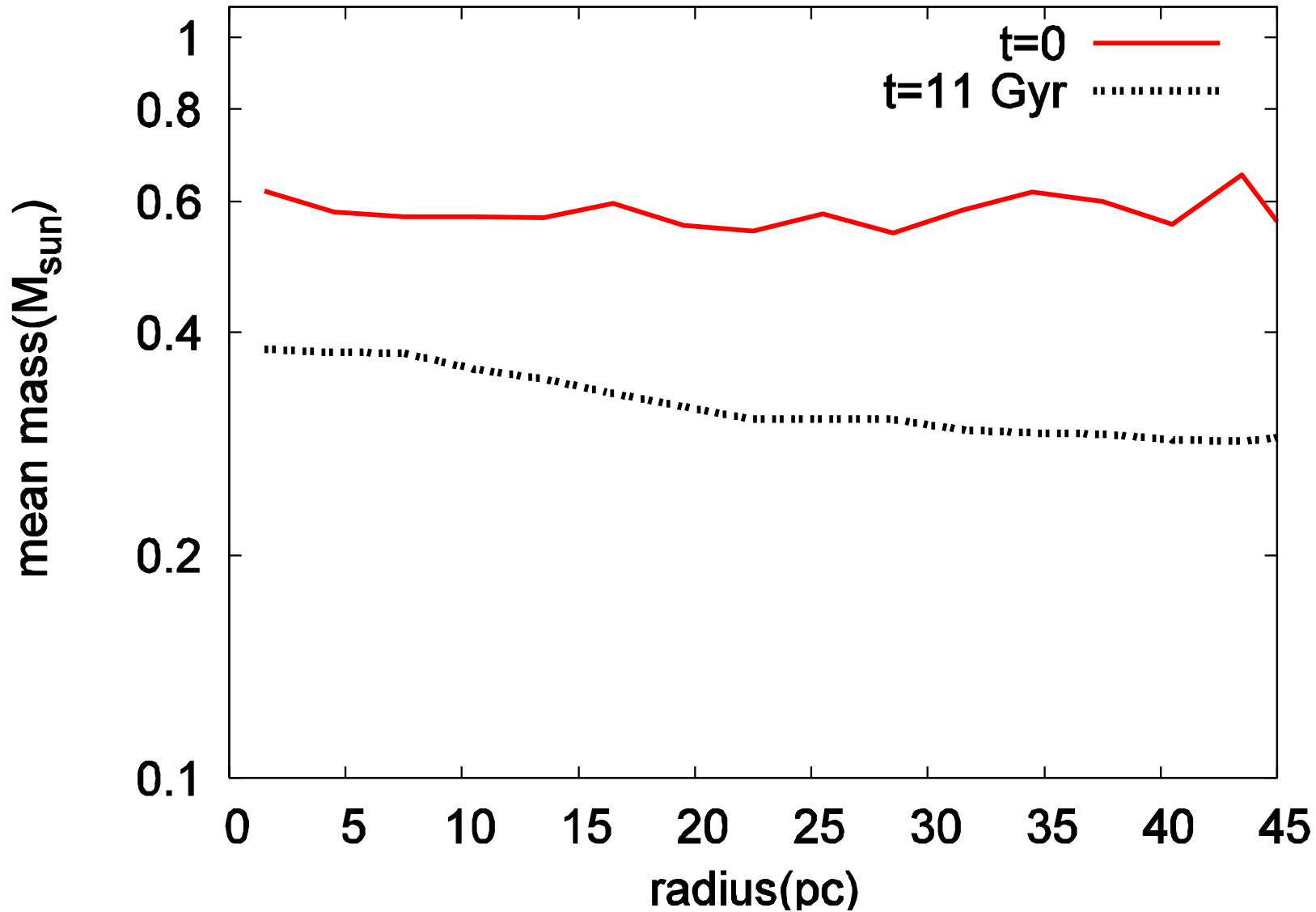


**Fig. 3.6.** In the Pleiades open cluster, stars with masses above  $M_{\odot}$  (dashed histogram) are more concentrated toward the center than stars with  $M < M_{\odot}$  (solid histogram) – J. D. Adams.

Taken from book by Spark & Gallagher, 2006



# Dynamical Mass Segregation

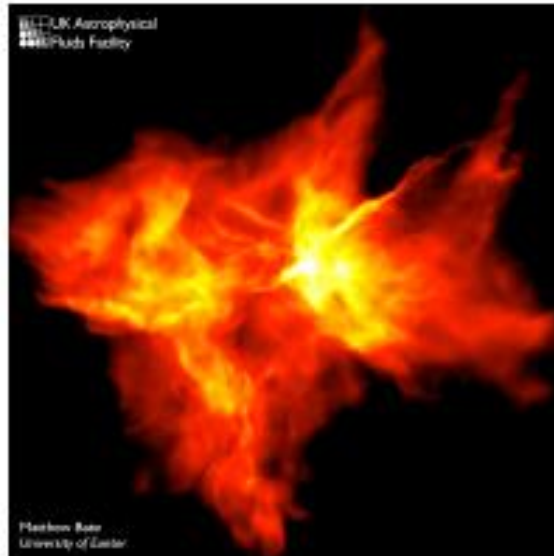
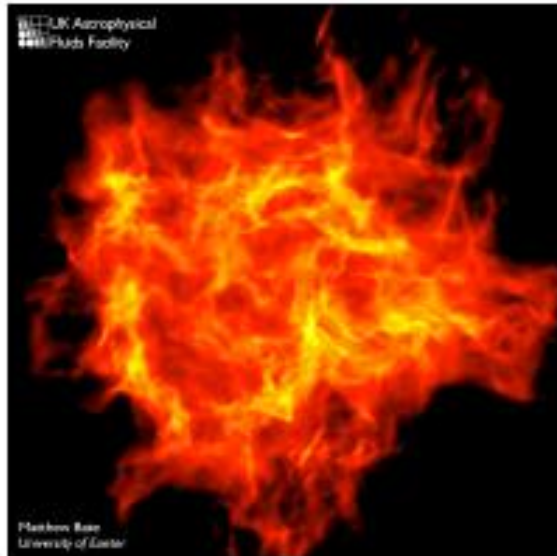


# Primordial Mass Segregation

- A number of observational studies have found evidence of mass segregation in clusters with **ages shorter** than the time needed to produce the observed segregation by **two-body relaxation** (de Grijs 2010)
- Observed segregation in young clusters would be primordial and **imprinted by the star-formation process**.

# Star Formation

- stars form in clumpy environments
- massive stars form preferentially at the centers of the clumps



(Bonnell & Bate 2006)

- segregation persists as small clumps merge to form larger ones (McMillan, Vesperini, & Portegies Zwart 2007)

# The importance of the **Distant Diffused** star clusters

1- Test of Gravity models, for example modified Newtonian dynamics (MOND; Milgrom 1983).

(Scarpa 2003, Baumgardt et al 2005, Haghi et al 2009 MNRAS, Haghi et al 2011, A&A, Haghi et al 2013 ApJ, Frank & Haghi et al 2012, MNRAS, Jordi & Haghi et al, 2011, AJ)

**Low** acceleration regime

## 2 - Direct N-body simulation

Using **GPU-accelerated N-body6 code** (Aarseth 2003; Nitadori & Aarseth 2012).

**Low** mass together with large radius make possible to simulate these clusters on a **star-by-star** basis with **GPU** computers.

# Numerical Modeling

## I. Modeling of Palomar 14

Zonoozi et al 2011

## II. Modeling of Palomar 4

Haghi et al 2013

# I. Modeling of Palomar 14

# Observational data of Palomar 14

- Spectroscopic and photometric data  
(Hilker 2006, A&A, Jordi, Hagi et al. 2009, AJ )
- Age of the cluster = 11.5 Gyr (young!!)
- Distance =  $71 \pm 1.3 \text{ kpc}$
- Half-light radius =  $1'.28'' = 26 \text{ pc}$
- Number of bright stars,  $N_{bs} = 2954 \pm 175$
- Number of giant,  $N_{(RGB+HB)} = 197 \pm 28$
- Velocity dispersion,  $V_r = 0.38 \pm 0.12 \text{ kms}^{-1}$   
**Velocity dispersion** (radial velocity of 17 giant stars)

# Palomar 14

- Mass function:

$$dN/dm = m^{-\alpha}$$

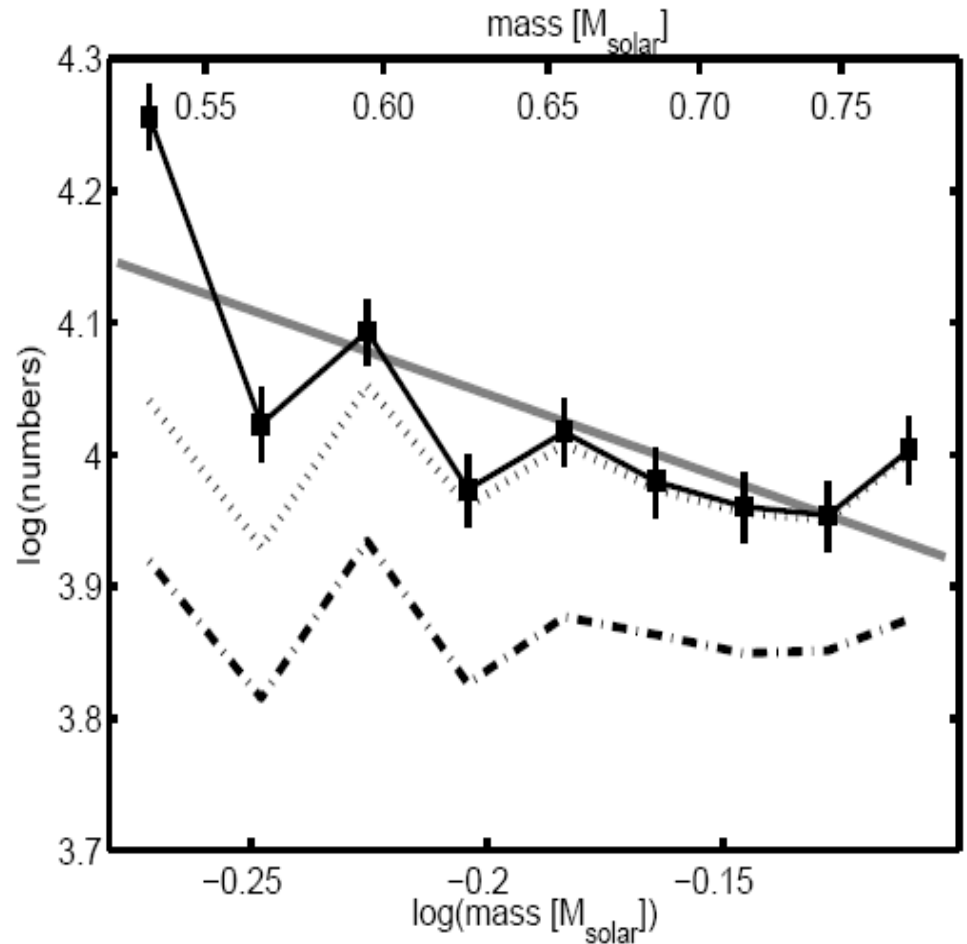
$$0.525 M_{sun} < M < 0.79 M_{sun}$$

$$\alpha = 1.27 \pm 0.44$$

- Flatter than Canonical  
Kroupa IMF

$$\alpha = 2.35$$

Jordi et al. (2009)





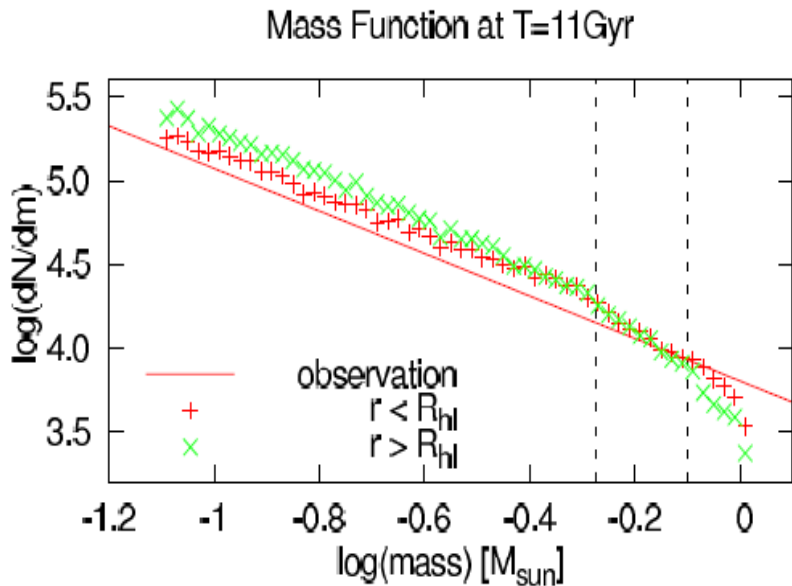
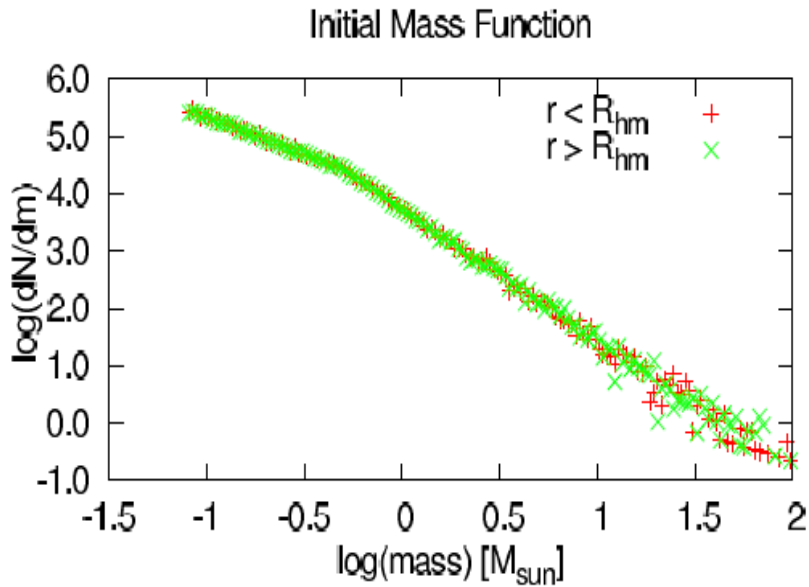
# Direct N-body simulation

## DESCRIPTION OF THE MODELS

- We use the collisional *N*-body code Nbody6 (Aarseth 2003; Nitadori & Aarseth 2012) on the GPU computers of the University of Bonn.
- Number of stars  $N \sim 100,000$  with Plummer model (Plummer 1911)
- Evolution time: 11 Gyr
- Stellar Evolution: SSE/BSE routines developed by Hurley et al.
- Tidal effect: galactic potential (Allen & Santillan (1991)).

# Canonical Kroupa IMF

Zonoozi et al 2011, MNRAS



$M_{r < R_{hm}}^f [M_{\odot}]$ ( $\pm 320$ )	$\alpha$ ( $\pm 0.15$ )	$\sigma_{los} [\text{km/sec}]$ ( $\pm 0.01$ )	$N_g$ ( $\pm 16$ )
1521	2.10	0.65	162
1402	1.85	0.51	136
1869	1.96	0.64	163
1530	1.90	0.64	161
1705	2.04	0.61	174
1542	2.08	0.58	188
1627	2.10	0.58	160
2127	1.88	0.66	174
1821	1.77	0.64	160
1689	2.13	0.61	172
1722	2.02	0.59	174
1970	2.14	0.61	171
1925	1.78	0.68	194
1695	1.87	0.65	193
1995	1.97	0.64	170
1788	2.14	0.61	178
1918	2.10	0.60	163
2038	2.06	0.68	181
2079	2.00	0.66	186
1910	1.90	0.65	166
1939	2.05	0.62	172
2155	1.98	0.61	171
2158	2.07	0.70	224
2279	2.00	0.66	195
2144	2.08	0.65	161
1935	1.96	0.63	190
1834	2.15	0.62	198
2367	2.22	0.62	170
$\Sigma$	$2200 \pm 90$	$1.27 \pm 0.44$	$198 \pm 19$
		$(0.64 \pm 0.15^*)$	

other scenarios:

Primordial mass segregation

Flattened IMF

Primordial binary fraction

# Results of primordial mass segregated clusters

Zonoozi et al 2011, MNRAS

Model	$R_{phm}$ [pc] $\pm 1.2$	$R_{phl}$ [pc] $\pm 1.9$	$N_{bs}$ $\pm 120$	$M_{r < R_{hm}}^f [M_{\odot}]$ $\pm 80$	$\alpha$ $\pm 0.10$	$\sigma_{los}$ [km/sec] $\pm 0.05$	$N_g$ $\pm 8$
S0.50M50R20	29.4	25.3	3184	2044	1.90	0.62	193
S0.60M50R20	32.2	27.5	3209	2061	1.9	0.58	207
S0.70M50R20	32.9	27.9	3267	2106	2.0	0.60	191
S0.80M50R20	35.6	29.6	3219	2066	1.95	0.57	222
S0.90M50R20	42.4	36.0	3257	2100	1.6	0.51	211
S0.91M50R20	43.8	38.5	3365	2169	1.63	0.46	206
S0.93M50R20	47.8	40.6	2887	1870	1.27	0.44	217
S0.95M50R20	48.7	41.5	3159	2045	1.28	0.47	215
<b>S0.95M50R15</b>	37.0	27.6	3077	1978	1.33	0.58	194
<b>S0.90M50R15</b>	33.9	26.3	3236	2085	1.67	0.55	210
B0.04M50R20	27.4	25.3	3132	2009	2.0	0.87	185
Observations		$26.4 \pm 0.5$	$2954 \pm 175$	$2200 \pm 90$	$1.27 \pm 0.44$	$0.38 \pm 0.12$ ( $0.64 \pm 0.15^*$ )	$198 \pm 19$

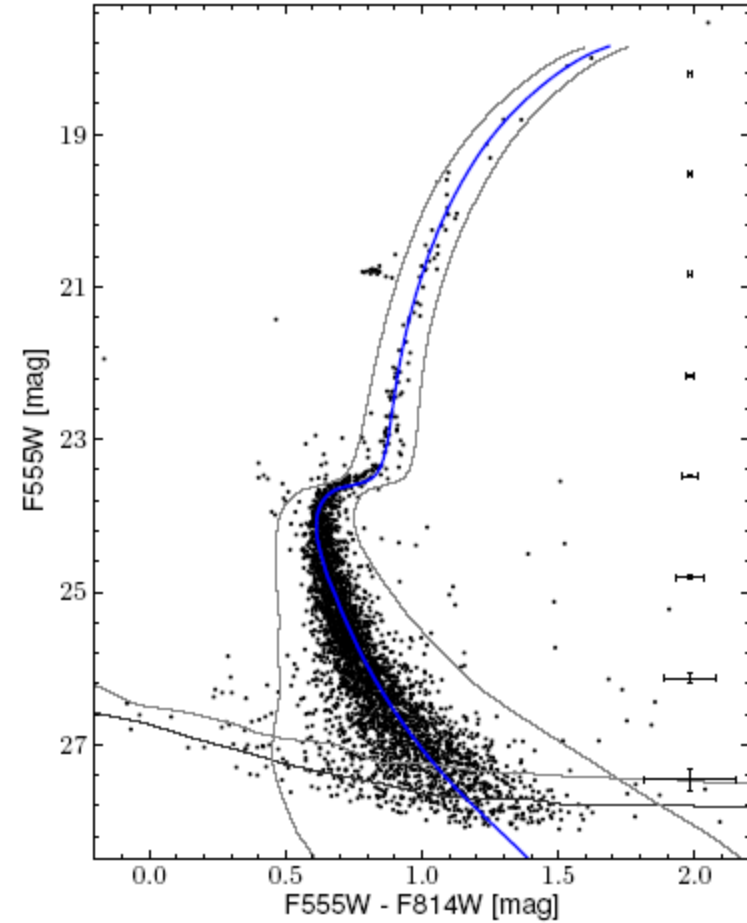
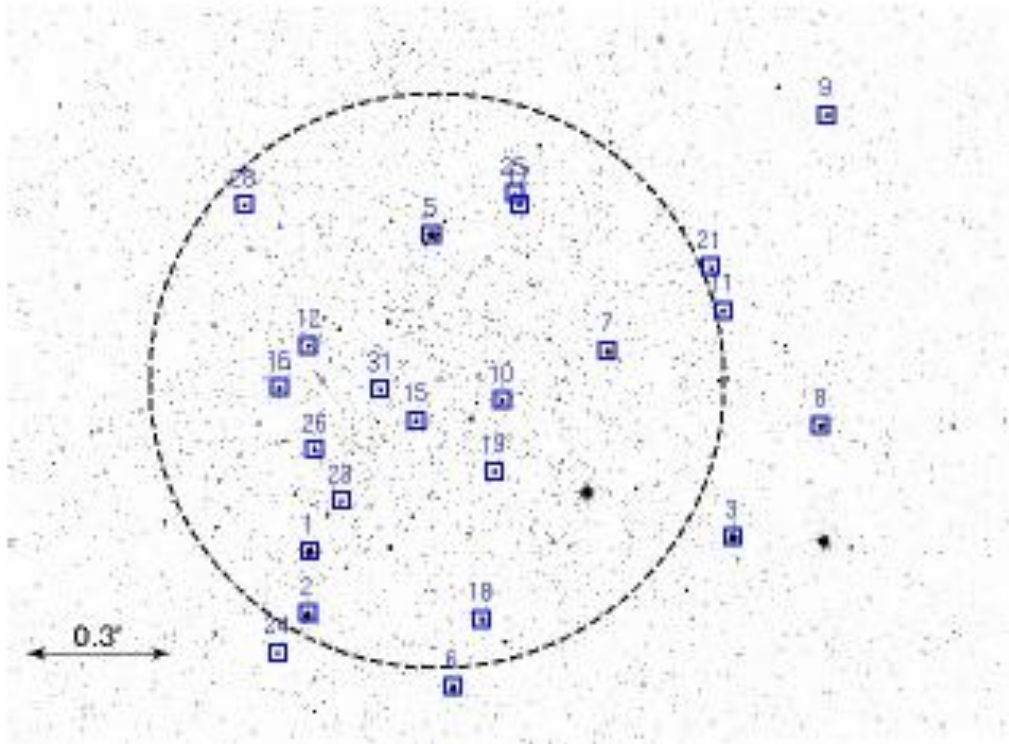
Clusters with such a strong degree of primordial mass segregation are able to reproduce the observed flat mass function inside the half-light radius.

Palomar 4

# Observation of Pal 4

(Frank et al. 2012)

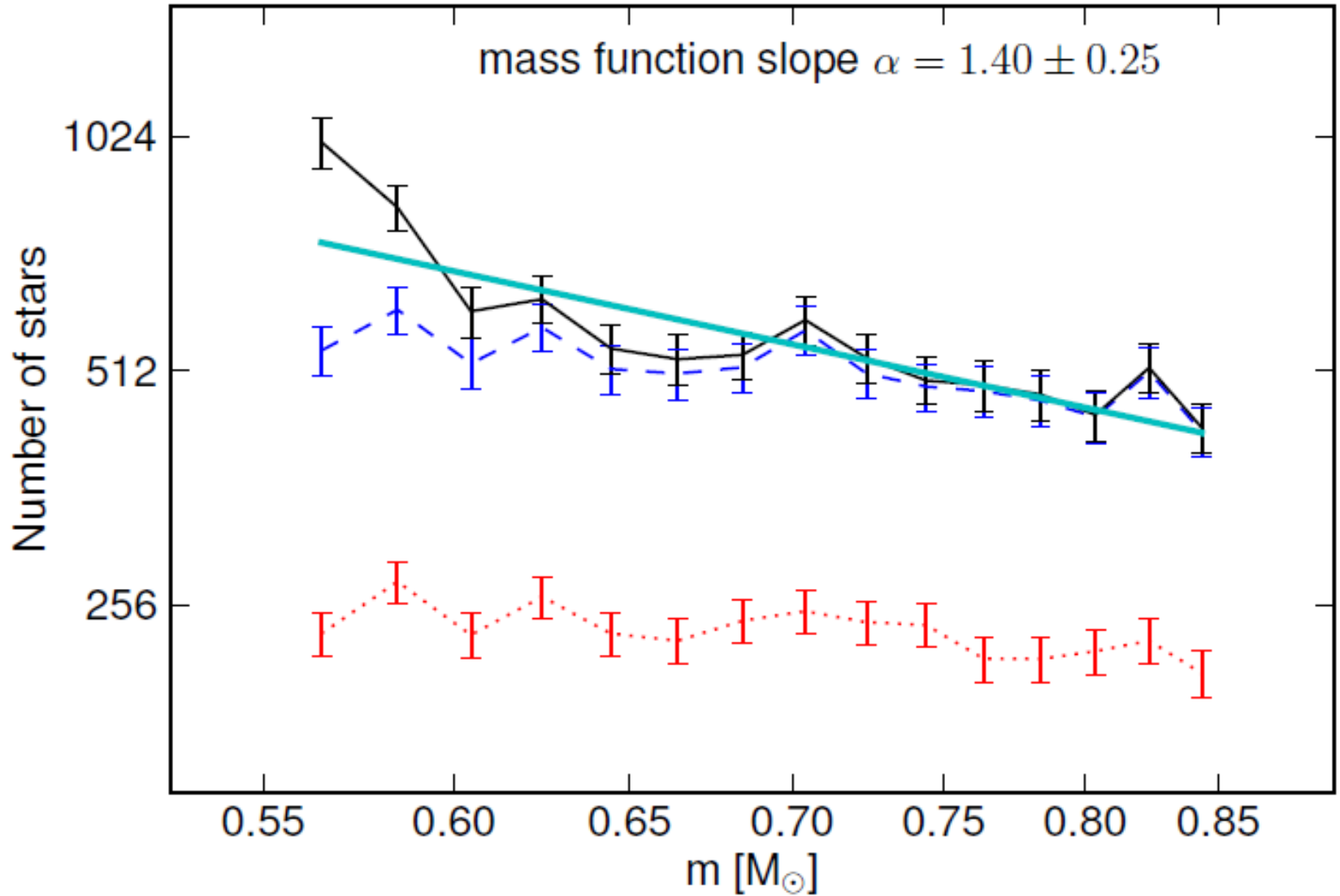
Position of spectroscopic target stars (24 candidates).



Observed color-magnitude diagram of Pal 4, containing 3878 stars

# Observation of Pal 4

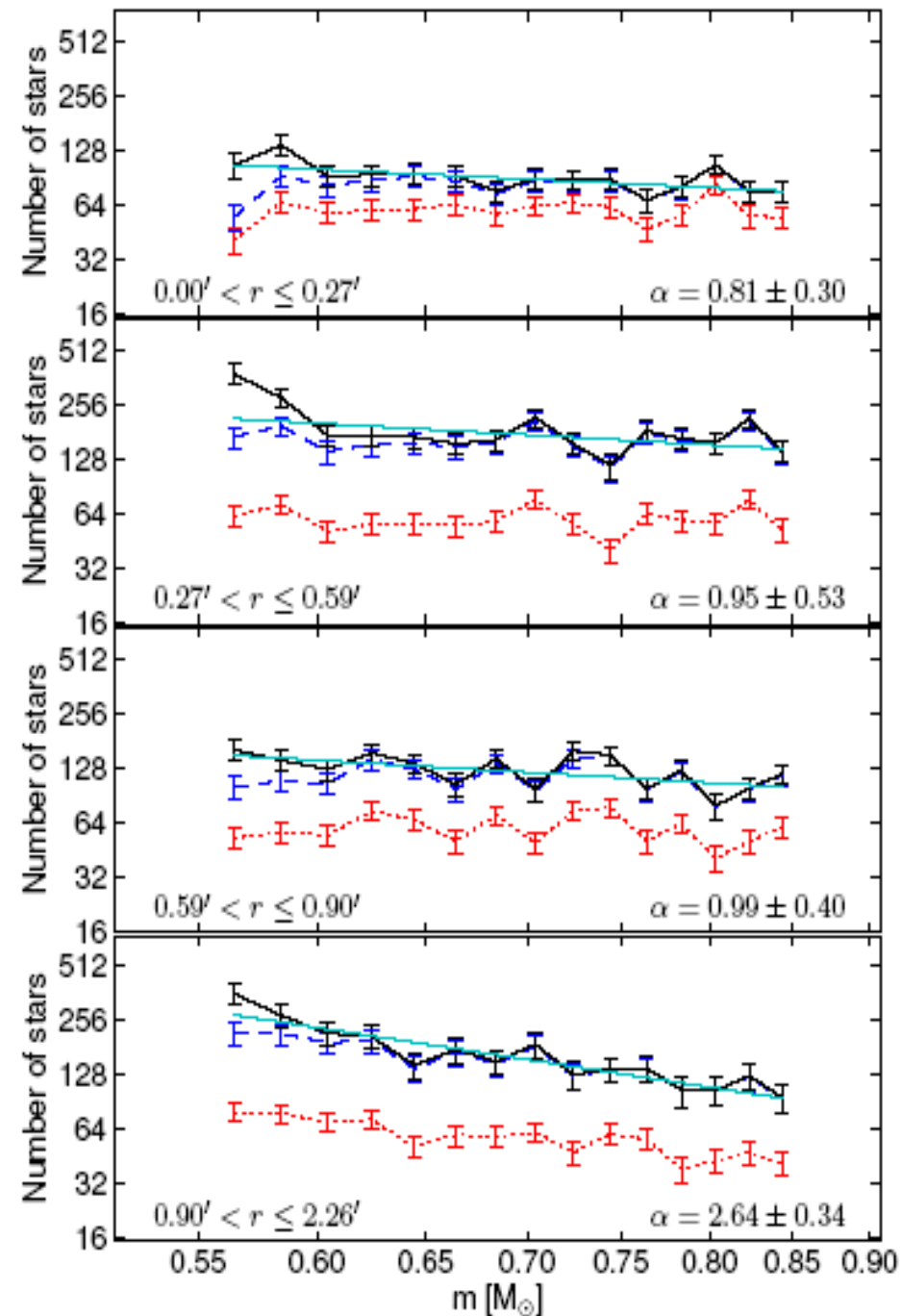
(Frank et al. 2012)



# Mass segregation

The mass-function as a function of radius

As the cluster's relaxation time is of the order of 20Gyr, this suggests primordial mass segregation.





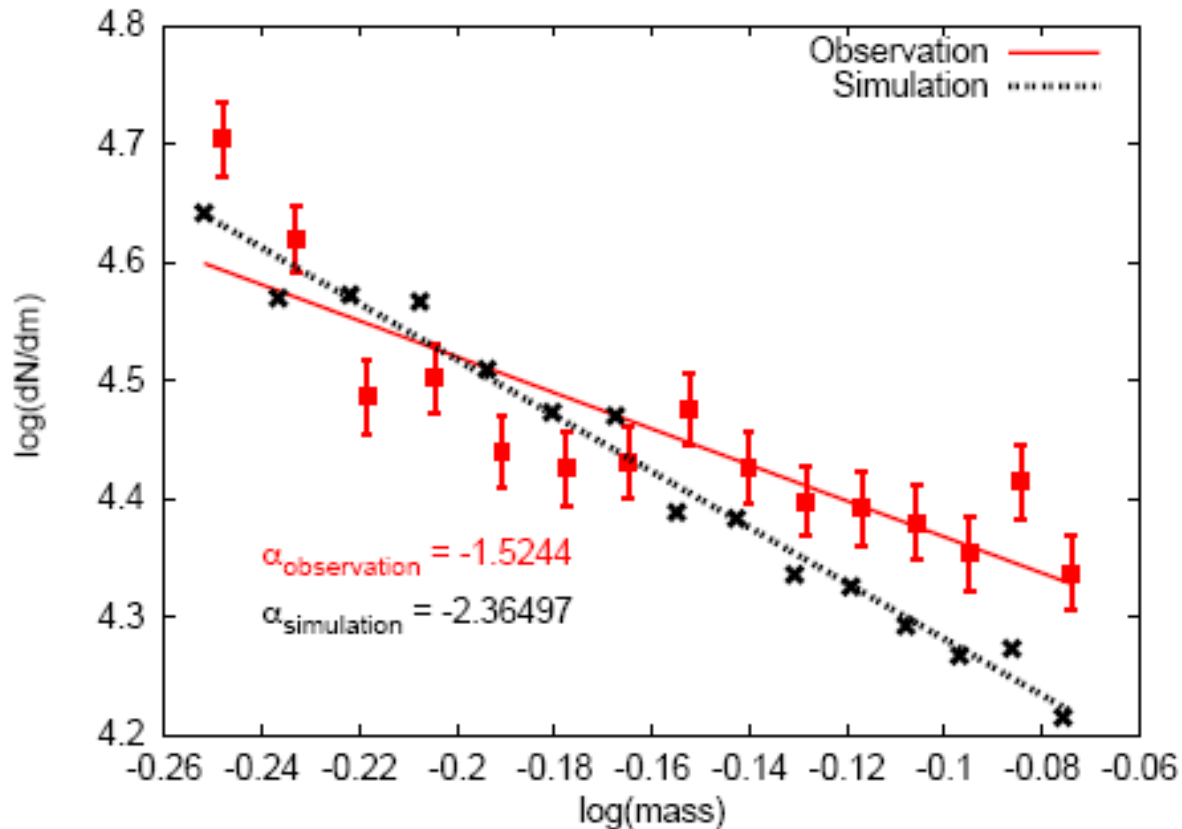
# The results of three sets of simulations:

- (i) Kroupa IMF without primordial mass segregation
- (ii) Kroupa IMF with different degrees of primordial mass segregation
- (iii) Flattened IMF with/without primordial mass segregation

# Canonical IMF without primordial mass segregation

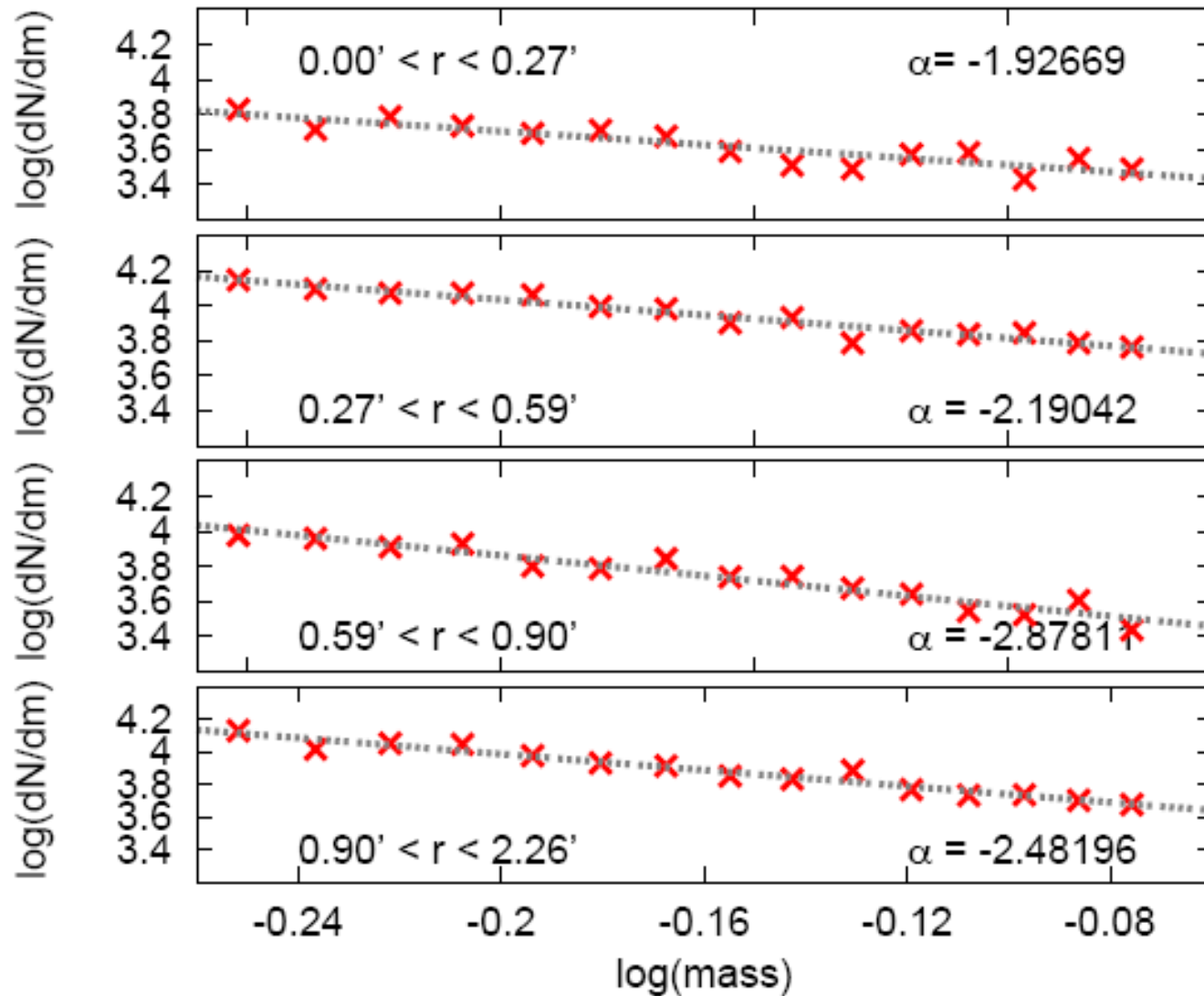
Model	$R_{phm}$ [pc]	$R_{phl}$ [pc]	$M_{r < R_t}^f$ [ $M_{\odot}$ ]	$\alpha_{tot}$	$\alpha_{in}$	$\alpha_{out}$	$\sigma_{los}$ [km/sec]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Canonical-NS							
M50R12	17.2	16.0	26695	2.20	2.00	2.56	0.81
M55R12	17.4	15.5	29299	2.21	2.04	2.75	0.84
M60R12	17.3	15.5	32418	2.25	1.95	2.72	0.90
M50R14	20.3	18.0	26702	2.23	2.00	2.46	0.76
M55R14	20.3	17.3	28436	2.01	1.67	2.54	0.77
M60R14	19.7	17.5	32246	2.25	2.10	2.58	0.84
M57R14.5	20.6	18.5	30484	2.35	2.00	2.70	0.80
Observation		$18.4 \pm 1.1$	$29800 \pm 800$	$1.4 \pm 0.25$	0.88	1.81	$0.87 \pm 0.18$

# I. Canonical IMF without primordial mass segregation

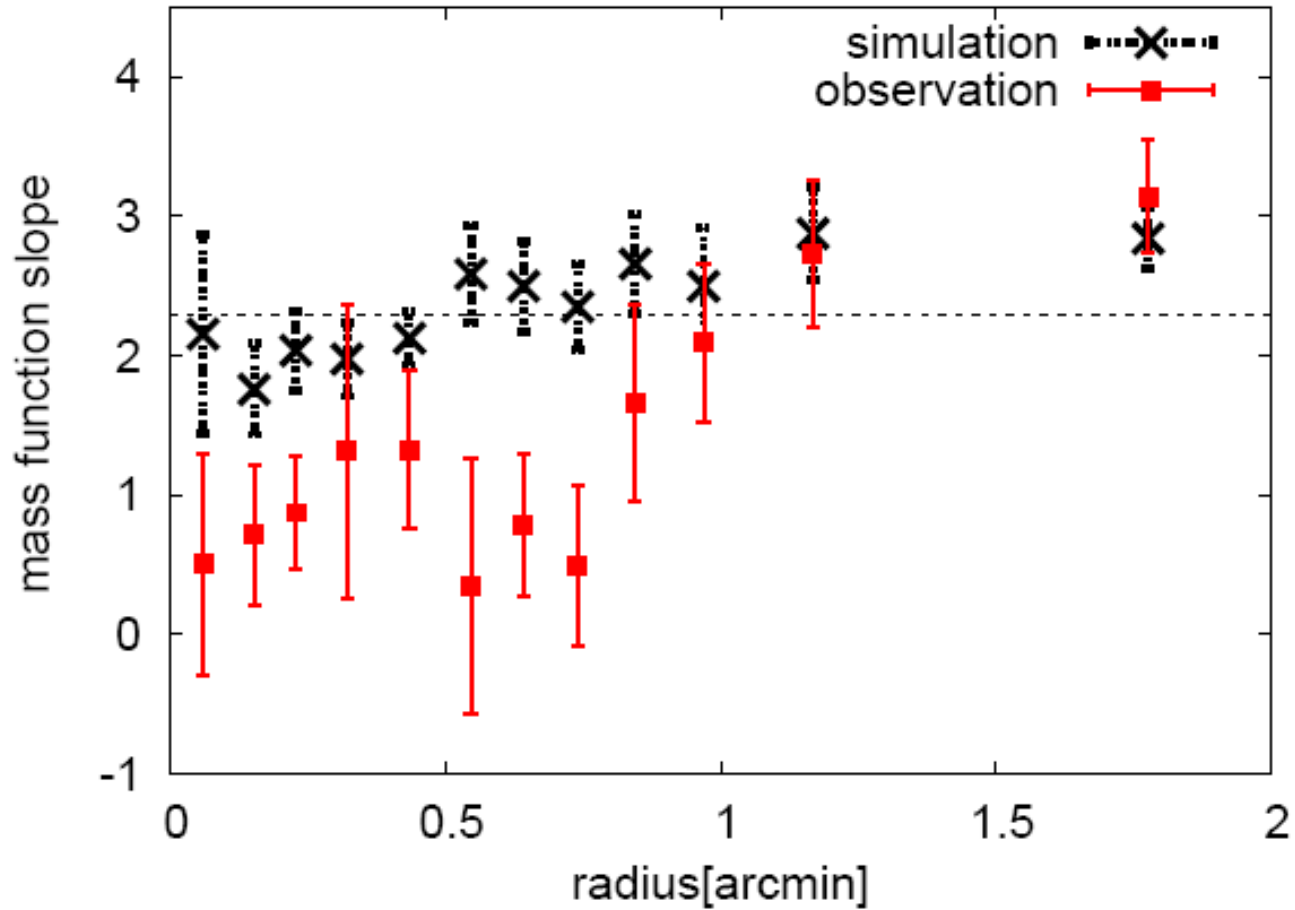


Global mass function of main sequence stars:  
It is steeper than the observed value

# Dynamical Mass Segregation



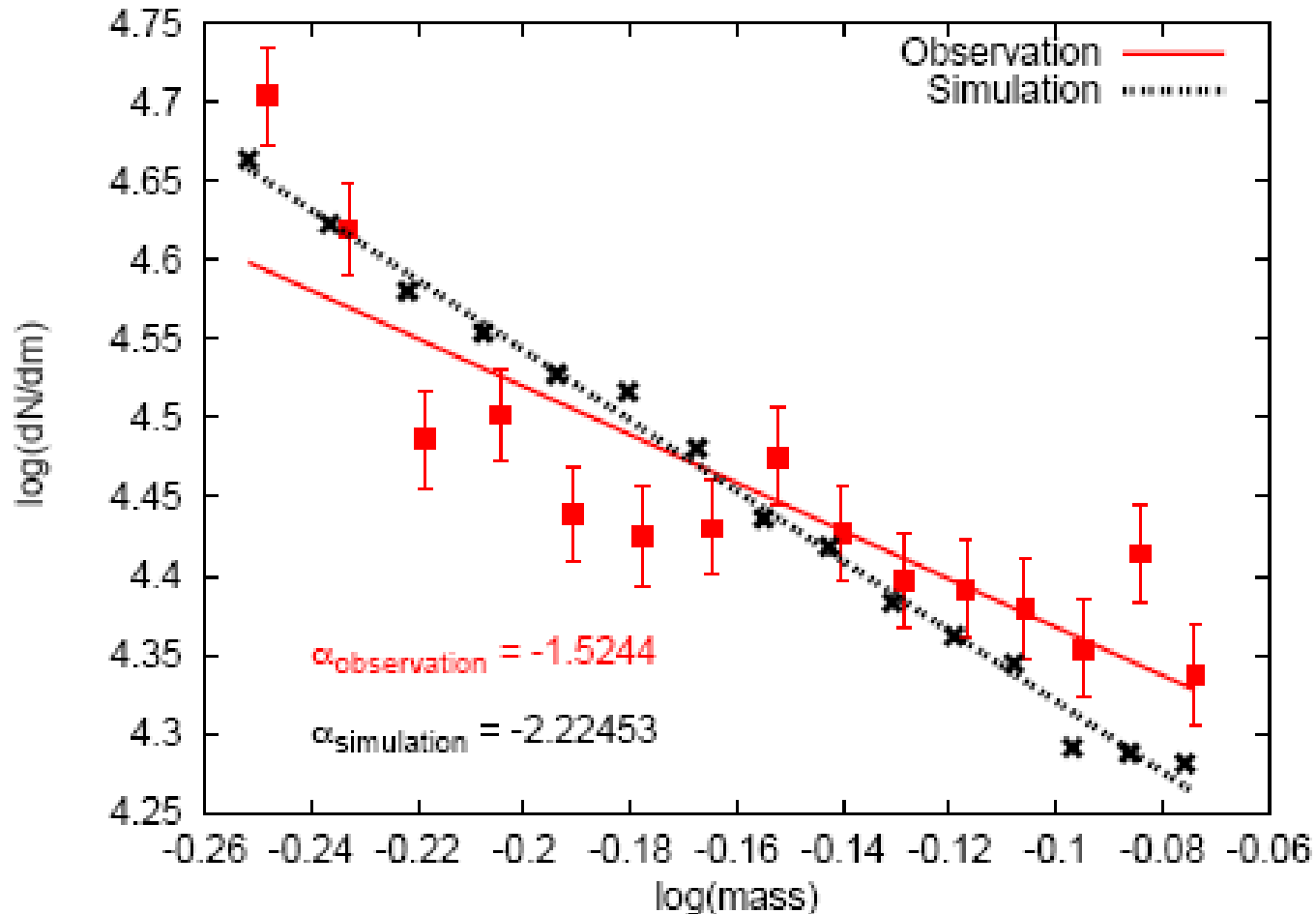
# Dynamical Mass Segregation



# Canonical IMF with primordial mass segregation

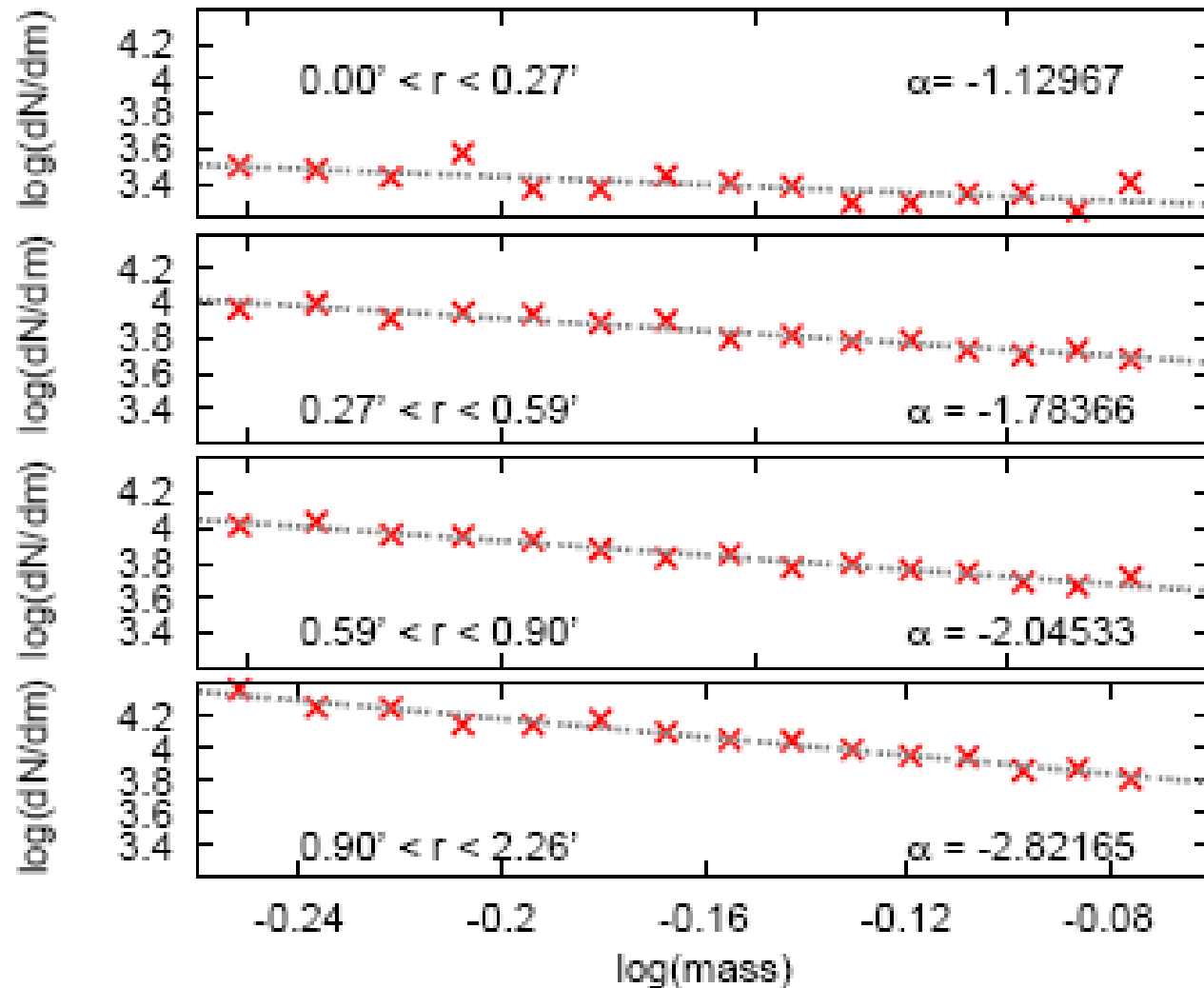
Model	$R_{phm}$ [pc]	$R_{phl}$ [pc]	$M_{r < R_t}^f$ [ $M_{\odot}$ ]	$\alpha_{tot}$	$\alpha_{in}$	$\alpha_{out}$	$\sigma_{los}$ [km/sec]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Canonical-S							
S0.50M60R10	16.8	13.3	32125	2.27	1.84	2.81	0.88
S0.80M60R8	17.1	12.3	32739	2.27	1.94	3.00	0.87
S0.95M60R8	21.0	16.5	32606	2.10	1.68	2.89	0.79
S0.95M60R10	28.2	20.8	31422	2.19	1.61	3.19	0.69
S0.95M55R9	26.5	20.8	27901	2.22	1.76	2.86	0.67
S0.95M57R10	27.8	19.1	29947	2.20	1.81	2.94	0.69
Observation		$18.4 \pm 1.1$	$29800 \pm 800$	$1.4 \pm 0.25$	0.88	1.81	$0.87 \pm 0.18$ ( $1.15 \pm 0.20^*$ )

# Models with Primordial Mass Segregation



Clusters with such a strong degree of primordial mass segregation are still not able to reproduce the observed flat mass function.

# Mass Function in different radial bins





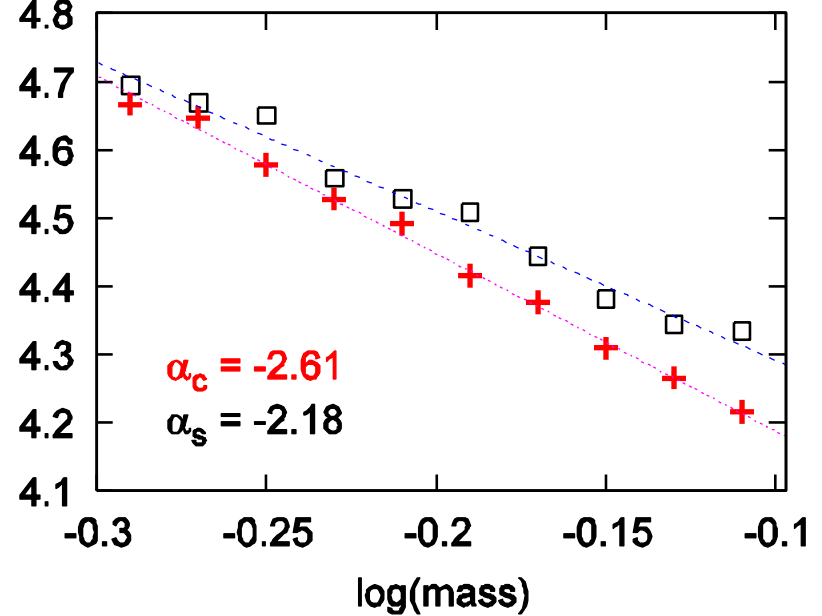
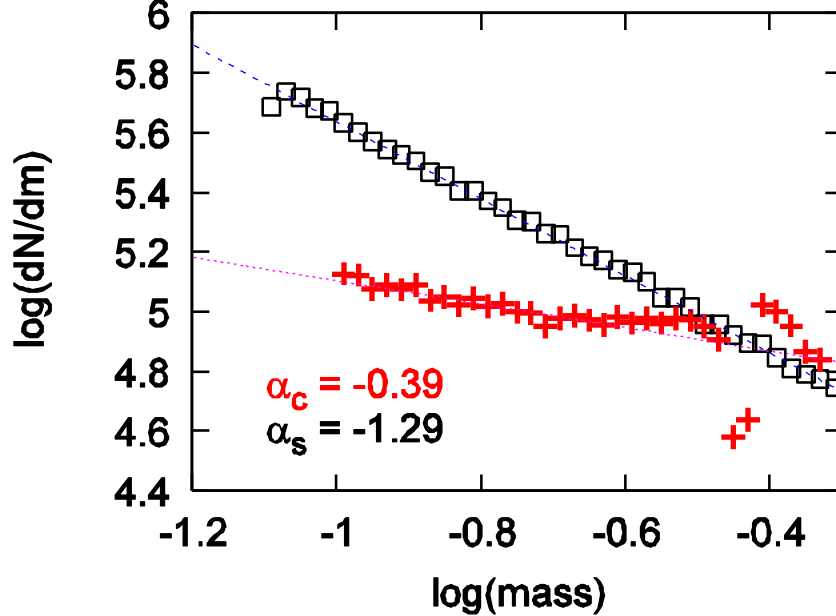
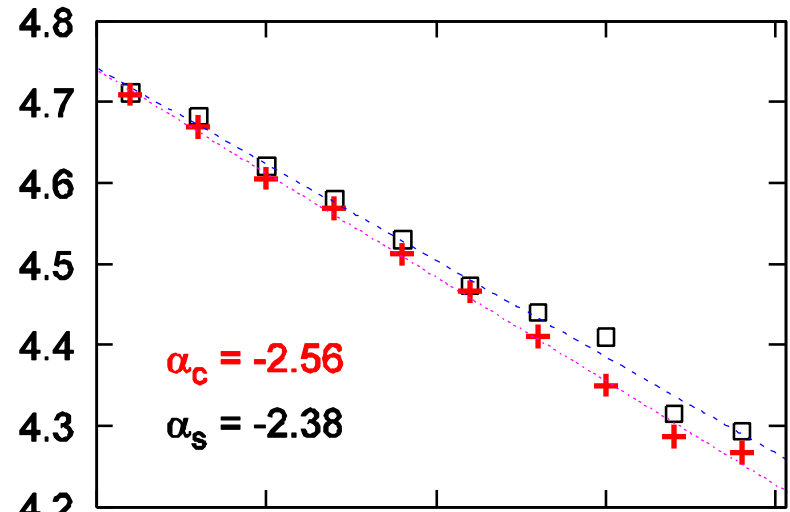
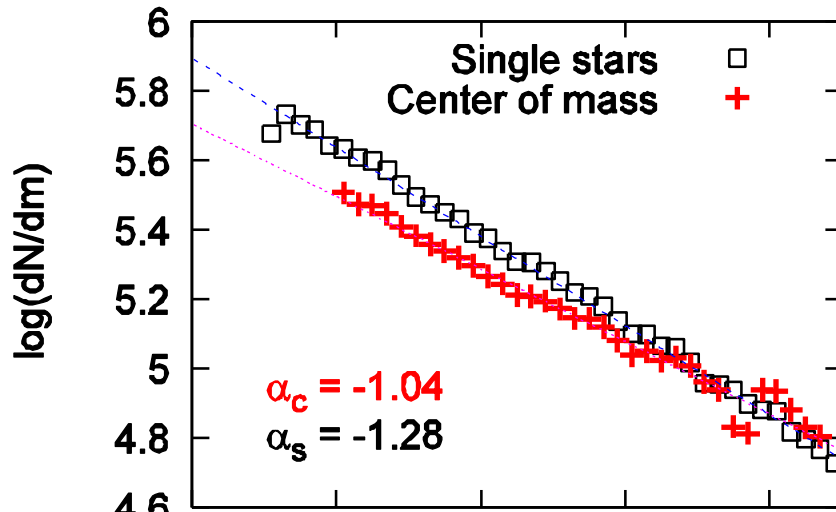
# The effect of unresolved binaries

- Binary stars, either primordial or dynamically formed during close encounters between single stars, can affect the observational parameters of a star cluster, such as velocity dispersion and mass function.
- If any of the stars in a sample are members of a binary system, their measured magnitude will be decreased and consequently yielded to the larger mass.

# The effect of unresolved binaries

$m < 0.5$

$0.5 < m < 0.8$



# 1 INTRODUCTION

The problem of the dynamical evolution of a globular cluster can be stated very simply. The only important force is the mutual attraction between the stars of the cluster; the other forces (like radiative pressure, electromagnetic forces, relativistic effects, etc.) are negligible. Therefore, the topic is the classical  $n$ -body problem: finding the motion of  $n$  points of given masses, mutually attracting themselves as the inverse square of their distance.

This exposition, whereas simple, relates to an extremely arduous mathematical problem. Despite a large number of studies, it has not been possible to find an explicit solution, which very likely does not exist. Hence, one can think of the numerical integration. This way, von Hoerner (1960) computed the evolution of artificial clusters comprising up to 16 stars, thanks to an electronic device. But high values of  $n$  are out of reach with such a method, as the computational time becomes rapidly extreme, even for a machine; the case of  $n = 16$  already corresponds to a system of 96 simultaneous differential equations.

In the globular clusters,  $n$  is of the order of magnitude of  $10^5-6$ . Such a high value naturally suggests to give up following the individual motions, and to use a statistical

It is possible  
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# Conclusion

The results are rather intriguing:

We suggest that the cluster must have been rather **LARGE** with an IMF depleted of **low-mass** stars at time=0. This may be interpreted to mean that it **was born mass segregated** and then went through a brief **violent gas-expulsion epoch** after being born which inflated the cluster and depleted its low-mass stellar content.

This is a highly **important constraint** on the very early processes that shaped GCs.

