Towards Lifshitz holography in 3-dimensional higher spin gravity

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FWF Der Wissenschaftsfonds.

Outline

Higher spin theories $D = 4$

- Higher spin $D = 4$
  - Massless bosonic fields $s \geq 2$
  - Non-interacting EOM, Lagrangian known [Fronsdal '78, Fang-Fronsdal '78]
  - Interacting?
    - Flat space $+$ Higher Spin fields $\rightarrow$ No-go theorems [Weinberg '64, Weinberg-Witten '80, ...]
  - Circumvent [Fradkin-Vasiliev '87, Vasiliev '92]
    - (A)dS background
    - Infinite tower of higher spins
    - Field equations (although proposals)

Why?

- Relation to String theory in tensionless limit
- Holography
  - Critical $O(N)$ vector model in 3 dimensions $\Longleftrightarrow$ bosonic higher spin theory in AdS_4 [Klebanov-Polyakov '02, Sezgin-Sundell '02]
  - Simple model
  - Various checks support conjecture [Giombi-Yin '09, '10, ...]

Outline

Higher spin theories $D = 3$

- Fradkin and Vasiliev ($D = 3$):
  - Infinite tower of higher spins $\rightarrow$ Spins of $s = 2, 3, ...N$
  - Field equations $\rightarrow$ Chern-Simons action
  - HS Fields have no propagating d.o.f.

Why $D = 3$?

- $D = 3 \rightarrow$ good balance between complexity and tractability
- Holography
  - 2D $\mathcal{W}_N$ minimal model CFTs in large-N 't Hooft limit $\Longleftrightarrow$ HS gravitational theories in AdS_3 [Gaberdiel-Gopakumar '11,'12]
  - CFT_2 high degree of analytical control
- HS Black holes [Gutperle-Kraus '11, Ammon et al. '11, Castro et al. '12]
  - Lifshitz black holes [Gutperle et al. '13]
- Non-AdS holography for higher spin gravity

  [Gary-Grumiller-Rashkov '12]

Outline

$sl(3,\mathbb{R}) \oplus sl(3,\mathbb{R})$ HS gravity

- Spin-3 gravity is $sl(3,\mathbb{R}) \oplus sl(3,\mathbb{R})$ Chern-Simons theory

$$\boxed{I[A,\bar{A}] = I_{CS}[A] - I_{CS}[\bar{A}]}$$

where

$$I_{CS}[A] = \frac{k}{4\pi}\int_{\mathcal{M}} \mathrm{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A) + B[A].$$

- Interpretation: Spin-3 field coupled to gravity

$$g_{\mu\nu} = \frac{1}{2}\mathrm{Tr}(e_\mu e_\nu) \qquad \phi_{\lambda\mu\nu} = \frac{1}{3!}\mathrm{Tr}(e_{(\lambda} e_\mu e_{\nu)})$$

$$e_\mu = \frac{\ell}{2}\left(A_\mu - \bar{A}_\mu\right)$$

Back to GR

- $sl(3,\mathbb{R})$-Algebra

$$
\begin{aligned}
[L_n,\, L_m] &= (n-m)\, L_{n+m} \\
[L_n,\, W_m] &= (2n-m)\, W_{n+m} \\
[W_n,\, W_m] &= \sigma\, (n-m)(2n^2+2m^2-nm-8)\, L_{n+m}
\end{aligned}
$$

- Restriction to subalgebra $sl(2,\mathbb{R})$:

$$
I_{CS}[A] - I_{CS}[\overline{A}] \propto \int \sqrt{|g|} \left(R + \frac{2}{\ell^2} \right) d^3x
$$

Outline

Lifshitz holography

- Lifshitz background is the proposed gravity dual to specific condensed matter systems [Kachru et al. '08] with phase transitions governed by fixed points which exhibit an anisotropic scale invariance between spatial and temporal scaling

$$t \to \lambda^z t \qquad \vec{x} \to \lambda \vec{x} \qquad (r \to r/\lambda) \qquad z \neq 1$$

- $z = 2$

$$ds^2_{\mathrm{Lif}^2_3} = \ell^2 \left(-e^{4\rho} dt^2 + d\rho^2 + e^{2\rho} dx^2\right)$$

Boundary conditions

- Ansatz

$$A = b^{-1}db + b^{-1}\left(\hat{a}^{(0)} + a^{(0)} + a^{(1)}\right)b$$
$$\overline{A} = bdb^{-1} + b\left(\hat{\bar{a}}^{(0)} + \bar{a}^{(0)} + \bar{a}^{(1)}\right)b^{-1}$$

with $b = e^{\rho L_0}$

- Generate background

$$\hat{a}^{(0)} = L_1 dx + \frac{4}{9} W_2 dt$$
$$\hat{\bar{a}}^{(0)} = L_{-1} dx + W_{-2} dt$$

- Leads to

$$g_{\mu\nu}\, dx^\mu dx^\nu = \ell^2 \left(-e^{4\rho} dt^2 + d\rho^2 + e^{2\rho} dx^2\right)$$
$$\phi_{\mu\nu\lambda}\, dx^\mu dx^\nu dx^\lambda = -\frac{5\ell^3}{4}\, e^{4\rho}\, dt\, (dx)^2$$

Fluctuations

- Add fluctuations (up to rescaling)

$$a^{(0)} = \big(4t\mathcal{W}(x)L_0 - \mathcal{L}(x)L_{-1}\big)\,dx$$
$$\qquad + \big(-\tfrac{16}{9}t^2\mathcal{W}(x)W_2 + \tfrac{16}{9}t\mathcal{L}(x)W_1 + \mathcal{W}(x)W_{-2}\big)\,dx$$
$$a^{(1)} = o\,(1)$$

and similar for the barred sector

  - States of metric + fluctuations one would typically not call asymptotically Lifshitz from pure metric point of view
  - t-depended

$\mathcal{W}_3 \oplus \mathcal{W}_3$ algebra

- Boundary preserving gauge transformations
- Finite, conserved Charges

$$\mathcal{Q} = \int dx \left(\mathcal{L}(x)\epsilon_L(x) + \mathcal{W}(x)\epsilon_W(x)\right)$$

$$\overline{\mathcal{Q}} = \int dx \left(\overline{\mathcal{L}}(x)\bar{\epsilon}_L(x) + \overline{\mathcal{W}}(x)\bar{\epsilon}_W(x)\right)$$

- ASA $\mathcal{W}_3 \oplus \mathcal{W}_3$ with $c = \frac{3l}{2G_N}$

$$\begin{aligned}
i\{\mathcal{L}_p, \mathcal{L}_q\} &= (p-q)\,\mathcal{L}_{p+q} + \frac{c}{12}(p^3 - p)\,\delta_{p+q,0} \\
i\{\mathcal{L}_p, \mathcal{W}_q\} &= (2p-q)\,\mathcal{W}_{p+q} \\
i\{\mathcal{W}_p, \mathcal{W}_q\} &= \chi\Big[(p-q)(2p^2+2q^2-pq-8)\,\mathcal{L}_{p+q} \\
&\quad + \frac{96}{c}(p-q)\,\Lambda_{p+q} + \frac{c}{12}p(p^2-1)(p^2-4)\,\delta_{p+q,0}\Big]
\end{aligned}$$

and identically, in the barred sector $\mathcal{L} \to \overline{\mathcal{L}}$ and $\mathcal{W} \to \overline{\mathcal{W}}$.

Outline

Motivation $D = 4$

Motivation $D = 3$

Higher spin gravity

Lifshitz holography

Conclusions

Conclusions

- Asymptotic symmetry algebra for boundary conditions which fulfill a well defined variational principle and lead to conserved charges
- $\mathcal{W}_3 \oplus \mathcal{W}_3$ algebra with a central charge
  - pure gravity [Brown-Henneaux '86]
  - asymptotically AdS in higher spin gravity [Campoleoni et al. '10, Henneaux-Rey '10]
- Perturbative spectrum equivalent to spin-3 gravity in AdS_3
  - How to distinguish from boundary perspective?

Conclusions

- Lifshitz spacetime has Killing vector fields with non-relativistic algebra

$$[\xi_H, \xi_P] = 0 \qquad [\xi_D, \xi_H] = z\xi_H \qquad [\xi_D, \xi_P] = \xi_P$$

- In HS theory it gets enhanced to full relativistic $\mathcal{W}_3 \oplus \mathcal{W}_3$
- Stay tuned for [Gary-Grumiller-Prohazka-Rey '14 (Work in progress)]

Thank you very much for your attention.