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STABILIZATION OF MODULUS IN RANDALL–SUNDRUM MODEL I BY BULK SCALAR FIELDS

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We consider the case of self-interacting scalar field for the stabilization of modulus in the Randall–Sundrum model I. This scalar field is non-minimally coupled to the Ricci curvature scalar. We find the compactification radius and we discuss the implications of our results for the cases of massive and massless scalars.

Keywords: Randall–Sundrum model; modulus stabilization.

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1. Introduction

In order to address the large hierarchy between the weak scale and the Planck scale, many attractive theories such as supersymmetry and higher-dimensional theories have been proposed. One of these attempts Randall–Sundrum I (Ref. 1) explains this hierarchy in terms of a small extra dimension. This proposal involves a "Planck brane" and a "TeV brane". And the space between the branes is a slice of anti-de sitter space. By solving the five-dimensional Einstein equations one obtains the metric for this space as

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}\, dx^\mu\, dx^\nu - r^2\, d\varphi^2\,, \tag{1}$$

where

$$\sigma = kr_c|\varphi| \qquad \text{and} \qquad \eta_{\mu\nu} = \text{diag}[-1, 1, 1, 1] \tag{2}$$

and $-\pi < \varphi < \pi$ is the extra-dimensional coordinate and r_c is the compactification radius, and k is a parameter which is assumed to be of order five-dimensional Planck scale, M.

The problem of stability of this extra dimension has been addressed by Goldberger and Wise.[2] Their solution involved a massive bulk scalar field with the usual kinetic term in the bulk and quartic interactions localized on the two branes.

Since then many studies have appeared on this subject.[3–10] The studies of Refs. 3 and 4 considers models for the stabilization of the modulus containing a bulk scalar field interacting with the spacetime curvature R.

Gunion and Grzadkowski[3] considered a class of generalizations of the Randall–Sundrum model containing a bulk scalar field Φ, interacting with the curvature R through the general coupling $Rf(\Phi)$. They show that by choosing a nontrivial background for the bulk scalar field it is possible to neglect the effect of the metric back reaction. And they obtain the general form of the scalar potential $V(\Phi)$.

Granda and Oliveros[4] considered the case of a massless scalar field but with non-minimal interaction with the curvature R. In this work by a suitable choice of the parameter one can neglect the effect of the back-reaction of the scalar field on the background geometry. Their work essentially corresponds to the work of Ref. 3, but with $V(\Phi) = 0$. Moreover, the discussion of Ref. 4 is restricted when the coupling of the bulk scalar field and the curvature is small in comparison with unity.

We propose a model where a massive bulk scalar field is non-minimally coupled to R. Hence, in our model $V(\Phi) = \frac{1}{2}m^2\Phi^2$.

The motivation for the present study is to fill some gaps in these studies.

(1) Our model allows positive effective mass squared for the bulk scalar field in the action.
(2) To present accurate expression for the effective potential.
(3) To discuss the parameter space of the model.

The plan of this paper is as follows: In Sec. 2, we describe our model and we obtain the effective potential and we express the extremization condition for this effective potential. In Sec. 3, we obtain the value of the stabilized modulus for two different cases where the squared effective mass of the bulk scalar field is either positive or negative. And finally in Sec. 4, we present our conclusions.

2. Effective Potential

The action for the bulk scalar field is

$$S_b = \int dx^4 \int_{-\pi}^{\pi} d\phi\sqrt{G}(G^{MN}\partial_M\Phi\partial_N\Phi - m^2\Phi^2 - \xi R\Phi^2)\,, \tag{3}$$

where $G = \det[G_{MN}]$, R is the bulk curvature for the metric (1) and it is given by

$$R = \frac{20\sigma' - 8\sigma''}{r_c^2}\,, \tag{4}$$

where $\sigma' = \partial_\phi\sigma$ and $\sigma'' = 2kr_c[\delta(\phi) - \delta(\phi - \pi)]$.

The interaction terms on the hidden and visible branes (at $\phi = 0$ and $\phi = \pi$, respectively) are given by the actions

$$S_h = -\int dx^4 \sqrt{-g_0}\lambda_0(\Phi^2 - v_0^2)^2 \tag{5}$$

and

$$S_v = -\int dx^4 \sqrt{-g_L}\lambda_L(\Phi^2 - v_L^2)^2 \,. \tag{6}$$

The ϕ dependent vacuum expectation value $\Phi(\phi)$ is determined from the classical equation of motion,

$$\begin{aligned}\partial_\phi(e^{-4\sigma}\partial_\phi\Phi) = e^{-4\sigma}[&m^2 r_c^2 + 4\lambda_L r_c \Phi(\Phi^2 - v_L^2)\delta(\phi-\pi)\\ &+ 4\lambda_0 r_c\Phi(\Phi^2 - v_0^2)\delta(\phi) + \xi(20\sigma' - 8\sigma'')\Phi]\,.\end{aligned} \tag{7}$$

Away from the boundaries (at $\phi = 0, \pi$) from this equation of motion one can obtain the solution for the scalar field as

$$\Phi = Ae^{(\nu+2)\sigma} + Be^{(-\nu+2)\sigma}\,, \tag{8}$$

where $\nu = \sqrt{4 + \frac{m^2}{k^2} + 20\xi}$. By integration over the extra dimension we obtain the effective potential which is

$$\begin{aligned}V_{\text{eff}} = k(\nu + 2 + 8\xi)A^2(e^{2\nu k r_c \pi} - 1) + k(\nu - 2 - 8\xi)B^2(1 - e^{-2\nu k r_c\pi})\\ + \lambda_0(\Phi^2(0) - v_0^2)^2 + \lambda_L e^{-4kr_c\pi}(\Phi^2(\pi) - v_L^2)^2\,.\end{aligned} \tag{9}$$

The stabilized size of the extra dimension is the minimum of the effective potential. The authors of Ref. 2, practically consider the case of infinite coupling, namely $\lambda_0 \to \infty$ and $\lambda_L \to \infty$. In this situation, we have

$$A + B = v_0\,, \qquad \text{and} \qquad Ae^{k\pi r_c(\nu+2)} + Be^{k\pi r_c(-\nu+2)} = v_L \tag{10}$$

with solution[2]

$$A = v_L e^{-(\nu+2)\pi k r_c} - v_0 e^{-2\nu\pi k r_c} \tag{11}$$

and

$$B = v_0(1 + e^{-2\nu\pi k r_c}) - v_L e^{-(\nu+2)\pi k r_c}\,. \tag{12}$$

Now, by assuming that $\frac{m}{k} \ll 1$, $|\xi| \ll 1$ and $\epsilon = \frac{m^2}{4k^2}$ we can express the effective potential as

$$V_{\text{eff}} = k[a_1 e^{\alpha_1 \pi k r_c} + b_1 e^{\beta_1 \pi k r_c} + c_1 e^{\gamma_1 \pi k r_c} + d_1]\,, \tag{13}$$

where

$$a_1 = (4 + 13\xi + \epsilon)v_L^2 \,, \qquad b_1 = -4(2 + 5\xi + \epsilon)v_0 v_L \,, \qquad c_1 = (4 + 10\xi + 2\epsilon)v_0^2 \tag{14}$$

and $d_1 = (\epsilon - 3\xi)v_0^2$. Moreover,

$$\alpha_1 = -4 \,, \qquad \beta_1 = -(4 + 5\xi + \epsilon) \,, \qquad \gamma_1 = -(4 + 10\xi + 2\epsilon) \,. \tag{15}$$

The effective mass squared of this scalar in the bulk is

$$m_{\rm eff}^2 = 4k^2(\epsilon + 5\xi) \,. \tag{16}$$

The first derivative of the effective potential is

$$\frac{dV_{\rm eff}}{dr_c} = \pi k^2 e^{\gamma_1 \pi k r_c}(a_1\alpha_1\Psi^2 + b_1\beta_1\Psi + c_1\gamma_1) \,, \tag{17}$$

where

$$\Psi = e^{(5\xi+\epsilon)\pi k r_c} \,. \tag{18}$$

Using the condition $\frac{dV_{\rm eff}}{dr_c} = 0$, we obtain the second derivative of the potential, namely:

$$\frac{dV_{\rm eff}^2}{dr_c^2} = \frac{-\pi}{4} k e^{\gamma_1 \pi k r_c} m_{\rm eff}^2 b_1\beta_1 \left[\Psi + \frac{2c_1\gamma_1}{b_1\beta_1}\right]. \tag{19}$$

The sign of R.H.S. of the above equation will determine the stability of the modulus.

3. Effective Mass of the Scalar and Modulus Stability

In order to investigate the effect of $\xi R\Phi^2$ of the action we investigate three different cases:

3.1. *The case of* $m_{\rm eff}^2 > 0$

In this case, the stabilized value of the modulus r_c is given by:

$$kr_c = \frac{1}{\pi(5\xi + \epsilon)} \ln\left[\frac{(4 + 15\xi + 3\epsilon - \sqrt{-12\xi + 4\epsilon - 35\xi^2 - 6\epsilon\xi + 5\epsilon^2})v_0}{(4 + 13\xi + \epsilon)v_L}\right], \tag{20}$$

where we have only kept the linear terms of parameters ξ and ϵ.

For $v_0 = 105$, $v_L = 1$, $k = 4$, $\epsilon = 0.12$ and $\xi = 0.001$, we obtain $kr_c = 11.5$.

And for the configuration $v_0 = 1.44$, $v_L = 1$, $k = 2$, $\epsilon = 0.1$ and $\xi = -0.019$, the value of $kr_c = 11.8$.

Since, we have chosen $\xi \ll 1$, $\epsilon \ll 1$ and as long as $v_0, v_L \ll M^{\frac{3}{2}}$ we can neglect the back reaction of the scalar field on the background geometry.[2,4]

3.2. *The case of* $m^2_{\text{eff}} = 0$

From Eq. (16) we see that in this case $\epsilon = -5\xi$, hence $\alpha_1 = \beta_1 = \gamma_1 = -4$. The effective potential is

$$V_{\text{eff}} = e^{-4\pi k r_c}[(4+8\xi)v_L^2 - 8v_0 v_L + 4v_0^2] - 8\xi v_0^2\,. \tag{21}$$

This effective potential does not have a minimum. Hence, when the effective mass squared of the bulk scalar field is zero we do not have a stable modulus.

3.3. *The case of* $m^2_{\text{eff}} < 0$

The value of r_c for the minimum of the effective potential in this case is

$$kr_c = \frac{1}{\pi(5\xi+\epsilon)} \ln\left[\frac{(4+15\xi+3\epsilon+\sqrt{-12\xi+4\epsilon-35\xi^2-6\epsilon\xi+5\epsilon^2})v_0}{(4+13\xi+\epsilon)v_L}\right] \tag{22}$$

and for the configuration $v_0 = 0.02$, $v_L = 1$, $k = 4$, $\epsilon = 0.1$ and $\xi = -0.04$, the value of $kr_c = 11.6$.

A more interesting situation is the case of a massless field non-minimally coupled to curvature of the spacetime. For this case $\epsilon = 0$. The minimum of the effective potential in this case is

$$kr_c = \frac{1}{5\pi\xi} \ln\left[\frac{(4+15\xi+\sqrt{-12\xi-35\xi^2})v_0}{(4+13\xi)v_L}\right]. \tag{23}$$

The expression inside the square root is positive if $\xi < 0$. Then from the action we see that we have a negative mass squared field in the five-dimensional AdS space. When $v_0 = 0.135$, $v_L = 1$, $k = 4$ and $\xi = -0.01$, the value of $kr_c = 12.2$.

In order to explain the physical basis for these cases we consider the model of Ref. 2. An analysis of this model in a case where the quartic couplings are finite[11] shows that in the large kr_c limit $\epsilon > 0$. Hence the squared mass of the bulk scalar is positive. In other word in our model when $\xi = 0$ then the effective squared mass is strictly positive.

In this work, we have shown that for a massless scalar field non-minimally coupled to gravity the effective mass squared of this scalar is negative. In other word in our model when $\epsilon = 0$ then the effective mass squared is strictly negative.

But for the more general case of a massive scalar field non-minimally coupled to gravity and in the limit of infinite quartic coupling the effective mass squared could be either positive or negative.

4. Conclusions

We proposed a simple model of a massive bulk scalar non-minimally coupled to gravity. And we obtained the effective potential and the value of stabilized modulus. We also show that if one utilizes a massless bulk scalar then the effective mass squared will be negative. The appearance of negative effective mass squared in the five-dimensional lagrangian will not induce an instability as long as tachyonic mode do not appear in the four-dimensional theory.[7,8]

There are several issues that deserves further investigations.

We have not discussed the mass of the radion in this framework. Our analysis was in the limit of infinitely large quartic couplings. Our preliminary numerical investigation shows that our results hold in the limit of finite values of quartic couplings. We plan to address these and other related issues in future work.

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