Holographic Duals of Black Rings and Rotating Black Holes

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- Black rings
- CFT duals of the Myers-Perry Black hole
- Holography in extremal black ring
- Discrimination of ring from hole in CFT

Black Rings

- Since the birth of general relativity in 1916, people have found many solutions for it. Untill 2001, all these solutions had a common feature: they all had topologically spherical horizons (S^2, S^3, ..., S^{D-2}).

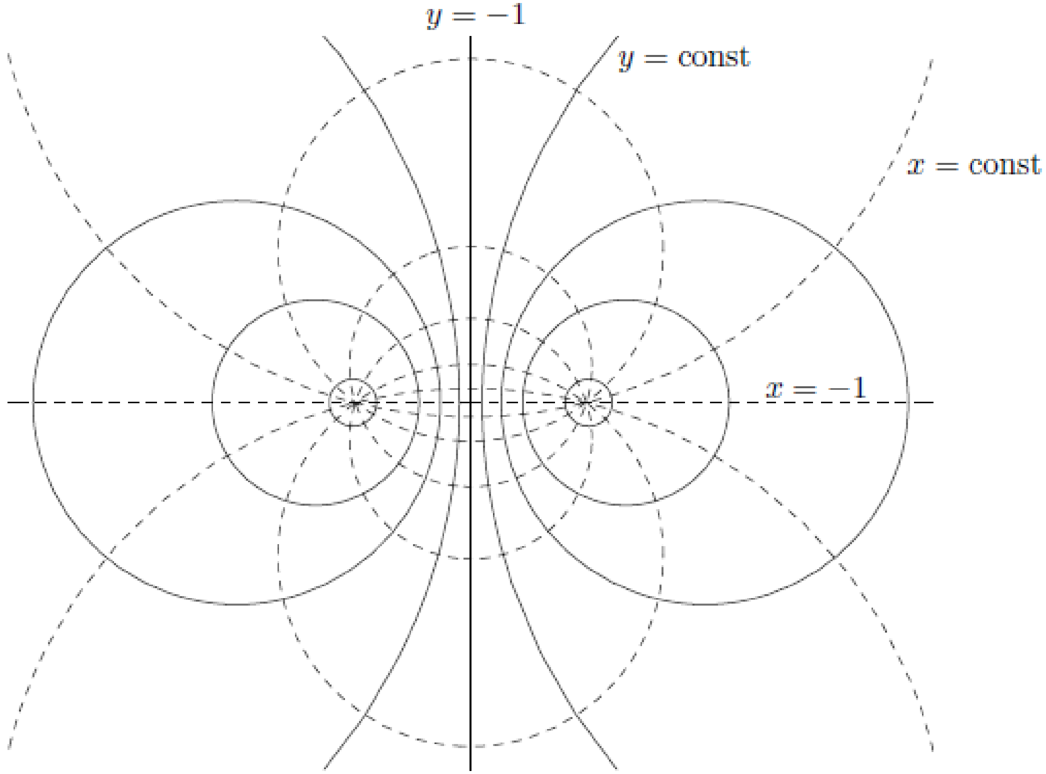
- In 2001, Emparan & Real, discovered new type of solutions of the Einstein theory. It is a 5 dimensional solution with the horizon topology of $S^1 \times S^2$. At higher dimensions the topology is $S^1 \times S^{(D-3)}$. They lives in $D \geq 5$ dimensions.

Black Rings

- Flat space in ring coordinates

$$d\mathbf{x}_4^2 = \frac{R^2}{(x-y)^2}\left[(y^2-1)d\psi^2 + \frac{dy^2}{y^2-1} + \frac{dx^2}{1-x^2} + (1-x^2)d\phi^2\right]$$

- This space can be depicted as



Black Rings

- Generically in D dimension an object can be rotating in more than one plane. The number of rotations is

$$\left\lfloor \frac{D-1}{2} \right\rfloor$$

Black Rings

- Single spin black ring
- The simplest black ring has only one rotation around S^1 (in ψ direction). This is a stationary asymptotically flat solution of vacuum Einstein gravity

$$ds^2 = -\frac{F(y)}{F(x)}\left(dt - C\,R\,\frac{1+y}{F(y)}\,d\psi\right)^2 + \frac{R^2}{(x-y)^2}\,F(x)\left[-\frac{G(y)}{F(y)}d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\phi^2\right]$$

(Emparan, Reall 0110260)

$$F(\xi) = 1 + \lambda\xi, \qquad G(\xi) = (1-\xi^2)(1+\nu\xi)\,, \qquad C = \sqrt{\lambda(\lambda-\nu)\frac{1+\lambda}{1-\lambda}}\,.$$

- This solution has some interesting properties:
- Its angular momentum is bounded from below; it can be arbitrarily large!!

$$\frac{J^2}{M^3} > 0.8437\frac{32G}{27\pi}$$

- It has a physical meaning: the minimum of angular momentum for a black ring can not be vanishing, otherwise it collapses.

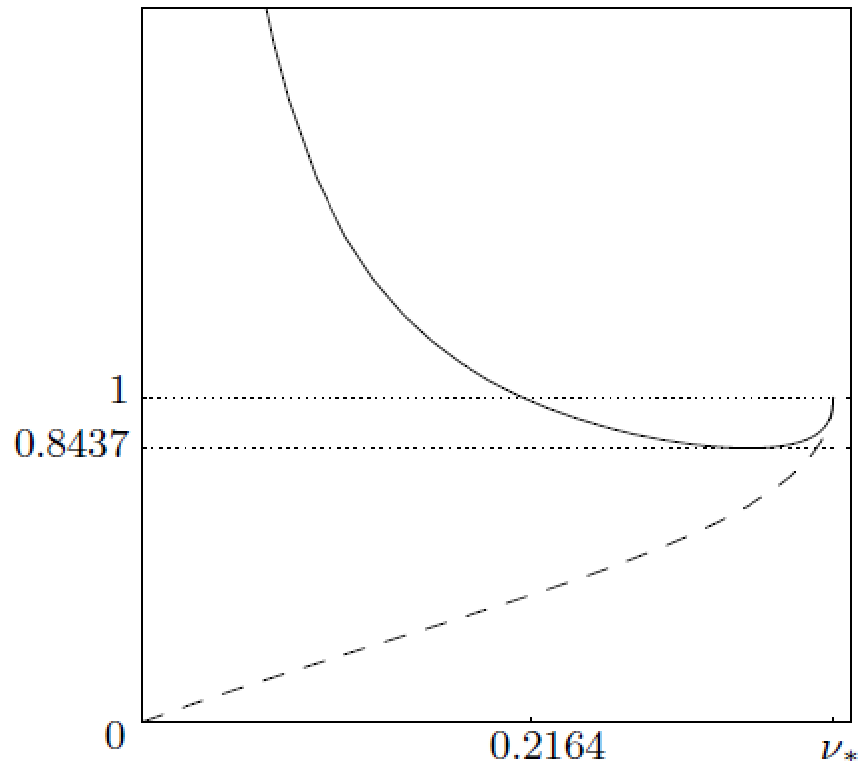
Black Rings

- Uniqueness violation!!
- The spin of the five-dimensional black holes is bounded from above as

$$\frac{J^2}{M^3} \leq \frac{32G}{27\pi}$$

- There is an overlapping region:

this figure shows $(27\pi/32G)J^2/M^3$ as a function of ν. the solid line corresponds to the black ring and the dashed line to the black hole. The dotted lines delimit the values for which a black hole and two black rings with the same mass and spin can exist.



Black Rings

- Double spin black ring
- In general a black ring can be spinning in both (φ,ψ) directions. Double rotating black ring solution (DRBR) is stationary and asymptotically flat too (Pomeransky, Sen'kov 0612005).

$$
\begin{aligned}
ds^2 \; &= \frac{H(y,x)}{H(x,y)}(dt+\Omega)^2 + \frac{F(x,y)}{H(y,x)}d\phi^2 + 2\frac{J(x,y)}{H(y,x)}d\phi d\psi \\
&- \frac{F(y,x)}{H(y,x)}d\psi^2 - \frac{2k^2 H(x,y)}{(x-y)^2(1-\nu)^2}\left(\frac{dx^2}{G(x)} - \frac{dy^2}{G(y)}\right)
\end{aligned}
$$

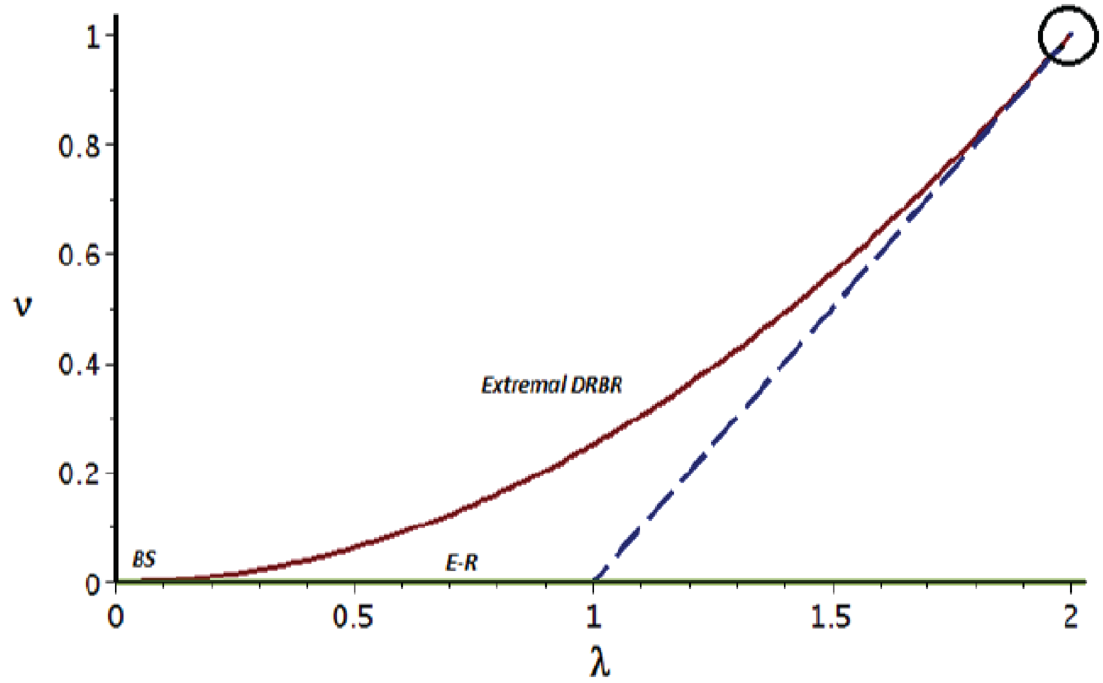
$$
\Omega = \frac{-2k\lambda\sqrt{(1+\nu)^2-\lambda^2}}{H(y,x)}\left(\sqrt{\nu}y(1-x^2)d\phi + \frac{1+y}{1-\lambda+\nu}(1+\lambda-\nu+\nu(1-\lambda-\nu)yx^2+2\nu x(1-y))d\psi\right)
$$

- This is a three parameter solution denoted by k,λ,ν. k corresponds to the ring radius and ν determines rotation of S^2. Ranges of these parameters is

$$
k > 0\,, \qquad 0 \le \nu < 1\,, \qquad 2\sqrt{\nu} \le \lambda < 1+\nu
$$

Black Rings

- Ignoring k, we can draw the space of parameters as (point $\lambda = 2, \nu = 1$ and line $\lambda = 1 + \nu$ are excluded from the parameter space):



- Entropy, Hawking temperature and mass of DRBR is given by

$$S_{BH} = \frac{8\pi^2 k^3 \lambda(1+\nu+\lambda)}{G_5(1-\nu)^2(y_h^{-1} - y_h)}, \qquad T_H = \frac{(y_h^{-1} - y_h)(1-\nu)\sqrt{\lambda^2 - 4\nu}}{8\pi k \lambda(1+\nu+\lambda)}, \qquad M = \frac{3\pi k^2 \lambda}{G_5(1+\nu-\lambda)}$$

- Where horizon (larger root of G(y)) lies at

$$y_h = \frac{-\lambda + \sqrt{\lambda^2 - 4\nu}}{2\nu}$$

CFT duals of the MP Black hole

- 5d Myers-Perry (MP) black holes form a three parameter family of vacuum Einstein gravity solutions

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi - b\cos^2\theta d\psi\right)^2 + \frac{\rho^2 dr^2}{\Delta} + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left(a\,dt - \left(r^2+a^2\right)d\phi\right)^2$$

$$+\frac{\cos^2\theta}{\rho^2}\left(b\,dt - \left(r^2+b^2\right)d\psi\right)^2 + \frac{1}{r^2\rho^2}\left(ab\,dt - b\left(r^2+a^2\right)\sin^2\theta d\phi - a\left(r^2+b^2\right)\cos^2\theta d\psi\right)^2$$

$$\Delta = \frac{1}{r^2}(r^2+a^2)(r^2+b^2) - 2M$$

$$T_H = \frac{r_+^4 - a^2b^2}{2\pi r_+(r_+^2+a^2)(r_+^2+b^2)}, \qquad S_{BH} = \frac{\pi^2(r_+^2+a^2)(r_+^2+b^2)}{2\,G_5 r_+}$$

- It has been shown that the near horizon of extremal Kerr black hole, by choosing proper boundary conditions, is dual to a chiral 2D CFT (Strominger, et al 0809.4266). This is generalized to many rotating solutions.
- For the MP black hole, extremality occurs at $r_+ = \sqrt{ab}$. The entropy is $S_{BH} = \frac{\pi^2}{2G_5}\sqrt{ab}(a+b)^2$

CFT duals of the MP Black hole

- The near horizon of extremal metirc is

$$ds^2 = \rho_0^2\left(-\frac{\hat{r}^2}{4}d\hat{t}^2 + \frac{d\hat{r}^2}{4\hat{r}^2}\right) + \rho_0^2 d\theta^2 + \frac{2M}{\rho_0^2}\Big[a^2\sin^2\theta\left(d\hat{\phi} + k_{\hat{\phi}}\hat{r}d\hat{t}\right)^2 + b^2\cos^2\theta\left(d\hat{\psi} + k_{\hat{\psi}}\hat{r}d\hat{t}\right)^2 + ab\left(\sin^2\theta(d\hat{\phi} + k_{\hat{\phi}}\hat{r}d\hat{t}) + \cos^2\theta(d\hat{\psi} + k_{\hat{\psi}}\hat{r}d\hat{t})\right)^2\Big]$$

  which has the isometry $SL(2,R) \times U(1)_\phi \times U(1)_\psi$

- This means that there is two dual chiral 2D CFT accompanied by rotation in ϕ and ψ directions with central charges and Frolov-Thorn temperatures (Lu, Mei, Pope 0811.2225)

$$c_\phi = \frac{3\pi}{2G_5}(a+b)^2 b\,, \quad c_\psi = \frac{3\pi}{2G_5}(a+b)^2 a\,, \qquad T_\phi = \frac{1}{2\pi k_\phi} = \frac{1}{\pi}\sqrt{\frac{a}{b}}\,, \quad T_\psi = \frac{1}{2\pi k_\psi} = \frac{1}{\pi}\sqrt{\frac{b}{a}}$$

- Entropy can be found from Cardy formula $S = \frac{\pi^2}{3}c_\phi T_\phi = \frac{\pi^2}{3}c_\psi T_\psi$ which is in agreement to the Beckenstein-Hawking entropy

CFT duals of the MP Black hole

- Another sign for the existence of 2D CFT appears in vanishing horizon limit of the extremal solution:
- Setting $a = 0$ in extremal MP solution kills its horizon area. Now taking the near horizon limit of this solution, one finds (Bardeen-Horowitz 9905099)

$$ds^2 = \sin^2\theta\left(-\frac{\tilde{r}^2}{b^2}\,dt^2 + \frac{b^2}{\tilde{r}^2}\,d\tilde{r}^2 + \tilde{r}^2 d\tilde{\phi}^2\right) + b^2(\sin^2\theta\, d\theta^2 + \cot^2\theta\, d\tilde{\psi}^2)$$

- There is an AdS3 part in this metric; due to the infinitesimal period of $\tilde{\phi}$ ($\tilde{\phi} \in [0, 2\pi\epsilon]$), let us call it *pinching* AdS3. Appearing this AdS3 is a signature of a 2D CFT with the Brown-Hennueax central charge $3b/2G_3$.

- In the limit $a = 0$ temperature and horizon area are vanishing while A/T remains finite; the other charges of the solution also remains finite. This is an example of extremal vanishing horizon (EVH) solutions.

CFT duals of the MP Black hole

- Such Ads3 throats appear in the near horizon of many EVH solutions. This provides a basis to propose the EVH/CFT duality (Sheikh-Jabbari, Yavartanoo 1107.5707, 1301.3387)
- One can check this proposal by rederiving central charges dual to the MP black hole: 3D Newton constant relates to the 5D one as $G_3 = \frac{G_5}{\pi b^2}$, so

$$c_{B.H.} = \frac{3b}{2G_3} = \frac{3\,\pi b^3}{2G_5}$$

- This is in complete agreement with the former result in the limit $a = 0$ so the proposal works.

- We may also investigate deviations from the EVH point by supposing infinitesimal value for a and writing other parameters as

$$a = \hat{a}\epsilon^2, \; b = b_0 + \delta b\,, \; M = \frac{b_0^2}{2} + (\delta M + b_0 \delta b)$$

CFT duals of the MP Black hole

- and taking the near horizon limit by $r = \hat{r}\,\epsilon,$ one can find that

$$ds^2 = \sin^2\theta\left[-\hat{\Delta}\,d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{\Delta}} + \hat{r}^2\left(d\hat{\phi} - \frac{\hat{a}}{\hat{r}^2}d\hat{t}\right)^2\right] + \ell^2(\sin^2\theta\,d\theta^2 + \cot^2\theta\,d\hat{\psi}^2)$$

- This is a BTZ geometry; again, due to the infinitesimal period of $\hat{\phi}$, it has pinching characteristic.

- Such a BTZ geometry appears in the near horizon limit of many near EVH solutions.

Holography in extremal DRBR

- In spite of single spin black ring, the DRBR admits the extremal limit. The location of the horizons are

$$y_{\pm} = \frac{-\lambda \pm \sqrt{\lambda^2 - 4\nu}}{2\nu}$$

- So one can obtain an extremal solution by choosing $\nu = \lambda^2/4$. The near horizon of this extremal solution takes to the form

$$\begin{aligned} ds^2 &= \frac{16k^2\Gamma(x)}{(\lambda-2)^2}\Big(-y^2dt^2 + \frac{dy^2}{y^2}\Big) + \frac{8\lambda^2k^2H(x)}{(\lambda x+2)^4(1-x^2)(4-\lambda^2)}dx^2 \\ &+ 4\frac{k^2(2+\lambda)^2}{(2-\lambda)^2}d\psi^2 + \frac{32\lambda^2k^2(1-x^2)}{H(x)(4-\lambda^2)}\Big(d\phi - y\,dt + \frac{(4+8\lambda+\lambda^2)}{4\lambda}d\psi\Big)^2 \end{aligned}$$

$$H(x) = \left(\lambda^2+4\right)\left(1+x^2\right) + 8\,\lambda\,x\,, \qquad \Gamma(x) = \frac{\lambda^2 H(x)}{2\,(2+\lambda\,x)^2\,(2+\lambda)^2}\,.$$

- Choosing proper boundary conditions which is respected by standard asymptotic killing vectors

$$\zeta_n = -e^{-in\phi}\partial_\phi - in\ r\ e^{-in\phi}\partial_r, \qquad \zeta_n = -e^{-in\psi}\partial_\psi - in\ r\ e^{-in\psi}\partial_r$$

Holography in extremal black ring

- We found the following result for central charges

$$c_\phi = -384 \frac{\pi \lambda^2 k^3}{(2+\lambda)(\lambda-2)^3} = 12 J_\phi, \qquad c_\psi = 0$$

- We can also read the Frolov-Thorn temperature as $T_\phi = \frac{1}{2\pi}$; so the Cardy entropy is

$$S = \frac{\pi^2}{3} c_\phi T = -64 \frac{\pi^2 \lambda^2 k^3}{(2+\lambda)(\lambda-2)^3}$$

- which is equal to the macroscopic entropy; so the Kerr/CFT analysis works here too.

Holography in extremal black ring

- It is also possible to study the holographic dual of DRBR in the EVH limit. DRBR becomes EVH only at point $\lambda = 2, \nu = 1$ in the parameter space, accompanied by rescaling k in a manner that keeps mass and other charges of DRBR finite. Since This point is excluded from the parameter space, we can only study the near EVH limit of DRBR.
- Around $\lambda = 2, \nu = 1$, there is some transformation to convert DRBR to an extremal MP black hole:

$$x = -1 + \frac{16\sqrt{a}k^3 \cos^2\theta}{(a+b)^{3/2}(r^2 - ab)}, \qquad \hat{y} = -1 - \frac{16\sqrt{a}k^3 \sin^2\theta}{(a+b)^{3/2}(r^2 - ab)}$$

- The rotation parameters a, b are $a = \sqrt{2\tilde{M}}\sigma\,,\ b = \sqrt{2\tilde{M}}(1-\sigma)$ where

$$\sigma = \frac{1+\nu-\lambda}{(1-\nu)^2}, \qquad \tilde{M} = \frac{8k^2}{1+\nu-\lambda}$$

- should remains finite

Holography in extremal black ring

- The EVH condition satsfies with the choice of parameters as

$$\nu = 1 - \hat{\nu}\epsilon\,, \qquad \lambda = 1 + \nu - \hat{\lambda}\epsilon^{2(1+\alpha)}\,, \qquad k = \hat{k}\epsilon^{1+\alpha}\,, \qquad \alpha > 0$$

- This kills the rotation parameter a of extremal MP black holes. More clearly, in the region around $\lambda = 2, \nu = 1$ we have black rings and extremal MP black holes with the same masses and spins; in this region, points which satisfies the above conditions, are correspond to EVH holes and rings.
- We found central charges of these EVH holes and rings as

$$c_\phi^{hole} = 18 J_\phi \qquad c_\phi^{ring} = 12 J_\phi$$

- Finally we found the near horizon of near EVH solution as

$$ds^2 = \sin^2\theta\left[-\frac{r^2}{4b^2}dt^2 + b^2\frac{dr^2}{4r^2} + p^{+2}\left(d\phi + \frac{1}{2\,p^+ b}\,r dt\right)^2\right] + b^2\left(\sin^2\theta\, d\theta^2 + \cot^2\theta d\psi^2\right)$$

- where $p^+ = \sqrt{ab}$. This is a self-dual AdS3 orbifold with radius b

Discrimination of ring from hole in CFT

- In the limit $\lambda = 2, \nu = 1$ we have a configuration of extremal black rings and extremal black holes of the same masses and spins. Their entropies are

$$S^{ring} = \frac{\pi^2 \tilde{M}\sqrt{2\tilde{M}}}{2G_5} \cdot \sqrt{\sigma}(1+\sqrt{1-4\sigma})\,, \qquad S^{hole} = \frac{\pi^2 \tilde{M}\sqrt{2\tilde{M}}}{2G_5} \cdot 2\sqrt{\sigma(1-\sigma)}$$

- These satisfy $S^{ring} < S^{hole} \leq \sqrt{3}S^{ring}$; so the hole has larger entropy.

- CFT viewpoint:
- The near horizon geometry of the near EVH hole is extremal BTZ while for the ring we deal with a self-dual AdS3 orbifold. So, the 2D CFT distinction between the hole and the ring lies within the 2d CFT distinction of an extremal BTZ from a self-dual orbifold. It has been shown that the former corresponds to the thermal state $|T_L = 0\rangle \bigotimes |T_R\rangle$, where $|T\rangle$ is a thermal state at temperature T, while the latter (the self-dual orbifold)

Discrimination of ring from hole in CFT

corresponds to $|c/24\rangle \otimes |T_R\rangle$, where $|c/24\rangle$ is the ground state of the 2D CFT with the energy of c/24 above the vacuum. (Balasubramanian, de Boer, Sheikh-Jabbari and Simon 0906.3272) On the other words, in the case of EVH MP black hole, dual CFT states have specified temperature (canonical description), while for EVH black ring the energy is specified (microcanonical description).

- Moreover, we remember that central charge of holes and rings does not match; This means that the ring and the hole are basically two different states in two distinct 2D chiral CFTs. In fact the hole CFT has a larger central charge.

- Summary

- We found holographic dual for the black ring. There is only one CFT in ϕ direction

- Pinching AdS3 geometries appears on the NH limit of EVH solutions. This is a signature of the EVH/CFT duality.

- Near the EVH point in the parameter space of DRBR there are configurations of holes and ring which can be identified by their central charges. Also in dual 2D CFT, holes and rings are corresponds to $|T_L = 0\rangle \bigotimes |T_R\rangle$, and $|c/24\rangle \bigotimes |T_R\rangle$, respectively.

References

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Thanks for your attention