Holographic Entanglement Entropy for 4D Conformal Gravity

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based on 1311.4329
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Outline

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Motivations

Studying HEE in the context of CG helps us to:

- Better understand the properties of the dual field theory
- Investigate the physical connection between EG and CG
- Improve our understanding of CG holography in 4D

Conformal Gravity Vs Einstein Gravity

Action

$$I_{\text{CG}} = -\frac{\kappa}{32\pi}\int d^4x\,\sqrt{-g}\,C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$$

$C_{\mu\nu\rho\sigma}$ is the Weyl tensor.

κ is the CG coupling constant.

Main Properties

1. CG respects Weyl Symmetry $\qquad g_{\mu\nu} \to \Omega(x) g_{\mu\nu}$
2. CG is renormalizable
3. CG has ghost

Conformal Gravity Vs Einstein Gravity

EG from CG

$$\text{Solution of CG}\Big|_{\text{NBC}} = \text{Solution of EG}$$

1. Fefferman-Graham Expansion

$$ds^2 = \tfrac{L^2}{r^2}\left(dr^2 + g_{ij}(x,r)dx^i dx^j\right), \qquad g_{ij}(x,r) = g_{ij}^{(0)} + g_{ij}^{(1)} r + g_{ij}^{(2)} r^2 + \cdots$$

2. Black-Brane solution

$$ds^2 = -b(r)\, dt^2 + \tfrac{dr^2}{b(r)} + dx^2 + dy^2, \qquad b(r) = 1 - \tfrac{a}{3} r \pm \sqrt{ma} r^2 - m r^3$$

"Einstein Gravity from Conformal Gravity", J. Maldacena, arXiv: 1105.5632.

Conformal Gravity Vs Einstein Gravity

Question

Can we extract the physical quantities of EG from the corresponding one's in CG using this proposal?

Holographic Entanglement Entropy

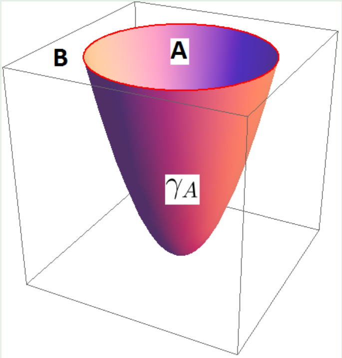
Einstein Gravity

- Action

$$I = \frac{1}{16\pi G_N}\int d^{d+1}x\sqrt{-g}(R-2\Lambda)$$

- HEE functional

$$S_{EE} = \frac{1}{4G_N}\int d^{d-1}y\sqrt{-h}$$



γ_A: Extension of the entangling region to the bulk

S. Ryu, T. Takayanagi, PRL 96, 2006.

HEE in Higher Derivative Gravity

Higher Derivative Gravity

- Action

$$I = -\int \sqrt{g} d^{d+1}x \left(\alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}\right)$$

- HEE functional

$$S_{EE} = 4\pi \int \sqrt{h} d^{d-1}y \left[2\alpha R + \beta \left(R_{ii} - \frac{K^2}{2}\right) + 2\gamma \left(R_{ijij} - (\mathrm{Tr}K)^2\right)\right]$$

$R_{ii} = R_{\mu\nu} n_i^\mu n_i^\nu, \qquad R_{ijij} = R_{\mu\nu\rho\sigma} n_i^\mu n_j^\nu n_i^\rho n_j^\sigma$

D. V. Fursaev, A. Patrushev and S. N. Solodukhin, arXiv: 1306.4000.

S_{EE} for Black-Brane in $(ml^3 \ll 1,\ al \ll 1)$ limit

	CG	EG
Strip	$\kappa L_y\left(\frac{\mathcal{S}_0}{l}+\mathcal{S}_1 ml^2\right)$	$\frac{L^2 L_y}{2G_N}\left(\frac{1}{\epsilon}+\frac{\mathcal{S}_0}{l}+\mathcal{S}_1 ml^2\right)$
Disk	$\kappa\pi\frac{m}{8}l^3$	$\frac{L^2\pi}{2G_N}\left(\frac{l}{\epsilon}-1+\frac{m}{8}l^3\right)$

Challenge

$$S_{EE}(\mathrm{CG})\big|_{\mathrm{NBC}} \neq S_{EE}(\mathrm{EG})$$

M. Alishahiha, A. F. Astaneh and M. M. M., arXiv:1311.4329.

Decomposition of the Weyl Action

$$I_{\text{Weyl}} = I_{\text{GB}} + I_{\text{dyn}}$$

- Topological Gauss-Bonnet term

$$I_{\text{GB}} = -\frac{\kappa}{32\pi} \int d^4x\sqrt{-g}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)$$

Gauss-Bonnet term is a total derivative → No contribution to the EoM

- Dynamical term

$$I_{\text{dyn}} = -\frac{\kappa}{16\pi} \int d^4x\sqrt{-g}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)$$

M. Alishahiha, A. F. Astaneh and M. M. M., arXiv:1311.4329.

HEE in 4D CG

S_{EE} for Black-Brane in $(ml^3 \ll 1,\ al \ll 1)$ limit

	Dynamical	GB
Strip	$\kappa L_y \left(\frac{1}{\epsilon} - \frac{a}{6} + \frac{\mathcal{S}_0}{l} + \mathcal{S}_1 m l^2\right)$	$\kappa L_y \left(-\frac{1}{\epsilon} + \frac{a}{6}\right)$
Disk	$\kappa\pi \left(\frac{l}{\epsilon} - 1 + \frac{ml^3}{8} - \frac{5al}{12} + \frac{\sqrt{ma}l^2}{6}\right)$	$\kappa\pi \left(-\frac{l}{\epsilon} + 1 + \frac{5al}{12} - \frac{\sqrt{ma}l^2}{6}\right)$

First observation

The Gauss-Bonnet term acts as a regulator

M. Alishahiha, A. F. Astaneh and M. M. M., arXiv:1311.4329.

HEE in 4D CG

S_{EE} for Black-Brane in $(ml^3 \ll 1,\ al \ll 1)$ limit

	Dynamical	EG
Strip	$\kappa L_y \left(\frac{1}{\epsilon} + \frac{\mathcal{S}_0}{l} + \mathcal{S}_1 ml^2 - \frac{a}{6}\right)$	$\frac{L^2 L_y}{2G_N}\left(\frac{1}{\epsilon} + \frac{\mathcal{S}_0}{l} + \mathcal{S}_1 ml^2\right)$
Disk	$\kappa\pi \left(\frac{l}{\epsilon} - 1 + \frac{ml^3}{8} - \frac{5al}{12} + \frac{\sqrt{ma}l^2}{6}\right)$	$\frac{L^2\pi}{2G_N}\left(\frac{l}{\epsilon} - 1 + \frac{ml^3}{8}\right)$

Second observation

$$\text{HEE for dynamical part}\Big|_{\text{NBC}} = \text{HEE in EG}$$

M. Alishahiha, A. F. Astaneh and M. M. M., arXiv:1311.4329.

HEE in 4D CG

Equivalence of HEE functional by imposing NBC

	$\text{Dynamical}\big|_{\text{NBC}}$	EG
Strip	$\kappa L_y \int dr \frac{r_t^2}{r^2\sqrt{b\,(r_t^4 - r^4)}}$	$\frac{L^2 L_y}{2G} \int dr \frac{r_t^2}{r^2\sqrt{b\,(r_t^4 - r^4)}}$
Disk	$\kappa L_y \int \frac{dr}{r^2}\sqrt{r^2 + \frac{r_t^2 - r^2}{b}}$	$\frac{L^2 L_y}{2G} \int \frac{dr}{r^2}\sqrt{r^2 + \frac{r_t^2 - r^2}{b}}$

Fixing CG coupling constant

$$\kappa = \frac{L^2}{2G_N}$$

M. Alishahiha, A. F. Astaneh and M. M. M., arXiv:1311.4329.

Counter-Example I: Non ALAdS Space-Time

$z = 4$ Lifshitz solution in CG

$$ds^2 = -\frac{b}{r^6}dt^2 + \frac{dr^2}{r^2 b} + dx^2 + dy^2, \qquad b = 1 + c_1 r^2 + \frac{c_1^2}{3} r^4 + c_2 r^6$$

S_{EE} for strip setting $b = 1$

Dynamical	GB
$\frac{3\kappa L_y}{2}\frac{1}{\epsilon}$	$-\frac{\kappa L_y}{2}\frac{1}{\epsilon}$

Third observation

The role of GB term depends on the asymptotic geometry.

M. Alishahiha, A. F. Astaneh and M. M. M., arXiv:1311.4329.

Counter-Example II

AdS Wave solution in CG

$$ds^2 = \frac{L^2}{r^2}\left(dr^2 - 2dx_+dx_- + k(r)\, dx_+^2 + dy^2\right), \qquad k(r) = c_0 + c_1 r + c_2 r^2 + c_3 r^3$$

Imposing NBC implies $c_1 = 0$ but c_2 remains arbitrary!

S_{EE} for strip setting $k = 1 + mr^3$

Dynamical	GB
$\kappa L_+ \left(\frac{1}{\epsilon} - \frac{2\pi\Gamma\left(\frac{3}{4}\right)^2}{\Gamma\left(\frac{1}{4}\right)^2}\,\frac{1}{\ell} + \frac{\Gamma\left(\frac{1}{4}\right)^2}{64\Gamma\left(\frac{3}{4}\right)^2}\, m\ell^2\right)$	$-\kappa\frac{L_+}{\epsilon}$

M. Alishahiha, A. F. Astaneh and M. M. M., arXiv:1311.4329.

Sum up

- HEE for 4D CG is finite and imposing NBC doesn't lead to EG results.
- The GB contribution to HEE makes it finite (for ALAdS).
- Imposing NBC on dynamical part of HEE leads to EG results.
- The CG coupling constant must be fixed as $\kappa = \frac{L^2}{2G_N}$.

Main References

- "Holographic derivation of entanglement entropy from AdS/CFT ", S. Ryu, T. Takayanagi, PRL 96, 2006.
- "Distributional Geometry of Squashed Cones", D. V. Fursaev, A. Patrushev and S. N. Solodukhin, arXiv: 1306.4000.
- "Einstein Gravity from Conformal Gravity", J. Maldacena, arXiv: 1105.5632.
- "Holographic Entanglement Entropy for 4D Conformal Gravity", M. Alishahiha, A. F. Astaneh and M. M. M., arXiv:1311.4329.

THANKS