Conformal gravity holography in four dimensions

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- EG: ghost-free, two-loop non-renormalizable
- CG: studied by Mannheim, 't Hooft, Maldacena, emerges from twistor string theory and as a counter term in AdS/CFT

Set of boundary conditions weaker than Starobinsky ones?

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Set of boundary conditions weaker than Starobinsky ones?

- the CG anolog to Schwarzschild - MKR (Mannheim-Kanzas-Riegert) solution - doesn't obey Starobinsky boundary conditions
- CG allows a Rindler-term D. Grumiller, 2011
- purpose: define set of boundary conditions weaker than Starobinsky ones that allow MKR solution and solutions with a condensate of partially massless gravitons.

- CG depends on Lorentz angles but not on distances
- $\Rightarrow$ it is invariant under local Weyl rescalings of the metric

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{2\omega}\, g_{\mu\nu} \tag{1}$$

- the bulk action is

$$I_{\rm CG} = \alpha_{\rm CG} \int \mathrm{d}^4x \sqrt{|g|}\, g_{\alpha\mu} g^{\beta\nu} g^{\gamma\lambda} g^{\delta\tau}\, C^\alpha{}_{\beta\gamma\delta} C^\mu{}_{\nu\lambda\tau} \tag{2}$$

- EOM require vanishing of Bach tensor:

$$\big(\nabla^\delta \nabla_\gamma + \frac{1}{2}\, R^\delta{}_\gamma\big)\, C^\gamma{}_{\alpha\delta\beta} = 0 \tag{3}$$

- simple classes of solutions to the EOM:
  - conformally flat metrics ($C^\gamma{}_{\alpha\delta\beta} = 0$)
  - Einstein metrics ($R_{\alpha\beta} \propto g_{\alpha\beta}$)

- solutions of EG belong to more general set of solutions of CG
- The most general spherically symmetric solution of CG

$$\mathrm{d}s^2 = -K(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{K(r)} + r^2\,\mathrm{d}\Omega^2_{S^2} \tag{4}$$

$$K(r) = \sqrt{1-12aM} - \frac{2M}{r} - \Lambda r^2 + 2ar \tag{5}$$

- $a = 0$ this reduces to Schwarzschild-(A)dS

- introduce ℓ, in EG $\Lambda = 3\sigma/\ell^2$, $\sigma = -1$ for AdS and $\sigma = 1$ for dS
- asymptotic expansion in $0 < \rho << \ell$ leads to line element $\Rightarrow$

$$\mathrm{d}s^2 = \frac{\ell^2}{\rho^2}\big(-\sigma\,\mathrm{d}\rho^2 + \gamma_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\big) \tag{6}$$

- partially fixed gauge, use of Gaussian coordinates.
- close to $\rho = 0$

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \frac{\rho}{\ell}\,\gamma_{ij}^{(1)} + \frac{\rho^2}{\ell^2}\,\gamma_{ij}^{(2)} + \frac{\rho^3}{\ell^3}\,\gamma_{ij}^{(3)} + \dots \tag{7}$$

- $\gamma_{ij}^{(0)}$ is invertible, $\gamma_{ij}^{(n)} = \gamma_{ij}^{(n)}(x^i)$

- specification of boundary conditions

$$\begin{array}{cc} \text{leading terms} & \text{first order terms} \\ \delta\gamma_{ij}^{(0)}|_{\partial\mathcal{M}} = 2\,\lambda\,\gamma_{ij}^{(0)} & \delta\gamma_{ij}^{(1)}|_{\partial\mathcal{M}} = \lambda\,\gamma_{ij}^{(1)} \end{array} \tag{8}$$

- λ - regular function on the boundary
- $\delta\gamma_{ij}^{(n)}|_{\partial\mathcal{M}} \neq 0$ for $n \geq 2$ - are allowed to vary freely
- inclusion of $\gamma_{ij}^{(1)}$ - vanishes by EG EOM
- Bach tensor=0 doesn't give such conditions

Consistency of the boundary conditions

- checked by: considering on-shell action and the variational principle
- general expectations: GHY and holographic counterterms $\Rightarrow$ desired boundary value problem

However, full action is the bulk action!

- on-shell action for our metric remains finite
- free-energy from the on-shell action leads to consistent thermodynamics
- the entropy derived in this way is the same as the entropy derived using Wald's Noether charge technique
- simplest possibility - added boundary terms are zero

⇒Consistency of the variational principle and the finiteness of the holographic response functions

- Rewrite action:

$$\Gamma_{\mathrm{CG}} = \int_{\mathcal{M}} \mathrm{d}^4x \sqrt{|g|}\,\big(2\,R^{\mu\nu}R_{\mu\nu} - \tfrac{2}{3}\,R^2\big) + 32\pi^2\chi(\mathcal{M}) + \int_{\partial\mathcal{M}} \mathrm{d}^3x\sqrt{|\gamma|}\,\big(-8\,\sigma\,\mathcal{G}^{ij}K_{ij} + \tfrac{4}{3}\,K^3 - 4\,KK^{ij}K_{ij} + \tfrac{8}{3}\,K^{ij}K_j{}^kK_{ki}\big) \quad (9)$$

- With action separated into a topological part - the Euler characteristic $\chi(\mathcal{M})$, and a Ricci-squared action
- $K_{ij} = -\frac{\sigma}{2}\mathcal{L}_n\gamma_{ij}$,

First variation of the action

The first variation of the rewritten action is

-
$$\delta\Gamma_{\mathrm{CG}} = \mathrm{EOM} + \int_{\partial\mathcal{M}} \mathrm{d}^3x \sqrt{|\gamma|}\left(\pi^{ij}\,\delta\gamma_{ij} + \Pi^{ij}\,\delta K_{ij}\right) . \qquad (10)$$

- boundary metric and extrinsic curvature vary independently
- Consistency of variational principle
  - for $\rho_c \le \rho$ using EOM $\Rightarrow$

$$\delta\Gamma_{\mathrm{CG}}\big|_{\mathrm{EOM}} = \int_{\partial\mathcal{M}} \mathrm{d}^3x \sqrt{|\gamma^{(0)}|}\left(\tau^{ij}\,\delta\gamma^{(0)}_{ij} + P^{ij}\,\delta\gamma^{(1)}_{ij}\right) . \qquad (11)$$

- tensors τ^{ij} and P^{ij} are finite as $\rho_c \to 0 \Rightarrow$ first variation of the action vanishes on-shell when the boundary conditions are satisfied
- τ^{ij} and P^{ij} - holographic response functions conjugate to the sources $\gamma^{(0)}_{ij}$ and $\gamma^{(1)}_{ij}$

Response functions τ^{ij} and P^{ij}

- $\tau^{ij} \propto$ to the usual Brown-York stress tensor.
- P^{ij} - "Partially massles response"
- the term $\gamma_{ij}^{(1)}$ plugged the linearized CG EOM around (A)dS background exhibits partial masslessness
- Response functions satisfy the trace conditions

$$\gamma_{ij}^{(0)}\tau^{ij} + \tfrac{1}{2}\,\psi_{ij}^{(1)}P^{ij} = 0 \qquad \gamma_{ij}^{(0)}P^{ij} = 0\ , \tag{12}$$

- For Starobinsky boundary conditions the τ^{ij} is traceless while in general only P^{ij} is traceless
- Summary of the section: Proposed boundary conditions are consistend and lead to well-defined variational principle for the CG action, i.e. 0- and 1- point functions are finite.

- Currents

$$J^i = (2\,\tau^i{}_j + 2\,P^{ik}\,\gamma^{(1)}_{kj})\,\xi^j \ , \tag{13}$$

w/ $\sigma = -1$ (AdS) case and on constant time surface $\mathcal{C}$ in $\partial\mathcal{M}$ give

$$Q[\xi] = \int_{\mathcal{C}} \mathrm{d}^2x\,\sqrt{h}\,u_i J^i \tag{14}$$

- u^i future pointing normal vector, J^i modified stress energy tensor

S. Hollands, A. Ishibashi and D. Marlof, 2005

- Charges generate asymptotic symmetries

$$\mathcal{D}_i\big(2\tau^{ij} + 2P^i{}_k\,\gamma^{(1)kj}\big) = P^{ik}\mathcal{D}^j\gamma^{(1)}_{ik} \tag{15}$$

- difference in charges on $\mathcal{C}_1$ and $\mathcal{C}_2$ that bound a region $\mathcal{V} \subset \partial\mathcal{M}$ vanishes

$$\Delta Q[\xi] = \int_{\mathcal{V}} \mathrm{d}^3x\sqrt{|\gamma^{(0)}|}\,\Big(\tau^{ij}\mathcal{L}_\xi\gamma^{(0)}_{ij} + P^{ij}\mathcal{L}_\xi\gamma^{(1)}_{ij}\Big) \ , \tag{16}$$

- $\Rightarrow$ charges are conserved

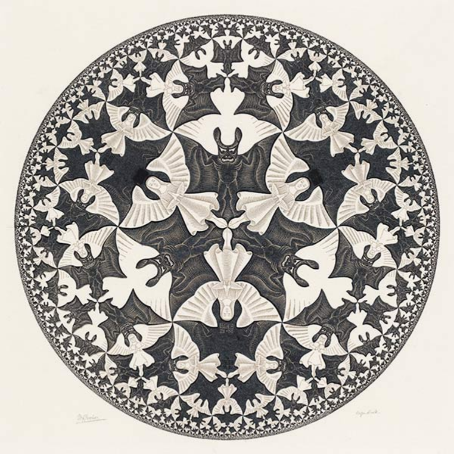
Legendre transformation $\Rightarrow$ exchanges the role of the PMR and its source

$$\tilde{\Gamma}_{\mathrm{CG}} = \Gamma_{\mathrm{CG}} + 8\int_{\partial\mathcal{M}} \mathrm{d}^3x\sqrt{|\gamma|}\,K^{ij}E_{ij}\,. \tag{17}$$

$$\delta\tilde{\Gamma}_{\mathrm{CG}} = \int_{\partial\mathcal{M}} \mathrm{d}^3x\sqrt{|\gamma|}\,\big(\tilde{\tau}^{ij}\,\delta\gamma_{ij}^{(0)} + \tilde{P}^{ij}\,\delta E_{ij}^{(2)}\big) \tag{18}$$

Vanishing trace $\tau_i^i = 0$

- CG holography in four dimensions
- MKR solutions and other solutions with an asymptotic Rindler term are viable



Thank you for the attention!

Examples

1. Special case with Starobinsky boundary conditions
   - $\gamma_{ij}^{(1)} = 0$
   - included: Einstein gravity with a cosmological constant
   - EOM demand $\Rightarrow E_{ij}^{(2)} = 0 \Rightarrow P^{ij} = 0$

$$\tau_{ij} = \frac{4\sigma}{\ell} E_{ij}^{(3)} \,. \tag{19}$$

   - $\Rightarrow$ traceless and conserverd stress tensor of Einstein gravity which agrees with J. M. Maldacena, 2011

2. MKR solution

$$\sigma = -1 \qquad p^i{}_j = \mathrm{diag}(1,\, -\tfrac{1}{2},\, -\tfrac{1}{2})^i{}_j$$
$$a_M = (1 - \sqrt{1 - 12aM})/6 \qquad m = M/\ell^2 \tag{20}$$

$$\tau^i{}_j = -8\tfrac{m}{\ell^2}\, p^i{}_j + 8\tfrac{a\, a_M}{\ell^2}\, \mathrm{diag}(1,\, -1,\, -1)^i{}_j$$
$$\mathrm{P}^i{}_j = 8\, \tfrac{a_M}{\ell^2}\, p^i{}_j \tag{21}$$

- $a = 0 \Rightarrow$ Einstein case
- $a \neq 0 \Rightarrow \tau^i_i \propto a^2$ and $P^{ij} \propto a \Rightarrow$ interpretation: partially massless graviton condensate

Conserved charge

$$Q[\partial_t] = m - a\, a_M \tag{22}$$

with $\alpha_{\mathrm{CG}} = \frac{1}{64\pi}$

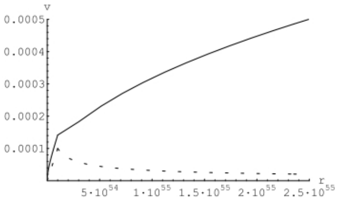
- Wald entropy: $S = A_h/(4\,\ell^2)$ with $A_h = 4\pi r_h^2$, $K(r_h) = 0$
- Area law obeyed!

3. Rotating black hole solutions in AdS

- parameters: μ - Rindler acceleration, $\tilde{a}$ - rotation parameter, mass=0

  H.-S. Liu and H. Lu, 2013
- mass=0 $\Rightarrow P^{ij} = 0$
- Nonvanishing $\gamma_{ij}^{(1)}$ is neccessary but not sufficient for a non-vanishing P_{ij}
- τ_{ij}
  - $\Rightarrow$ conserved energy $E = -\tilde{a}^2\mu/[\ell^2(1-\tilde{a}^2/\ell^2)^2]$
  - $\Rightarrow$ conserved angular momentum $J = E\ell^2/\tilde{a}$

Motivation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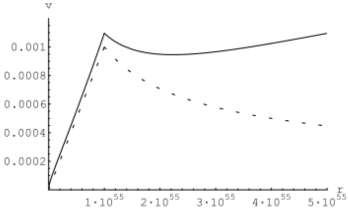
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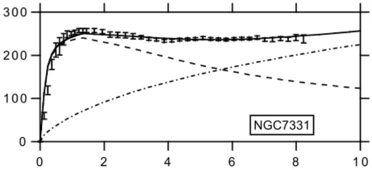
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