Hot and Magnetized Pions

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Introduction

QCD Phases Under Extreme Conditions

Quantum Chromodynamics

- QCD is the fundamental theory of strong interaction
- Main Properties:
  - Asymptotic freedom
  - Confinement
  - Chiral symmetry
- At different scales of energies $\longrightarrow$ Different degrees of freedom
  - High energy scales: “Quarks and gluons”
  - Low energy scales: “Weakly interacting mesons and baryons”

Question: How to explore QCD at different energy scales?

- **Experimentally:**
  - Back to Early Universe $\longrightarrow$ Big Bang
  - Relativistic Heavy Ion Experiments $\longrightarrow$ Little Bang

Question: How to explore QCD at different energy scales?

- **Experimentally:**
  - Back to Early Universe $\longrightarrow$ Big Bang
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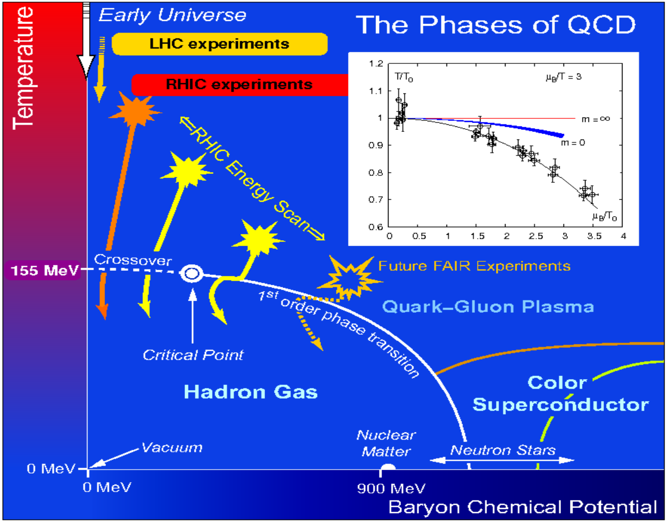
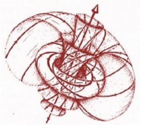
- **Theoretically:**
  - High energy scales: Perturbative QCD at $T = 0$ and $T \neq 0$
  - Low energy scales: Nonperturbative QCD at $T = 0$ and $T \neq 0$
    - Lattice QCD
    - Effective models describing the dynamics of mesons (Chiral perturbation theory, NJL like models, Relativistic Hydrodynamics, **AdS/CFT**, etc.)
    - $\cdots$

Quark Matter: QCD Phase Diagram



Strong Magnetic Fields in Nature and Lab



- The Early Universe: $10^{47} - 10^{48}$ G
  - Primordial magnetic fields from superconducting cosmic strings

    [e.g. T. Kibble, E. Witten]

- Compact Stars: $10^{14} - 10^{15}$ G (s) and $10^{18} - 10^{20}$ G (c)
  - Magnetized quark matter in neutron stars

    [e.g. A. Shabad et al., E. Ferrer et al., K. Rajagopal et al.]

- Heavy Ion Experiments: $10^{18} - 10^{19}$ G
  - Charge asymmetry fluctuations in HIC

    [e.g. D. Kharzeev et al.]

Questions

- The effect of constant B fields on QCD phases? χSB and CS

  [-] *Sh. Fayazbakhsh and N.S., **PRD 82** (2010)*

  [-] *Sh. Fayazbakhsh and N.S., **PRD 83** (2011)*

- The effect of constant B fields on meson properties at finite T?

  [-] *Sh. Fayazbakhsh, S. Sadeghian and N.S., **PRD 86** (2012)*

  [-] *Sh. Fayazbakhsh and N.S., **PRD 88** (2013)*

Effects of Constant B Field on Quark Matter

- Magnetic Catalysis **[Klimenko 1992, Miransky 1995]**
  - Bound state formation even at the weakest attractive interaction
  - Dynamical mass generation $\rightarrow$ Dynamical chiral SB

$$m_{dyn} \sim \sqrt{eB}\exp\left(-\frac{2\pi^2}{GeB}\right) \qquad \rightarrow \qquad \langle\bar{\psi}\psi\rangle \neq 0$$

- Chiral Magnetic Effect **[Kharzeev et al. 2008]**
  - Exploring the strong CP violation in HIC
- Magneto-Sono-Luminescence in HIC **[Kharzeev et al. 2013 and 2014]**
  - Photon and dilepton production in HIC

Magnetic Catalysis (MC): Applications

- Cosmology
- Condensed Matter Physics
- Particle Physics
  - Spontaneous creation of axial currents [Son et al 2005]
  - Formation of π^0 domain walls [Stephanov et al 2008]
  - Anisotropy in thermodynamic quantities [Rischke et al 2009, N.S. 2009]
  - Paraelectricity in magnetized (massless) QED [Ferrer et al 2011]
  - Effects of MC on various phases of QCD [Klimenko 1992, Miransky 1995]

    Chiral and Color-Superconductivity phase transitions

    [Shovkovy et al 2007, Fukushima et al 2008, Fayazbakhsh and N.S. 2010 and 2011]
  - Effects of MC on meson dynamics [Andersen 2012, Fayazbakhsh and N.S. 2012, 2013]

The Model

Quantum Effective Action of a **Magnetized** Two-Flavor NJL Model

Two-Flavor NJL Model in a Constant B Field

- Lagrangian density of two-flavor (gauged) NJL model

$$\mathcal{L}_{\text{NJL}} = \bar{q}(i\gamma\cdot D - m_0)q + G\big[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\big] + \cdots$$

F: $q_f^c, f = 1, 2$ and $c = r, g, b$

G: $D_\mu \equiv \partial_\mu + ieA_\mu^{\text{ext.}}$ with $A_\mu^{\text{ext}} = (0, 0, xB, 0)$ leading to $\mathbf{B} = B\mathbf{e}_3$

- Introduce bosonic fields

$$\sigma(x) \equiv -2G\,\bar{q}q, \qquad \vec{\pi}(x) \equiv -2G\,\bar{q}i\gamma_5\vec{\tau}q$$

- Semi-bosonized Lagrangian density

$$\mathcal{L}_{\text{s.b.}} = -\frac{\sigma^2 + \vec{\pi}^2}{4G} + \bar{q}\big[i\gamma\cdot D - m_0 - \sigma(x) - i\gamma_5\vec{\tau}\cdot\vec{\pi}(x)\big]q$$

Quantum Effective Action of Mesons

- Integrate out the fermions:

  Quantum effective action for mesons

$$\Gamma_{\text{eff}}[\sigma,\vec{\pi}] = \mathrm{Tr}\ln[i\gamma\cdot D - m_0 - \sigma(x) - i\gamma^5\vec{\tau}\cdot\vec{\pi}(x)]$$

- Choose a SSB direction for $\Phi(x) = (\sigma, \vec{\pi})$
  - Expand around mean fields $\Phi_0 = \langle\Phi(x)\rangle$: $\Phi(x) = \Phi_0 + \bar{\Phi}(x)$
  - Mean fields: $\langle\sigma(x)\rangle \equiv \sigma_0,\ \langle\vec{\pi}(x)\rangle = 0$

In a second order derivative expansion, we obtain:

Quantum Effective Action of Mesons in a Derivative Expansion

$$\boxed{\Gamma_{\text{eff}}[\bar{\Phi}] = -\frac{1}{2}\int d^4x\, \bar{\Phi}(x)\left(\bar{\Box} + m_\Phi^2\right)\bar{\Phi}(x) - \Omega[m]}$$

- In the effective kinetic term $[\Phi = (\sigma, \vec{\pi})]$:

$$\bar{\Phi}(x) \equiv |\mathcal{F}_\Phi^{00}|^{1/2}\Phi(x) \quad \text{and} \quad \bar{\Box} \equiv \partial_0^2 - u_\Phi^{(i)2}\partial_i^2$$

$$\text{with} \quad m_\Phi^2 \equiv \left|\frac{M_\Phi^2}{\mathcal{F}_\Phi^{00}}\right| \quad \text{and} \quad u_\Phi^{(i)2} \equiv \left|\frac{\mathcal{F}_\Phi^{ii}}{\mathcal{F}_\Phi^{00}}\right|$$

$$M_\Phi^2 \equiv -\int d^4z \frac{\delta^2\Gamma_{\text{eff}}}{\delta\Phi(0)\delta\Phi(z)}\bigg|_{\Phi_0} \quad \text{and} \quad \mathcal{F}_\Phi^{\mu\nu} \equiv -\int d^4z\, z^\mu z^\nu \frac{\delta^2\Gamma_{\text{eff}}}{\delta\Phi(0)\delta\Phi(z)}\bigg|_{\Phi_0}$$

Dispersion Law of σ and $\vec{\pi}$ mesons

- For σ meson

$$q_0^2 = u_\sigma^{(1)2} q_1^2 + u_\sigma^{(2)2} q_2^2 + u_\sigma^{(3)2} q_3^2 + m_\sigma^2$$

- For $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ mesons

$$q_0^2 = u_{\pi_\ell}^{(1)2} q_1^2 + u_{\pi_\ell}^{(2)2} q_2^2 + u_{\pi_\ell}^{(3)2} q_3^2 + m_{\pi_\ell}^2, \qquad \ell = 1, 2, 3$$

$\pi^0 \equiv \pi^3$ and $\pi^\pm \sim (\pi^1 \pm i\pi^2)$

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$\pi^0 \equiv \pi^3$ and $\pi^\pm \sim (\pi^1 \pm i\pi^2)$

- Later we will show

$$\begin{cases} u_\pi^{(1)} = u_\pi^{(2)} = u_\pi^{(3)} < 1 & \text{at} \quad T \neq 0,\ eB = 0 \\ u_\pi^{(1)} = u_\pi^{(2)} > u_\pi^{(3)} = 1 & \text{at} \quad T \neq 0,\ eB \neq 0 \end{cases}$$

Effective Potential of Mesons

$$\boxed{\Gamma_{\text{eff}}[\bar{\Phi}] = -\frac{1}{2}\int d^4x\, \bar{\Phi}(x)\left(\bar{\Box} + m_\Phi^2\right)\bar{\Phi}(x) - \Omega[m]}$$

- In the effective potential $\Omega(m)$, m is the constituent quark mass

$$m = m_0 + \sigma_0$$

- At finite (T, μ, eB): $\Omega(m) = \Omega(m; T, \mu, eB)$ is the thermodynamic potential leading to the phase diagram of our two-flavor NJL model

Results (I)

Phases of Magnetized Quark Matter and Meson Properties in an External B Field

[1] *Sh. Fayazbakhsh and N.S.,* ***PRD 82*** *(2010)*

[2] *Sh. Fayazbakhsh and N.S.,* ***PRD 83*** *(2011)*

[3] *Sh. Fayazbakhsh, S. Sadeghian and N.S.,* ***PRD 86*** *(2012)*

Two-Flavor NJL Model: Phase Diagram

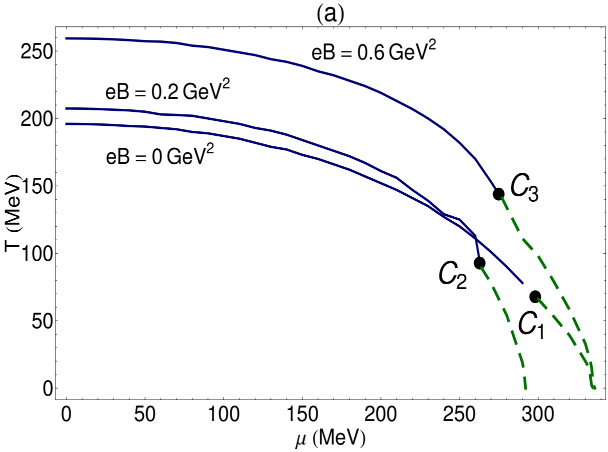
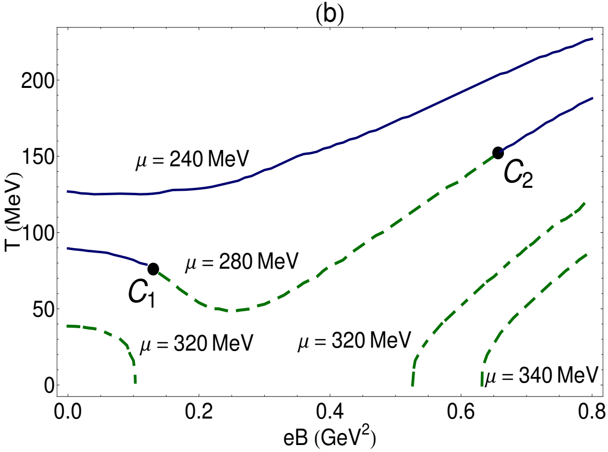


Figure: Phase portrait of a two-flavor magnetized NJL model [2,3]

Two-Flavor NJL Model: Phase Diagrams



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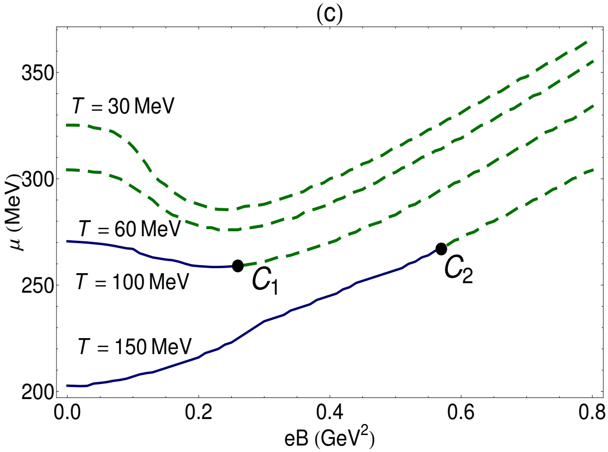

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Conclusion (I)

Effects of Constant B Field on QCD Phase Diagram

1. Shift of critical T and μ

   - NJL model [Shovkovy et al., and Ebert et al. (2006)]

$$T_c = m_{dyn}(T=0) \sim \sqrt{eB}\exp\left(-\frac{2\pi^2}{GeB}\right)$$

   - (Inverse) Magnetic catalysis

2. Changing the type of the phase transition from 2nd to 1st order

   - Ordinary $U(1)$ superconductivity [Well-known result]
   - Electroweak phase transition [Elmfors et al. 1998, Bordag et al. 2000, N.S. et al 2008]
   - Chiral phase transition [Sh. Fayazbakhsh and N.S. 2010 and 2011]
   - Color-superconductivity [Sh. Fayazbakhsh and N.S. 2010 and 2011]

     In a 2-flavor magnetized NJL model: CS to normal transition always 2nd order [perhaps a model dependent result]

Two-Flavor NJL Model: Meson Masses

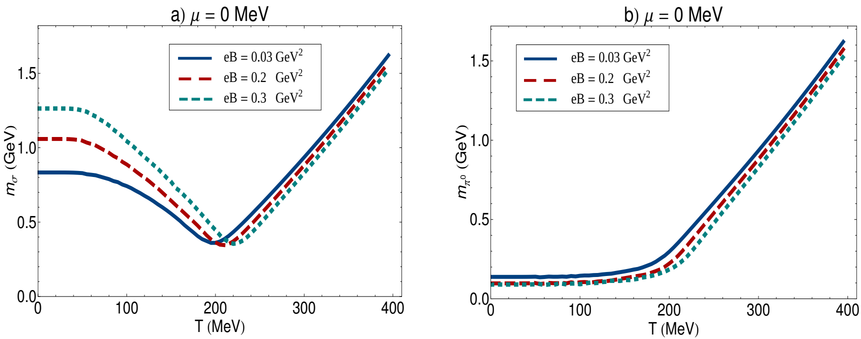

Figure: T dependence of m_σ and m_{π^0} [3]

Two-Flavor NJL Model: Directional Refraction Indices

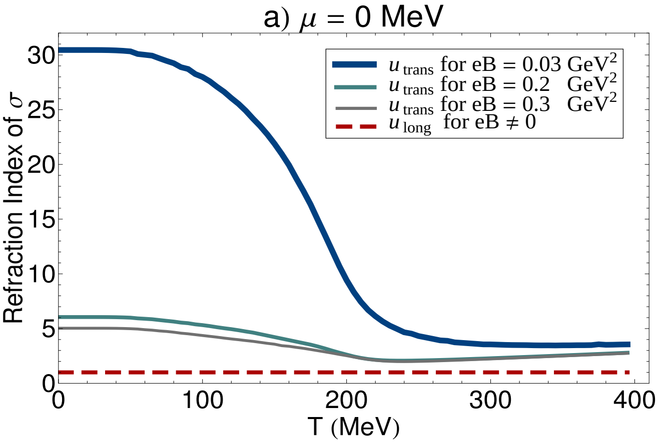

Figure: T dependence of directional refraction indices of σ meson [3]

Two-Flavor NJL Model: Directional Refraction Indices

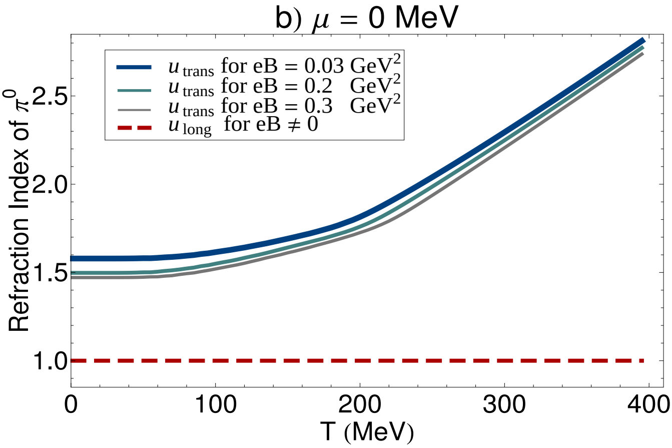

Figure: T dependence of refraction indices of π^0 meson [3]

Conclusion (I)

Effects on Meson Dynamics

- T dependence of meson masses for various B
- T dependence of directional refraction indices for various B
  - Anisotropy in $u_{\pi^0}^{(i)}, i = 1, 2, 3$: $u_{\pi^0}^{(1)} = u_{\pi^0}^{(2)} \neq u_{\pi^0}^{(3)}$
  - It turns out: $u_{\pi^0}^{(3)} = 1$ but $u_{\pi^0}^{(1)} = u_{\pi^0}^{(2)} > 1$
    (Superluminal velocities ...???)

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Effects on Meson Dynamics

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Note:

- External B fields $\Rightarrow$ Superluminal velocities [J. Alexandre, J. Ellis and N. E. Mavromatos (2012)]

Questions

$$\boxed{u_\pi^{(i)}, i = 1, 2, 3 \leftrightarrow f_\pi}$$

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Modify:

- Low energy QCD theorems at finite T and eB
- PCAC relation at finite T and eB

Decay Constant of Hot and Magnetized π^0

[4] *Sh. Fayazbakhsh and N.S.,* ***PRD 88*** *(2013)*

- **Zero T:** NJL Model **[Buballa et al. (2004)]**

PCAC Relation and Pion Decay Constant at $(T, eB) = 0$

Reminder: PCAC Relation

- Global $SU_A(2)$ transformation

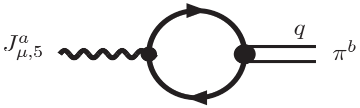
$$q(x) \to e^{i\gamma_5 \vec{\alpha}_5 \cdot \vec{\tau}} q(x) \qquad \text{and} \qquad \bar{q}(x) \to e^{i\gamma_5 \vec{\alpha}_5 \cdot \vec{\tau}} \bar{q}(x)$$

- In the chiral limit $m_0 \to 0$ $\qquad \delta_{\vec{\alpha}_5} \mathcal{L}_{\text{QCD}} = 0$

- Conserved Noether current

$$\mathcal{J}^a_{5,\mu} = \bar{q}\gamma_\mu \gamma_5 \tau^a q \qquad \text{with} \qquad \partial^\mu \mathcal{J}^a_{5,\mu} = 0, \quad a = 1, 2, 3$$

PCAC Relation and Pion Decay Constant at $(T, eB) = 0$



- PCAC relation $\qquad \langle 0|\mathcal{J}^a_{5,\mu}(0)|\pi^b(q)\rangle = f_\pi q_\mu \delta^{ab}$
  - This ansatz is compatible with Lorentz invariance
  - Pions are Goldstone bosons of SB of chiral invariance ($m_\pi = 0$)

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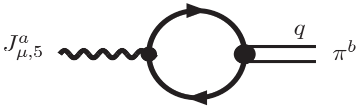

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Check: Using $\partial^\mu \mathcal{J}^a_{5,\mu} = 0$ and $q_0^2 = \mathbf{q}^2 + m_\pi^2$, we obtain:

$$\langle 0|\overbrace{\partial^\mu \mathcal{J}^a_{5,\mu}(0)}^{=0}|\pi^b(q)\rangle = f_\pi \underbrace{q^2}_{=m_\pi^2}\delta^{ab}$$

and therefore, $m_\pi^2 = 0$ $\quad\blacksquare$

Low Energy Theorems of QCD at $(T, eB) = 0$

Combinig the PCAC relation: $\langle 0|\mathcal{J}^a_{5,\mu}(0)|\pi^b(q)\rangle = f_\pi q_\mu \delta^{ab}$

and the BS relation: $-\dfrac{g^2_{\pi qq}}{q^2 - m^2_\pi} = \dfrac{2G}{1 - 2G\Pi_\pi(q^2)}$

Low Energy Theorems of QCD at $(T, eB) = 0$

We obtain low enegy theorems of QCD

1. The Goldberger-Treiman (GT) relation

$$\boxed{g_{\pi qq} f_\pi = m + \mathcal{O}(m_0^2)}$$

with the quark-meson coupling $\quad g_{\pi qq}^{-2} = \left.\frac{d\Pi_\pi}{dq^2}\right|_{q^2=m_\pi^2}$

with constituent quark mass $\quad m = m_0 + \sigma_0$

2. The Gell-Mann-Oakes-Renner (GOR) relation

$$\boxed{m_\pi^2 f_\pi^2 = \frac{m_0 \sigma_0}{2G} + \mathcal{O}(m_0^2)}$$

Questions

- Modifications: PCAC and low energy GT and GOR theorems at finite T and eB, that explicitly break the Lorentz invariance
  - **Zero T:** NJL Model [Buballa et al. (2004)]
  - **Finite T:** Massless QCD [Pisarski (1996)]
  - **Finite B:** Chiral perturbation theory [Agasian (2001)]
  - **Finite T and B:** NJL Model [Fayazbakhsh and N.S. (2013)]

Modified PCAC Relation at $(T, eB) \neq 0$

- **Keypoint:** Modified pion dispersion relation

$$\boxed{q_0^2 = u_\pi^{(i)2} q_i^2 + m_\pi^2}$$

arising from

$$\Gamma_{\text{eff}}[\bar{\Phi}] = -\frac{1}{2}\int d^4x\, \bar{\Phi}(x)\left(\bar{\Box} + m_\Phi^2\right)\bar{\Phi}(x) - \Omega(m)$$

with

$$\bar{\Box} + m_\Phi^2 \equiv \partial_0^2 - u_\Phi^{(i)2}\partial_i^2 + m_\Phi^2$$

Modified PCAC Relation at $(T, eB) \neq 0$

- Define: $$u_\pi^{(\mu)} = (1, u_\pi^{(i)})$$
- *Modified PCAC relation at $(T, eB) \neq 0$*

$$\boxed{\langle 0|\mathcal{J}_{5,\mu}^{a}(0)|\pi^b(q)\rangle = f_b u_\pi^{(\mu)2} q_\mu \delta^{ab}, \qquad \forall\mu}$$

This is correct, because using $\partial^\mu \mathcal{J}_{5,\mu}^{a} = 0$ and $q_0^2 - u_\pi^{(i)2} q_i^2 = m_\pi^2$

$$0 = \langle 0|\partial^\mu \mathcal{J}_{5,\mu}^{a}(0)|\pi^b(q)\rangle = f_b m_\pi^2 \delta^{ab} \Longrightarrow m_\pi^2 = 0 \qquad \blacksquare$$

Combining

- *Modified PCAC relation at* $(T, eB) \neq 0$

$$\langle 0|\mathcal{J}^{a}_{5,\mu}(0)|\pi^{b}(q)\rangle = f_b u_{\pi}^{(\mu)2} q_\mu \delta^{ab}$$

- *Modified Bethe-Salpeter relation at* $(T, eB) \neq 0$

Keeping in mind: $[\bar{\pi}_a = |\mathcal{F}^{00}_{aa}|^{1/2}\pi_a]$

$$-\frac{g^2_{\pi_a qq}|\mathcal{F}^{00}_{aa}|^{-1}}{q_0^2 - u_{\pi_a}^{(i)2} q_i^2 - m_{\pi_a}^2} = \frac{2G}{1 - 2G\Pi_{\pi_a}(q)}$$

Modified Low Energy Theorems of QCD at $(T, eB) \neq 0$

we obtain

1. The modified Goldberger-Treiman (GT) relation

$$\boxed{f_\pi^{(\mu)} g_{qq\pi}^{(\mu)} = m + \mathcal{O}(m_0^2)}$$

2. The modified Gell-Mann-Oakes-Renner (GOR) relation

$$\boxed{m_\pi^2 f_\pi^{(\mu)2} = u_\pi^{(\mu)2} \frac{m_0 \sigma_0}{2G} + \mathcal{O}(m_0^2)}$$

Definitions:

- *Directional decay constant of pions:*

$$f_\pi^{(\mu)} \equiv f_a |\mathcal{F}_{aa}^{\mu\mu}|^{1/2}$$

- *Directional quark-pion coupling:*

$$(g_{qq\pi}^{(\mu)})^{-2} \equiv g^{\mu\mu} \frac{d\Pi_\pi(q)}{dq_\mu^2}\bigg|_{\tilde{q}_\pi=(m_\pi,\mathbf{0})} = g_{qq\pi}^{-2} |\mathcal{F}_{aa}^{\mu\mu}|$$

Results (II)

Sh. Fayazbakhsh and N.S., ***PRD 88*** *(2013)*

Directional Quark-Pion Coupling

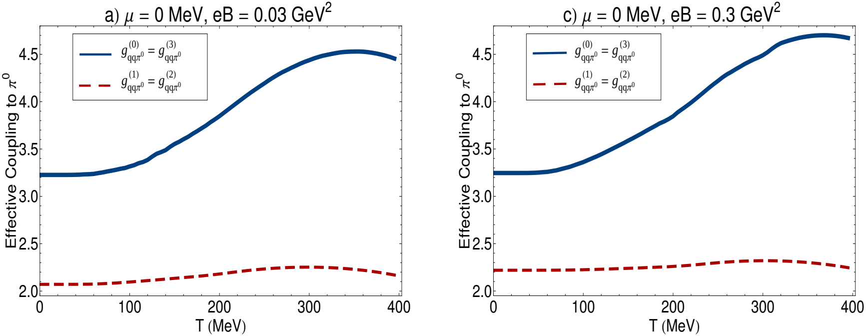

Figure: T dependence of longitudinal and transverse $g^{(\mu)}_{qq\pi^0}$

$$g^{\|}_{qq\pi} > g^{\perp}_{qq\pi} \qquad \text{related to} \qquad u_{\|} < u_{\perp}$$

Directional weak decay constant

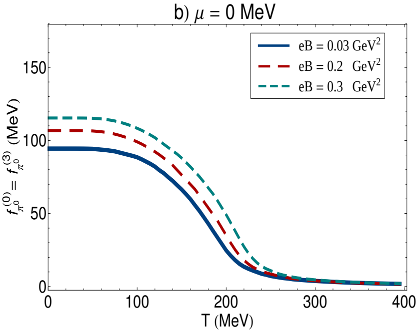

Figure: T dependence of longitudinal and transverse $f_{\pi^0}^{(\mu)}$

$f_\pi^{\|} < f_\pi^{\perp}$ related to $u_{\|} < u_{\perp}$

In Summary

$$\boxed{u_\pi^{(i)}, i = 1, 2, 3 \leftrightarrow f_\pi}$$

We have shown, analytically and numerically, that the results for the refraction indices $u_\pi^i, i = 1, 2, 3$ at finite T and/or B are compatible with the modified low energy theorems of QCD [Remember: For $T \neq 0$ and $B = 0$, $u_\pi^{(1)} = u_\pi^{(2)} = u_\pi^{(3)} < 1$ and for $T \neq 0$ and $B \neq 0$, $u_\pi^{(1)} = u_\pi^{(2)} > u_\pi^{(3)} = 1$].