The challenging application of the IBP method in the two-loop calculation of the single top quark production

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in collaboration with

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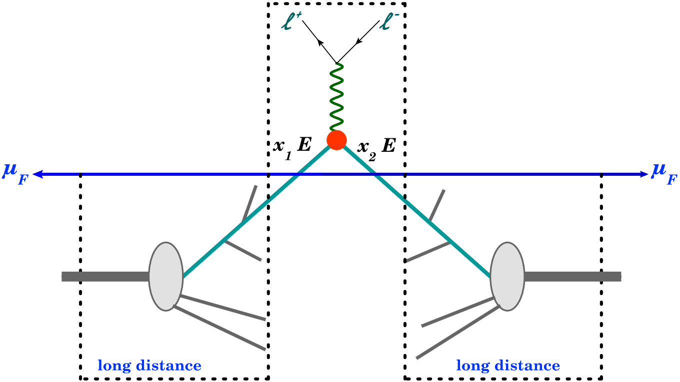
Contents

Master formula for hadron collisions



$$\sum_{a,b} \int_0^1 dx_1 dx_2 \underbrace{f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2)}_{\text{Parton density functions}} \quad \underbrace{\hat{\sigma}_{a,b}(\mu_r^2,\mu_F^2)}_{\text{Parton-level cross section}}$$

Perturbative expansion

$\hat{\sigma}_{a,b}(\mu_r^2, \mu_F^2)$ *Parton-level cross section*

- *The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter*

$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \ldots \right)$$

LO predictions | NLO corrections | NNLO corrections | NNNLO corrections

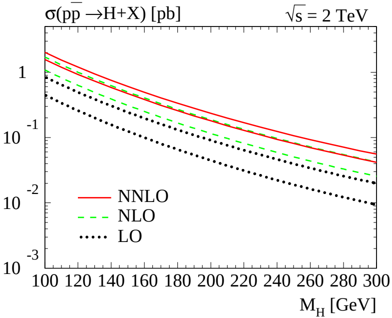
Higher order calculation

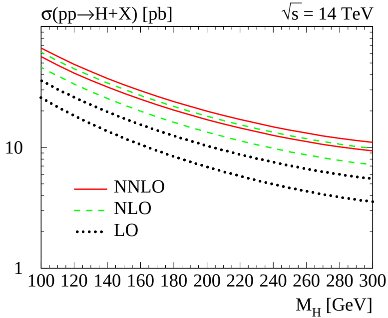
Higher order calculation

- *More precision in calculated results*

More precision

Ex.: Total cross section for Higgs production in gluon fusion





[R. Harlander, W. Kilgore Nov. '02]

- *Perturbative convergence* LO $\rightarrow$ NLO$(\approx 70\%)$ *and* NLO $\rightarrow$ NNLO$(\approx 30\%)$

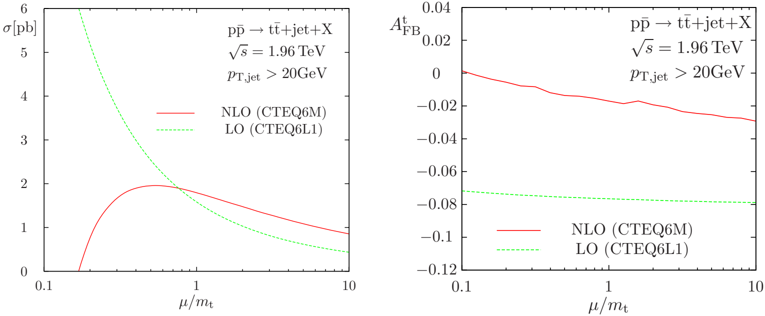
Higher order calculation

Higher order calculation

- *More precision in calculated results*
- *New effects*

New effects

Ex.: forward–backward charge asymmetry of the top quark



*[**S. Dittmaier, P. Uwer , S. Weinzierl Apr. ’08**]*

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Exact NLO or NNLO calculations of σ_{hard} *needed because of:*

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Accurate and reliable predictions of parton–level observables.

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Backgrounds for New Physics Searches

NNLO

When do we need NNLO?

- *When NLO corrections are large*

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- *When truly high precision is needed*

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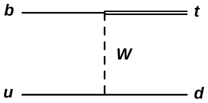
For instance ***single top quark*** *production :*

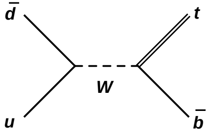
Single top quark production

The three main hadronic production modes for single top quark in the Standard Model:

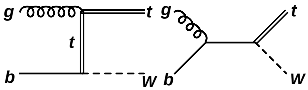
- *t-channel*



- *s-channel*



- *associated t+W*



NNLO single top quark production

The theoretical status of the single top quark production:

Process	$\sqrt{S}$	$\sigma_{\mathrm{LO}}(\mathrm{pb})$	$\sigma_{\mathrm{NLO}}(\mathrm{pb})$
t–channel	2.0 *TeV* $p\bar{p}$	1.068	1.062
	14.0 *TeV* pp	152.7	155.9

*[**B.Harris, E. Laenen, L.Phaf, Z. Sullivan, S. Weinzierl ’02**]*

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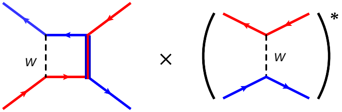
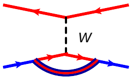


$\propto \; tr[T_a]\; tr[T_a] = 0$

Only vertex corrections contribute:



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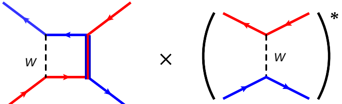

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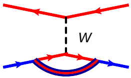

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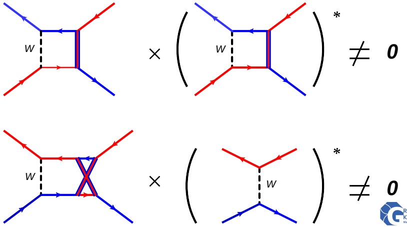

Phenomenology of the single top quark production

Single top quark production

1. *to study the nature of the weak interaction*
2. *is a source of polarized top quarks*

   (Polarization accessible through angular distributions of decay products)
3. *to measure directlly the Cabibbo-Kobayashi-Maskawa (CKM) matrix element* V_{tb}

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

without assumption of unitarity and three families

$\Rightarrow |V_{tb}| = 0.92 \pm 0.10$

*[**C. Schwanenberger, Moriond QCD and High Energy Interactions, 13 March 2013**]*

4. *to access to the b-quark PDFs*

   *[**Alwall, Frederix, Gerard, Giammanco, Herquet, Kalinin, Kou, Lemaitre, Maltoni '07**]*

Tensor integral

In the NNLO–corrections occur tensor integrals

$$\mathcal{J}(d, a_1, \cdots, a_n)[1, k_1^{\mu}, k_2^{\nu}, \cdots] = \int d^d k_1 \int d^d k_2 \frac{\prod_{ij} k_1^{\mu_i} k_2^{\nu_j}}{P_1^{a_1} \cdots P_n^{a_n}}$$

Possibilities to reduce tensor integrals to scalar integrals:

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- *By projection method*

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Tensor reduction $\Rightarrow$ *various scalar integrals with the same structure of the integrand however with different powers of propagators*

Integration by parts

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[**Chetyrkin, Tkachov '81**]

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[**Chetyrkin, Tkachov '81**]

Reduction techniques:

- *Laporta: efficient algorithm to solve linear system of IBP–Identities*

AIR	[**Anastasiou, Lazopoulos '04**]
FIRE	[**Smirnov '08**]
Crusher	[**Marquard, Seidel (to be published)**]
REDUZE 1&2	[**Studerus '09; Manteuffel, Studerus '12**]

Tarasov's method

Tensor reduction leads to a very large number of scalar integrals which are shifted in dimension and have other powers of propagators

$$\mathcal{J}(d, a_1, \cdots, a_n)_{[k_1^\mu k_2^\nu, \cdots]} \rightarrow g^{\mu\nu} \sum_i \mathcal{J}(d + x_i, a_1^i, \cdots, a_n^i)_{[1]}$$

Example for two loop corrections to Axial Vector Form Factors

$$\mathcal{J}(d, 1, 1, 1, 1, 1, 1)_{[1, k_1^{\mu_1} k_1^{\mu_2} k_2^{\nu_1} k_2^{\nu_2}]} \rightarrow \mathcal{J}(2 + d, 2, 1, 1, 1, 1, 2) +$$

$$\cdots + \mathcal{J}(4 + d, 1, 1, 1, 2, 3, 1) + \cdots + \mathcal{J}(8 + d, 3, 3, 3, 2, 1, 2)$$

Shift in the dimension

Shift in the dimension

An arbitrary scalar Feynman integral:

$$\mathcal{J}^{(d)}(\{s_i\},\{m_s^2\}) \propto \prod_{j=1}^{N} c_j \int_0^\infty \cdots \int_0^\infty \frac{d\alpha_j \, \alpha_j^{a_j-1}}{[D(\alpha)]^{\frac{d}{2}}} e^{i\left[\frac{Q(\{s_i\},\alpha)}{D(\alpha)} - \sum_{l=1}^{N} \alpha_l (m_l^2 - i\epsilon)\right]}$$

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$D\left(\frac{\partial}{\partial m_j^2}\right)$ *(polynomial differential operator) obtained from* $D(\alpha)$ *by substituting* $\alpha_i \to \partial_j \equiv \partial/\partial m_j^2$. *The application of* $D(\partial_j)$ *to the scalar integral:*

$$\mathcal{J}^{(d-2)}(\{s_i\},\{m_s^2\}) \propto D(\partial_j)\ \mathcal{J}^{(d)}(\{s_i\},\{m_s^2\}),$$

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$$\mathcal{J}^{(d-2)}(\{s_i\},\{m_s^2\}) \propto D(\partial_j)\ \mathcal{J}^{(d)}(\{s_i\},\{m_s^2\}),$$

apply this to master integrals

$$\mathcal{J}_{master}(d-2, a_1, \cdots, a_n) = \sum_i c_i \mathcal{J}(d, a_1^i, \cdots, a_n^i),$$

Tarasov's method

all scalar integrals in rhs. of that equation have to be replaced by master integrals.
i.e.

$$\mathcal{I}_{master}(d-2, a_1, \cdots, a_n) = \sum_j D_j \mathcal{I}_{master}(d, a_1^j, \cdots, a_n^j),$$

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$$\mathcal{J}_{master}(d-2, a_1, \cdots, a_n) = \sum_j D_j \mathcal{J}_{master}(d, a_1^j, \cdots, a_n^j),$$

we have for all master integrals:

$$\begin{pmatrix} \mathcal{J}_1^{d-2} \\ \vdots \\ \mathcal{J}_l^{d-2} \end{pmatrix}_{master} = D_{ll} \cdot \begin{pmatrix} \mathcal{J}_1^{d} \\ \vdots \\ \mathcal{J}_l^{d} \end{pmatrix}_{master}$$

where l is the number of master integrals.

Projection method

By this method we get scalar products between loop momenta and external momenta and no shift in the dimension of the integrals

$$\mathcal{I}(d, a_1, \cdots, a_n)_{[1, k_1^{\mu}, k_2^{\mu}, \cdots]} \rightarrow g^{\mu\nu} \sum_{ij} \mathcal{I}(d, a_1, \cdots, a_n)_{[1]} k_i p_j$$

Example for two-loop corrections to Axial Vector Form Factors

$$\mathcal{I}(d, 1, 1, 1, 1, 1, 1)_{[1, k_1^{\mu_1} k_1^{\mu_2} k_2^{\nu_1} k_2^{\nu_2}]} \rightarrow \mathcal{I}(d, -2, 1, 1, 1, 1, 1) + \cdots$$

$$+ \mathcal{I}(d, -1, 0, 1, 1, 1, 1) + \cdots + \mathcal{I}(d, 0, 1, 1, 1, -2, -2)$$

Projection method

The general tensor structure for the amplitude $\mathcal{A}$:

$$\mathcal{A} = \sum_{i=1}^{n} B_i(t,u,s)\mathcal{S}_i,$$

where t, u *and* s *are the Mandelstam variables and* $\mathcal{S}_i$ *are the Dirac structures.*

Projectors for the tensor coefficients:

$$\mathcal{S}_j^\dagger \mathcal{A} = \sum_{i=1}^{n} B_i(t,u,s)\underbrace{\left(\mathcal{S}_j^\dagger \mathcal{S}_i\right)}_{\mathcal{M}_{ji}} \Rightarrow B_i(t,u,s) = \sum_i \mathcal{M}_{ij}^{-1}\left(\mathcal{S}_j^\dagger \mathcal{A}\right)$$

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essential for this method : *to be able to calculate the inverse matrix* $\mathcal{M}_{ij}^{-1}$

Tarasov's method vs. Projection method

Tarasov's method

- *Positive powers for propagators (the sum of the powers of all propagators is large)*
- *Calculate the inverse matrix in order to shift back the dimension*

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Projection method

- *Negative powers for propagators*
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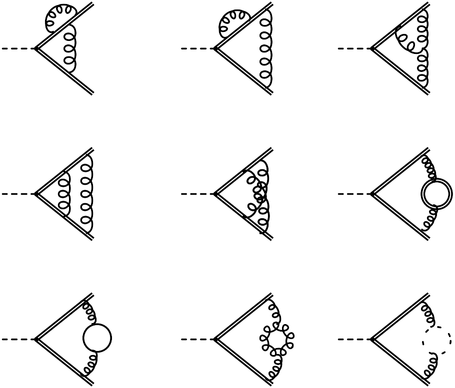
Two-loop corrections to heavy quark form factors

We implemented both methods to calculate the two loop corrections to Heavy Quark Vector and axial Vector Form Factors:



[W. Bernreuther,R. Bonciani, T. Gehrmann, R. Heinesch, T. Leineweber, P. Mastrolia, E. Remiddi '04]

[J. Gluza, A. Mitov, S. Moch, T. Riemann '09]

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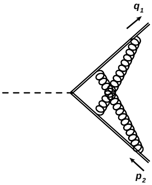

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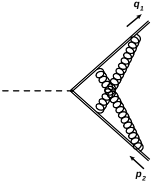

There are 6 Dirac structures

(Heavy Quark Vector and axial Vector

Form Factors):

$$\mathcal{S}_1 = \bar{u}(q_1)(1+\gamma_5)u(p_2)p_2^{\mu}$$
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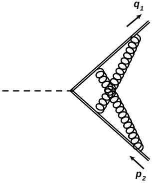

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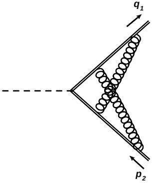
	Projection	*Tarasov*
number of integrals	*564*	*671*
max sum of powers of propagators	*6*	*14*
max sum of negative powers of propagators	*4*	*0*
reduction time	*7500 s*	*433260 s*



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Now one may come to the conclusion:

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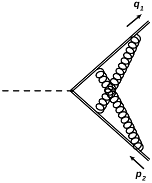

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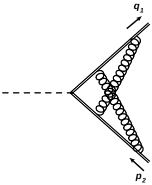
reduction time	*7500 s*	*433260 s*

Now one may come to the conclusion:
⇒*The projection method is an alternative method for the multi-loop calculation!*

Two-loop corrections to heavy quark form factors



There are 6 Dirac structures

(Heavy Quark Vector and axial Vector Form Factors):

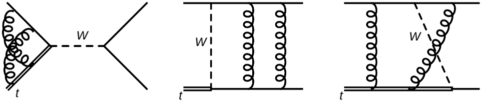
$$\mathcal{S}_1 = \bar{u}(q_1)(1+\gamma_5)u(p_2)p_2^{\mu}$$
$$\mathcal{S}_2 = \bar{u}(q_1)(1-\gamma_5)u(p_2)p_2^{\mu}$$
$$\mathcal{S}_3 = \bar{u}(q_1)(1+\gamma_5)u(p_2)q_1^{\mu}$$
$$\mathcal{S}_4 = \bar{u}(q_1)(1-\gamma_5)u(p_2)q_1^{\mu}$$
$$\mathcal{S}_5 = \bar{u}(q_1)(1+\gamma_5)\gamma_{\mu}u(p_2)$$
$$\mathcal{S}_6 = \bar{u}(q_1)(1-\gamma_5)\gamma_{\mu}u(p_2)$$

	Projection	*Tarasov*
number of integrals	*564*	*671*
max sum of powers of propagators	*6*	*14*
max sum of negative powers of propagators	*4*	*0*
reduction time	*7500 s*	*433260 s*

Now one may come to the conclusion:
⇒The projection method is an alternative method for the multi-loop calculation!

but $\cdots$

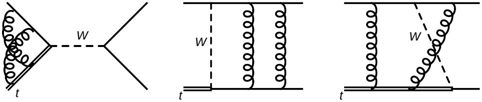
There are three topological families:



Vertex corrections **Planar double boxes** **Non Planar double boxes**

Two-loop corrections to single top quark production

There are three topological families:



Vertex corrections ***Planar double boxes*** ***Non Planar double boxes***

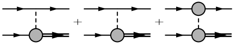
and 11 Dirac structures:

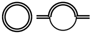
$$
\begin{aligned}
S_1 &= \overline{u}(q_1)\, \gamma_7\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{q_1}\, u(p_1) \\
S_2 &= \overline{u}(q_1)\, \gamma_6\gamma_{p_1}\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{q_1}\, u(p_1) \\
S_3 &= \overline{u}(q_1)\, \gamma_6\gamma_{\mu_1}\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{\mu_1}\, u(p_1) \\
S_4 &= \overline{u}(q_1)\, \gamma_7\gamma_{\mu_1}\gamma_{p_1}\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{\mu_1}\, u(p_1) \\
S_5 &= \overline{u}(q_1)\, \gamma_7\gamma_{\mu_1}\gamma_{\mu_2}\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{q_1}\, u(p_1) \\
S_6 &= \overline{u}(q_1)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{p_1}\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{q_1}\, u(p_1) \\
S_7 &= \overline{u}(q_1)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\, u(p_1) \\
S_8 &= \overline{u}(q_1)\, \gamma_7\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{p_1}\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\, u(p_1) \\
S_9 &= \overline{u}(q_1)\, \gamma_7\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{q_1}\, u(p_1) \\
S_{10} &= \overline{u}(q_1)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{p_1}\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{q_1}\, u(p_1) \\
S_{11} &= \overline{u}(q_1)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}\, u(p_2)\overline{u}(q_2)\, \gamma_6\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}\, u(p_1)
\end{aligned}
$$

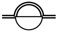
Two-loop corrections to single top quark production

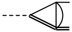
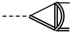
- *Vertex corrections: both methods*

Vertex corrections

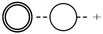
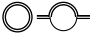


$= \Big[N_C^2 T C_F g_s^4 \Big\{ S_1 \Big[\frac{1}{m_t^5}\Big(\frac{-8-11t+t^2}{2\epsilon(t-1)^3} - \frac{-71-275t+44t^2}{12(t-1)^3} + \mathcal{O}(\epsilon)\Big)$  $- \Big(\frac{44t}{3m_t^3(t-1)^3}+\mathcal{O}(\epsilon)\Big)$ 

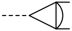
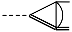
$-\Big(\frac{2n_f}{3m_t^3(t-1)}+\mathcal{O}(\epsilon)\Big)$  $+\frac{1}{m_t^3}\Big(-\frac{-8-11t+t^2}{3\epsilon(t-1)^3}+\frac{-103-226t+41t^2}{18(t-1)^3}+\mathcal{O}(\epsilon)\Big)$ 

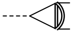
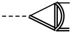
$+\Big(\frac{2n_f}{3m_t(t-1)}+\mathcal{O}(\epsilon)\Big)$  $+\Big(\frac{12t}{m_t(t-1)^3}+\mathcal{O}(\epsilon)\Big)$  $-\Big(\frac{2m_t(t+1)}{3(t-1)}+\mathcal{O}(\epsilon)\Big)$  $\Big]$

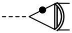
$+S_3\Big[\frac{1}{m_t^4}\Big(-\frac{11+12t+13t^2}{24\epsilon(t-1)^2}+\frac{-72+169t-324t^2+303t^3+76t^4-24t^5}{72(t-1)^3 t}+\mathcal{O}(\epsilon)\Big)$ 

$+\frac{1}{m_t^2}\Big(\frac{2}{3\epsilon}-\frac{46+28t}{9t}+\mathcal{O}(\epsilon)\Big)$  $+\frac{1}{m_t^2}\Big(\frac{2t}{3\epsilon(t-1)}+\frac{2t(1+5t-14t^2)}{9(t-1)^3}+\mathcal{O}(\epsilon)\Big)$ 

$+\frac{1}{m_t^2}\Big(\frac{n_f(5-19t)}{36(t-1)}-\frac{n_f(t+1)}{6\epsilon(t-1)}+\mathcal{O}(\epsilon)\Big)$  $+\frac{1}{m_t^2}\Big(\frac{5t^2+12t+19}{36\epsilon(t-1)^2}+\frac{29t^2-86t-87}{72(t-1)^2}+\mathcal{O}(\epsilon)\Big)$ 

$+\big(-n_f/9+n_f/(3\epsilon)+\mathcal{O}(\epsilon)\big)$  $+\Big(\frac{n_f t}{3\epsilon(t-1)}-\frac{n_f(3+t)}{9(t-1)}+\mathcal{O}(\epsilon)\Big)$ 

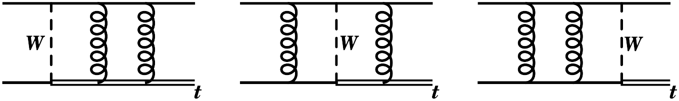
$+\big(2/t-t/3+\mathcal{O}(\epsilon)\big)$  $+\Big(\frac{(1+3t+3t^2-t^3)}{3(t-1)^2}+\mathcal{O}(\epsilon)\Big)$ 

$-\Big(\frac{m_t^2(3t^2+19t+28)}{9}+\mathcal{O}(\epsilon)\Big)$  $-\Big(\frac{m_t^2 t(3t^2+4t+1)}{9(t-1)}+\mathcal{O}(\epsilon)\Big)$  $\Big]\Big\}$

$+N_C^2 C_F^2\Big\{\cdots\Big\}+N_C^2 C_F C_A\Big\{\cdots\Big\}\Big]\frac{1}{m_t^2\left(t-m_W^2\right)}$

- *Vertex corrections: both methods* ✓
- *Planar double boxes : projection method*

Planar double boxes



- ✓ *reduction up to 4 scalar products with* REDUZE2 *and* CRUSHER
- ✓ *consistency checks of the reduction tables*
- ✓ *calculation of the corresponding diagrams*
- ✓ *check of the end results*

Two-loop corrections to single top quark production

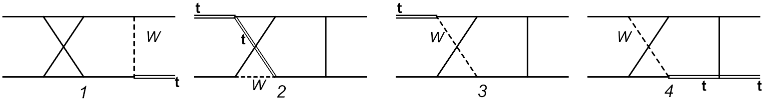
- *Vertex corrections: both methods* ✓
- *Planar double boxes : projection method* ✓

Two-loop corrections to single top quark production

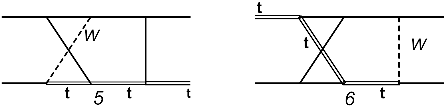
- *Vertex corrections: both methods* ✓
- *Planar double boxes : projection method* ✓
- *Non Planar double boxes: a challenge !*

Non planar double boxes

- *Reduction to master integrals: need four scalar products*
- *Rather simpler topologies :*



- *Topologies 3 and 4 are completely reduced*
- REDUZE2 *reduced up to 3 scalar products, fourth is challenging!*
- *More complicated topologies:*



Conclusions

- *It is important to consider higher order contributions in the perturbation series*
- *We have seen two possibilities to reduce tensor integrals to scalar integrals*
- *The choice of the reduction method determines how difficult the next step (IBP) is*
- *We calculated the two loop vertex corrections to the single top quark production*
- *As a test of our setup, we have calculated the* $\mathcal{O}(\alpha_s^2)$ *contributions to the Heavy Quark Vector and Axial Vector Form Factors, confirming the results of* `Bernreuther et al.` *and* `Gluza et al.`
- *We computed also the double box contributions, however the application of the IBP method is not trivial*

Backup

Non Planar double boxes

Using standard software to reduce $\mathcal{I}(1,1,1,1,1,1,1,\,-a_8,-a_9)$ $|a_8|+|a_9|=4$, *will generate large number of seeds in sub sectors 6 (allowed 1 dot).* $\Rightarrow$ *bottleneck for reduction! One alternative approach:*

1. *Consider the given integrals* $\mathcal{I}(1,1,1,1,1,1,1,\,-a_8,-a_9)$, $|a_8|+|a_9|=4$ *as seeds*
2. *Determine all IBP-Relations, in which these integrals occur*

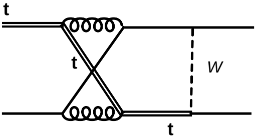
Example

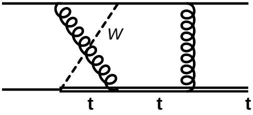
```
In[17]:= t1ibp[[7]] /. {a1 ->1,a2->1,a3->1,a4->1,a5->1,a6->1,a7->1,a8->-2,a9->-2}
Out[17]= -INT[0, 1, 1, 1, 1, 1, 2, -2, -2] +  2 INT[1, 0, 1, 1, 1, 1, 1, -2, -1] -
 INT[1, 0, 1, 1, 1, 1, 2, -2, -2] - INT[1, 0, 1, 1, 1, 2, 1, -2, -2] -
 INT[1, 0, 2, 1, 1, 1, 1, -2, -2] + INT[1, 1, 1, 0, 1, 1, 2, -2, -2] +
 INT[1, 1, 1, 0, 1, 2, 1, -2, -2] + (-3 + d) INT[1, 1, 1, 1, 1, 1, 1, -2, -2] +
 2 t INT[1, 1, 1, 1, 1, 1, 1, -2, -1] +  INT[0, 1, 1, 1, 1, 1, 2, -3, -2] -
 (mw^2 + s + t) INT[1, 1, 1, 1, 1, 1, 2, -2, -2]
```

3. *Solve each IBP relation for such integrals* $\mathcal{I}(1,1,1,1,1,1,1,\,-a_8,-a_9)$ *with 4 scalar products*
4. *Consider all other integrals with 4 scalar products and 1 dot in the sub sectors with 6 propagators as seeds*

Topology	*# Diagrams*	*reduction*	*performed checks*
Vertex corrections	*29*	✓	✓
Planar double boxes	*6*	✓	✓
Non Planar double boxes	*12*	*work in progress*	–

There are two most complicated topologies, which could not be reduced completely until now :





Backup

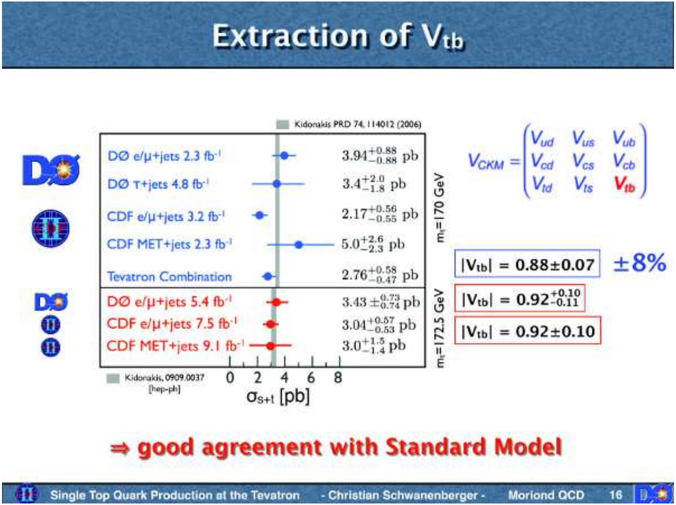
```
       identities:
        ibp:
          - \{ r: [t, t+1], s:  [3, 4] \}
 topo2 6 125  6 3  1 -3 1 1 1 1 1 0 0
.
.
.
topo2 6 125  6 4  1 -1 1 1 1 1 1 -1 -2
topo2 6 125  6 4  1 -1 1 1 1 1 1 0 -3
topo2 6 125  6 4  1 0 1 1 1 1 1 -4 0
.
.
.
topo2 6 125  7 4  1 0 1 1 1 1 2 -4 0
topo2 6 125  7 4  1 0 1 1 1 1 2 -3 -1
topo2 6 125  7 4  1 0 1 1 1 1 2 -2 -2
topo2 6 125  7 4  1 0 1 1 1 1 2 -1 -3
topo2 6 125  7 4  1 0 1 1 1 1 2 0 -4


grep topo2 seeds_topo2_6_125_ibp|wc
    175     2450     6636
```

Backup

```
topo2 6 125  6 3  1 -3 1 1 1 1 1 0 0
topo2 6 125  6 3  1 -2 1 1 1 1 1 -1 0
topo2 6 125  6 3  1 -2 1 1 1 1 1 0 -1
topo2 6 125  6 3  1 -1 1 1 1 1 1 -2 0
topo2 6 125  6 3  1 -1 1 1 1 1 1 -1 -1
topo2 6 125  6 3  1 -1 1 1 1 1 1 0 -2
topo2 6 125  6 3  1 0 1 1 1 1 1 -3 0
topo2 6 125  6 3  1 0 1 1 1 1 1 -2 -1
topo2 6 125  6 3  1 0 1 1 1 1 1 -1 -2
topo2 6 125  6 3  1 0 1 1 1 1 1 0 -3
topo2 6 125  6 4  1 -4 1 1 1 1 1 0 0
topo2 6 125  6 4  1 -3 1 1 1 1 1 -1 0
topo2 6 125  6 4  1 -3 1 1 1 1 1 0 -1
topo2 6 125  6 4  1 -2 1 1 1 1 1 -2 0
topo2 6 125  6 4  1 -2 1 1 1 1 1 -1 -1
topo2 6 125  6 4  1 -2 1 1 1 1 1 0 -2
topo2 6 125  6 4  1 -1 1 1 1 1 1 -3 0
topo2 6 125  6 4  1 -1 1 1 1 1 1 -2 -1
topo2 6 125  6 4  1 -1 1 1 1 1 1 -1 -2
topo2 6 125  6 4  1 -1 1 1 1 1 1 0 -3
topo2 6 125  6 4  1 0 1 1 1 1 1 -4 0
topo2 6 125  6 4  1 0 1 1 1 1 1 -3 -1
topo2 6 125  6 4  1 0 1 1 1 1 1 -2 -2
topo2 6 125  6 4  1 0 1 1 1 1 1 -1 -3
topo2 6 125  6 4  1 0 1 1 1 1 1 0 -4
topo2 6 125  7 4  1 0 1 1 1 1 2 -2 -2
topo2 6 125  7 3  1 0 1 1 1 2 1 -2 -1
```

Backup


Extraction of Vtb
Kidonakis PRD 74, 114012 (2006)
DØ e/μ+jets 2.3 fb⁻¹ $3.94^{+0.88}_{-0.88}$ pb
DØ τ+jets 4.8 fb⁻¹ $3.4^{+2.0}_{-1.8}$ pb
CDF e/μ+jets 3.2 fb⁻¹ $2.17^{+0.56}_{-0.55}$ pb
CDF MET+jets 2.3 fb⁻¹ $5.0^{+2.6}_{-2.3}$ pb
Tevatron Combination $2.76^{+0.58}_{-0.47}$ pb
DØ e/μ+jets 5.4 fb⁻¹ $3.43 \pm^{0.73}_{0.74}$ pb
CDF e/μ+jets 7.5 fb⁻¹ $3.04^{+0.57}_{-0.53}$ pb
CDF MET+jets 9.1 fb⁻¹ $3.0^{+1.5}_{-1.4}$ pb
m_t=170 GeV
m_t=172.5 GeV
Kidonakis, 0909.0037 [hep-ph]
0 2 4 6 8
σ_{s+t} [pb]
$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$
$|V_{tb}| = 0.88 \pm 0.07$ ±8%
$|V_{tb}| = 0.92^{+0.10}_{-0.11}$
$|V_{tb}| = 0.92 \pm 0.10$
⇒ good agreement with Standard Model
Single Top Quark Production at the Tevatron - Christian Schwanenberger - Moriond QCD 16