Observable local non-Gaussianity in single field inflationary models

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Based on:
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Outline

- Inflation / Cosmological perturbation theory
- Non-Gaussainity and consistency relation
- Models with large local non-Gaussainity

Dynamics of Inflation

Friedmann equations

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{m_{P\ell}^2}\rho - \frac{K}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3m_{P\ell}^2}(\rho + 3P) \longrightarrow$$

Accelerating the universe needs extraordinary content

FRW metric:

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -dt^2 + a^2 \left[\frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] \end{aligned}$$

- Matter content of universe at inflationary phase is inflaton, a scalar field

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad , \quad P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

conditions for inflation:

- Slow-Roll conditions:

$$\epsilon = -\frac{\dot{H}}{H^2} \quad , \quad \eta = \frac{\dot{\epsilon}}{H\epsilon}$$

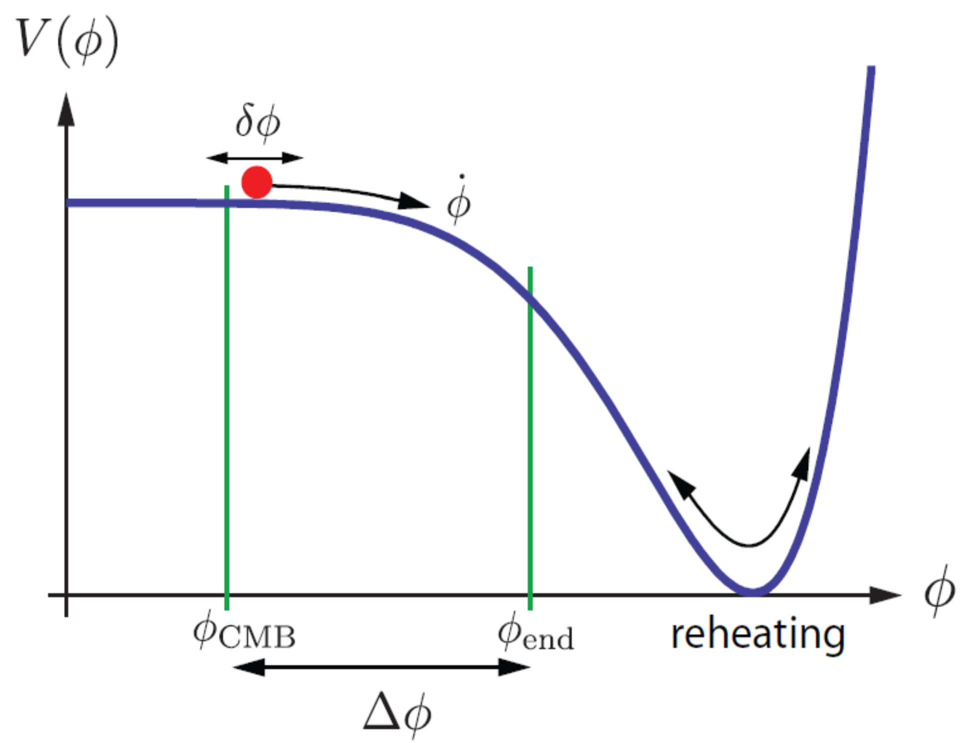
$$\epsilon \ll 1 \text{ and } |\eta| \ll 1$$

• In order to inflation occurs and lasts sufficiently long time.

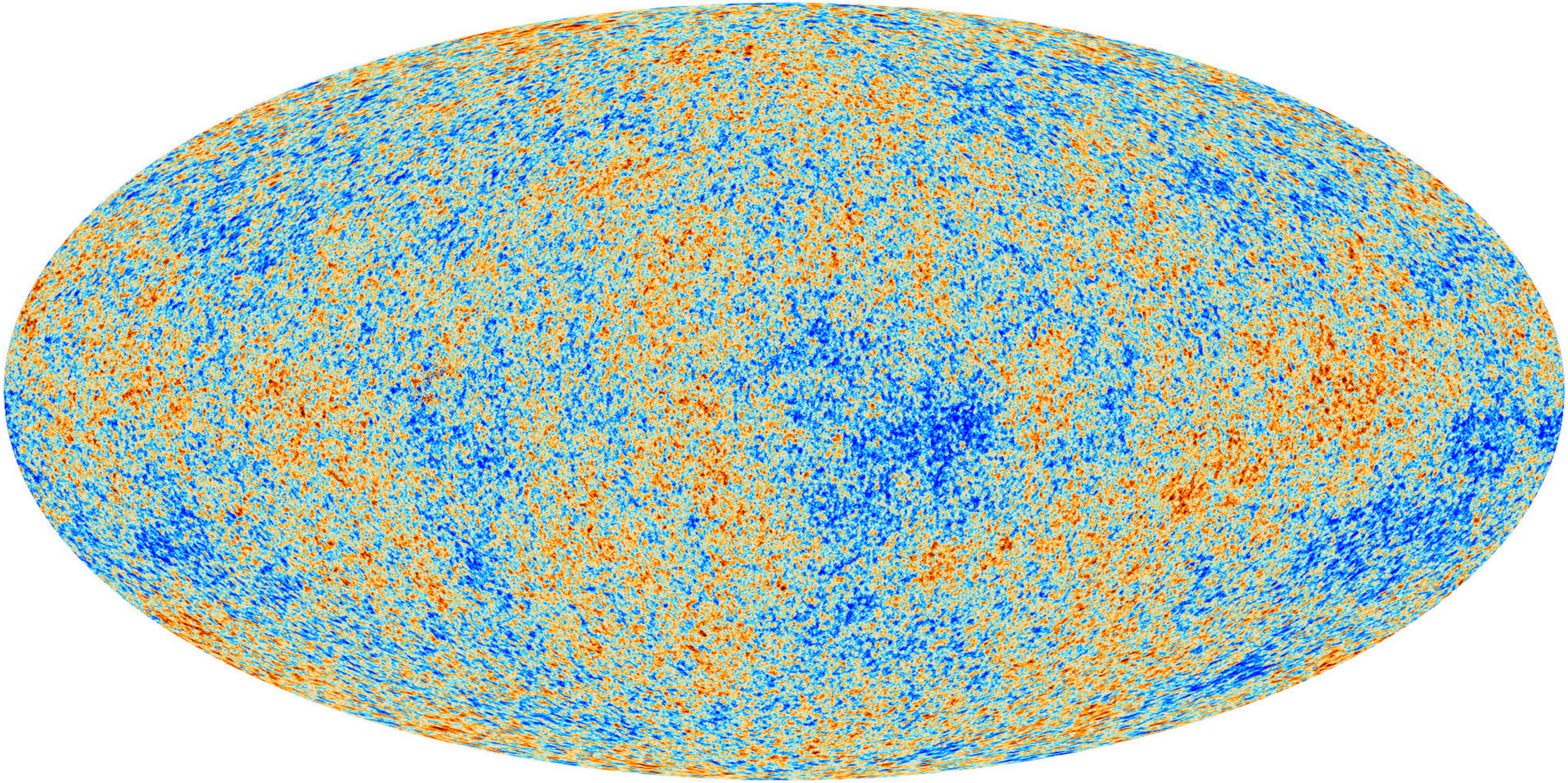
• Minimum number of efolds required:

$$\frac{a_f}{a_i} \equiv e^N$$

$$N > 60$$



Fluctuations on CMB temperature



How can we model the CMB data?

Perturbation theory in cosmology

- Perturbation of metric around FRW background:

$$\begin{aligned} ds^2 &= -(1+2A)dt^2 + 2a(\partial_i B - S_i)dx^i dt \\ &+a^2\left[(1-2\psi)\delta_{ij} + 2\partial_{ij}E + 2\partial_{(i}F_{j)} + h_{ij}\right]dx^i dx^j \end{aligned}$$

+ Perturbation of matter:

$$\phi = \phi_0(t) + \delta\phi(t,x)$$

Perturbed Einstein Equation

Perturbations are not gauge invariant, in general.

- Gauge invariant quantity:

Curvature perturbation: $\mathcal{R} \equiv \psi + \frac{H}{\dot{\phi}}\delta\phi$, Bardeen Potential: $\Phi = \frac{3}{5}\mathcal{R}$

Power spectrum and CMB

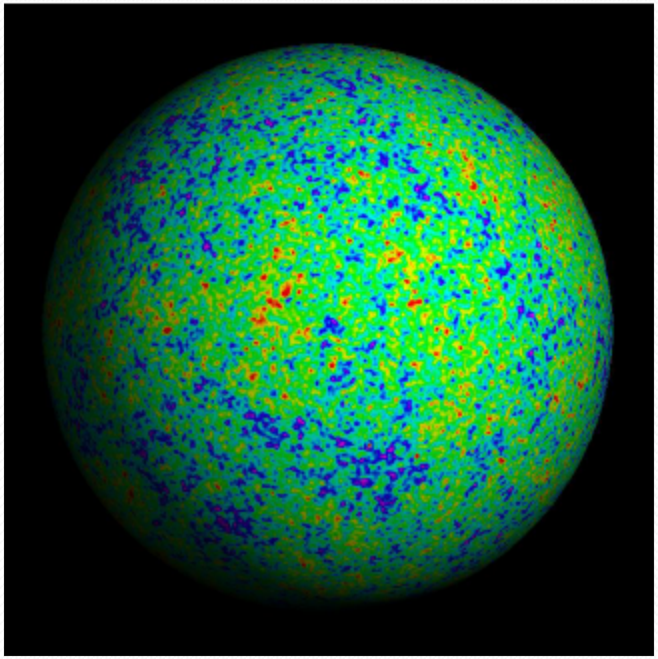
$$\frac{\delta T}{T} \to \Phi(t) \to \Phi_0 \quad , \mathcal{R}_0$$

Power spectrum of curvature perturbation:

$$< \mathcal{R}_{k_1} \mathcal{R}_{k_2} > \longrightarrow \mathcal{P}_{\mathcal{R}} \equiv \frac{4\pi k^3}{(2\pi)^3} |\mathcal{R}^2|$$

COBE normalization: $\mathcal{P}_R \simeq 10^{-10}$

Spectral index: $n_s - 1 = \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k} \simeq -0.04$



For simple slow-roll models

$$1 - n_s = 2\epsilon + \eta$$

Non-Gaussianity

- The three point function of perturbations

$$\langle \Phi(\mathbf{k}_1)\,\Phi(\mathbf{k}_2)\,\Phi(\mathbf{k}_3)\rangle \equiv (2\pi)^3 \delta_D(\mathbf{k}_{123})\; B_\Phi(k_1, k_2, k_3)$$

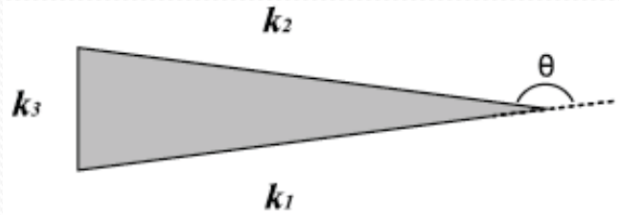
$$\Phi = \frac{3}{5}\mathcal{R}$$

- Local non-Gaussianity

$$\Phi(x) = \Phi_G + f_{NL}\,\Phi(x)^2$$

Planck: $f_{\mathrm{NL}}^{\mathrm{local}} = 2.7 \pm 5.8$

Consistency relation

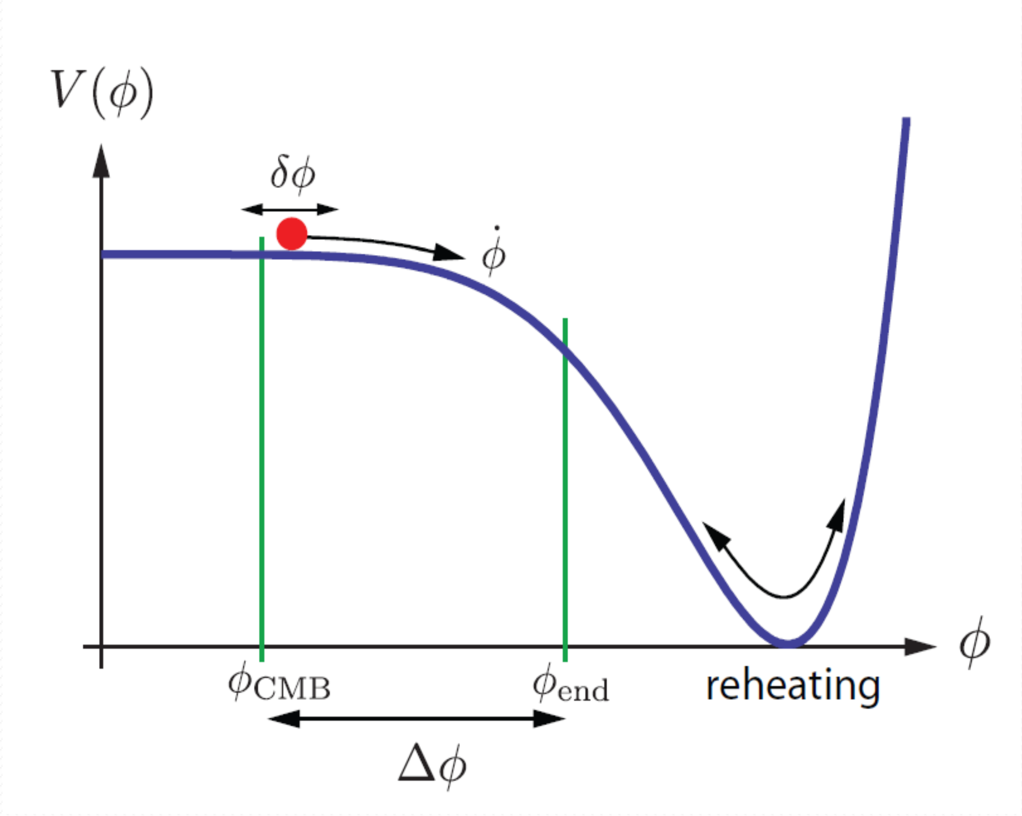


Maldacena '03

$$B_{\Phi}^{\text{single-field}}(k_1 \to 0, k_2, k_3 = k_2) \to \frac{5}{3}(1-n_s)P_{\Phi}(k_1)P_{\Phi}(k_2)$$

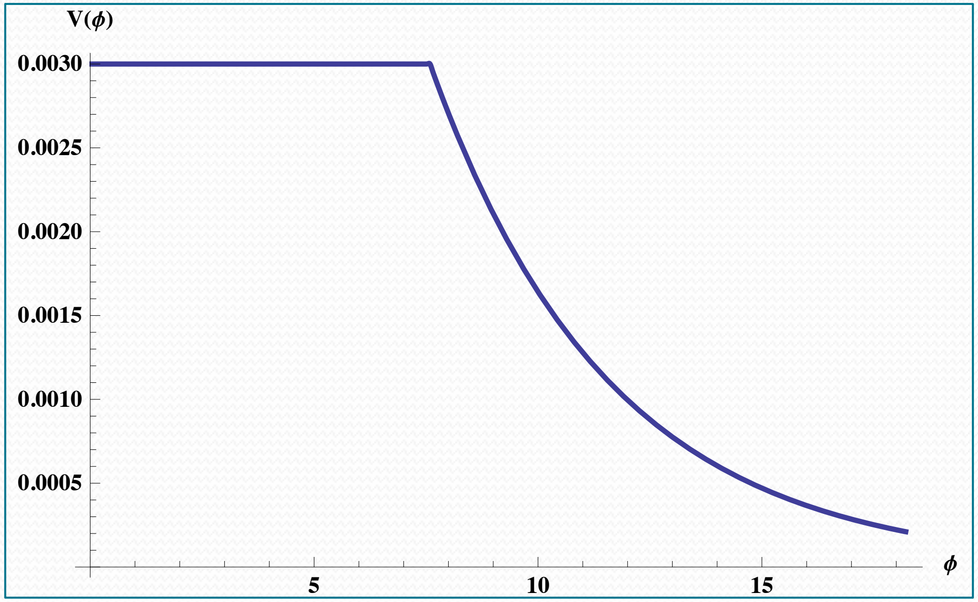
- Any detection of local type/squeezed limit non-Gaussianity can rule out most single field models of inflation
- Assumptions:
  - BD-vacuum
  - Attractor phase/ conservation of curvature perturbation at large scales.

Attractor / non-attractor inflation



The model

- Consider a simple single field model with constant potential as the first phase of inflation



- We need a second stage of inflation to have a graceful exit as well as stable perturbations.

Back-ground equations of motion and slow-roll parameters:

$$\ddot{\phi} + 3H\dot{\phi} = 0\,, \quad 3{M_{\rm Pl}}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V_0 \simeq V_0$$

$$\epsilon \propto a^{-6}\,, \quad \eta \simeq -6$$

$$\epsilon = -\frac{\dot{H}}{H^2}\,, \quad \eta = \frac{\dot{\epsilon}}{H\epsilon}$$

- It is not a slow-roll inflation

Results

- We have a scale invariant power spectrum!

$$\mathcal{P}_{\mathcal{R}} \equiv \frac{k^3}{2\pi^2} P_k \simeq \frac{H^2}{8\pi^2 {M_{\rm Pl}}^2 \epsilon_e} = \frac{H^2}{8\pi^2 {M_{\rm Pl}}^2 \epsilon_k} e^{6N_k}$$

$$n_s - 1 = \eta + 6 \simeq 0$$

$$f_N L = \frac{5}{2}$$

The model

$$\mathcal{L} = \frac{X^\alpha}{M^{4\alpha-4}} - V(\phi)\,,\, V(\phi) = V_0 + v\left(\frac{\phi}{M_P}\right)^\beta$$

$$X = -\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

- Observational predictions:

$$n_s - 1 = \eta + 6 \simeq 0$$

$$f_{NL}^{\text{loc}} \simeq \frac{5}{4c_s^2}$$

$$c_s^2 \simeq 1/(2\alpha - 1)$$

Conclusion

- Non-Gaussianity can rule out / constrain inflationary models

- Most single field inflationary models predict small local non-Gaussianity but there are some counter examples.

Thank you