Chiral Alfven Waves in an Anomalous Charged Fluid

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IPM

Thermodynamics:

State of equ.

$$T^{\mu\nu}_{\mathrm{RF}} = \begin{pmatrix} \epsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

$$J^{\mu}_{\mathrm{RF}} = (n, 0, 0, 0)$$

General Lorentz frame:

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + p)u^{\mu}u^{\nu} + p\eta^{\mu\nu} \\ J^{\mu} &= nu^{\mu} \end{aligned}$$

Out of Equilibrium

$T^{\mu\nu}(x) \qquad J^{\mu}(x)$

Equations of motion:

$$\partial_\mu T^{\mu\nu}(x) = 0$$
$$\partial_\mu J^{\mu}(x) = 0$$

In presence of an electromagnetic field:

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu$$
$$\partial_\mu J^{\mu} = 0$$

Equations Variables

$$4 + 1 \neq 10 + 4$$

Hydrodynamics: Local thermal equilibrium

Idea of Hydrodynamics:

$$T(x),\ \mu(x),\ u^\mu(x)$$

Constitutive relations in **LTE**:

$$T^{\mu\nu}(x) = (\varepsilon(x) + P(x))u^\mu(x)u^\nu(x) + P(x)\eta^{\mu\nu}$$
$$J^\mu(x) = n(x)u^\mu(x)$$

EoM:

$$\partial_\mu T^{\mu\nu}(x) = 0$$
$$\partial_\mu J^\mu(x) = 0$$

Out of local equilibrium:

Derivative expansion:

$$T^{\mu\nu}(x) = T^{\mu\nu}_{(0)}(x) + T^{\mu\nu}_{(1)}(x) + \ldots$$
$$J^{\mu}(x) = J^{\mu}_{(0)}(x) + J^{\mu}_{(1)}(x) + \ldots$$

First order hydrodynamics:

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + p)u^{\mu}u^{\nu} + p\eta^{\mu\nu} + \tau^{\mu\nu} \\ J^{\mu} &= nu^{\mu} + \nu^{\mu} \end{aligned}$$

First order derivative corrections:

Landau-Lifshitz frame:

$$\tau^{\mu\nu} = -\eta P^{\mu\alpha} P^{\nu\beta} \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha\right) - \left(\zeta - \frac{2}{3}\eta\right) P^{\mu\nu} \partial . u$$

$$\nu^{\mu} = -\sigma T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T}\right) + \sigma E^{\mu}$$

$$P^{\mu\nu} = u^\mu u^\nu + \eta^{\mu\nu}$$

Transport coefficients:

$$\eta \quad \zeta \quad \sigma$$

Constraints on Hydrodynamics

Entropy current: $S^{\mu} = s\, u^{\mu}$

Second Law of thermodynamics: $\partial_{\mu} S^{\mu} \geq 0$

1) Local equ.: $\partial_{\mu} S^{\mu} = 0$

2) Out of local equ.: $\eta \geqslant 0\,, \quad \zeta \geqslant 0\,, \quad \sigma \geqslant 0$

Parallel developments

Fluid-Gravity duality:

Fluid dynamical flow in a 4-dim conformal sys

is mapped on to

Long wave-length perturbations of Einstein eqs in an asymptotically AdS 5-dim space-time

Dictionary:

$$S_5 = \int R \qquad \equiv \qquad \text{uncharged free fluid}$$

$$S_5 = \int (R + F^2) \qquad \equiv \qquad \text{free charged fluid}$$

$$S_5 = \int (R + F^2 + \kappa F \tilde{F}) \qquad \equiv \qquad \text{free charged fluid with } \textbf{vorticity}$$

Vorticity term:

Nabamita Banerjee[a], Jyotirmoy Bhattacharya[b], Sayantani Bhattacharyya[b], Suvankar Dutta[a],
R. Loganayagam[b], and P. Surówka[c,d].
arXiv:0809.2596

Parity violating term

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$$

Adding to the current:

$$J^\mu = nu^\mu - \sigma T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu + \xi\omega^\mu$$

Second law:

$$\partial_\mu S^\mu = \sigma(\ )^2 + \eta(\ )^2 + \zeta(\ )^2 + \xi(\ )$$

$$\downarrow$$

$$+ \text{ or } -$$

Non-dissipative character

Dam T. Son[1] and Piotr Surówka[2]
Phys.Rev.Lett. 103 (2009) 191601

Parity violating terms in hydrodynamics

are related to

Anomalies

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$
$$\partial_\mu J^\mu = \mathcal{C} E_\mu B^\mu$$

$$\nu^\mu = -\sigma T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T}\right) + \sigma E^\mu + \xi \omega^\mu + \xi_B B^\mu$$

Anomalous transport coefficients:

$$\xi = \mathcal{C}\mu^2 \left(1 - \frac{2}{3}\frac{\bar{n}\mu}{\bar{\epsilon} + \bar{p}}\right) + \mathcal{D}T^2 \left(1 - \frac{2\bar{n}\mu}{\bar{\epsilon} + \bar{p}}\right)$$
$$\xi_B = \mathcal{C}\mu \left(1 - \frac{1}{2}\frac{\bar{n}\mu}{\bar{\epsilon} + \bar{p}}\right) - \frac{\mathcal{D}}{2}\frac{\bar{n}T^2}{\bar{\epsilon} + \bar{p}}$$

Hydrodynamic perturbations

State of equilibrium:

$$u^\mu = (1,0,0,0),\ T = \text{Const.},\ \mu = \text{Const.},\ \boldsymbol{B} = \boldsymbol{0}$$

Linearized equations:

$$\partial_t \delta\epsilon + ik^j \pi_j = 0$$

$$\partial_t \pi_i + ik_i v_s^2 \delta\epsilon + \mathcal{M}_{ij}\pi_j = -iDF_{im}k^m n + F^{im}\left(\frac{\sigma}{\bar{w}}F_{mj} + i\frac{\xi}{2\bar{w}}\epsilon_{mlj}k^l\right)\pi^j$$

$$\partial_t n + \left(k^2 D - \frac{i}{2}\left(\frac{\partial \xi_B}{\partial n}\right)_\epsilon \epsilon^{ijm} F_{ij} k_m\right) n + \frac{i\sigma}{\bar{w}} k_j F^{jk} \pi_k = 0$$

Non-dissipative fluid at B=0

Linearized EoM:

$$\partial_t \delta\epsilon + ik_j \pi^j = 0$$
$$\partial_t \pi^j + ik_j v_s^2 \delta\epsilon = 0$$
$$\partial_t n = 0$$

Hydrodynamic modes:

Ordinary sound waves $\omega_{1,2}(\mathbf{k}) = \pm v_s k$

Dissipative fluid at B=0

Linearized EoM:

$$\partial_t n + \mathbf{k}^2 D n = 0$$
$$\partial_t \pi^j + i k_j v_s^2 \delta\epsilon - \mathcal{M}^{ij}\pi_i = 0$$
$$\partial_t \delta\epsilon + i k_j \pi^j = 0$$

Hydrodynamic modes:

Hydrodynamic modes at $\mathbf{B} = 0$
$\omega_{1,2}(\mathbf{k}) = \pm v_s k - \frac{i}{2}\mathbf{k}^2\gamma_s$
$\omega_3(\mathbf{k}) = -iD\mathbf{k}^2$,
$\omega_4(\mathbf{k}) = -i\gamma_\eta \mathbf{k}^2$

Dissipative fluid in presence of B

Navid Abbasi[1] and Ali Davody[1]
1508.06879

Linearized EoM:

$$\partial_t n + \mathbf{k}^2 D n + \frac{i\sigma}{\bar{w}} k_j F^{jk} \pi_k = 0$$

$$\partial_t \pi^j + i k_j v_s^2 \delta\epsilon - \mathcal{M}^{ij} \pi_i = i D F^{jm} k_m n + \frac{\sigma}{\bar{w}} F^{jk} F_{km} \pi^m$$

$$\partial_t \delta\epsilon + i k_j \pi^j = 0$$

Hydrodynamic modes:

Hydrodynamic modes in presence of magnetic field
$\omega_{1,2}(\mathbf{k}) = \pm v_s k - \frac{i}{2}\left(\mathbf{k}^2 \gamma_s + \frac{\sigma}{\bar{w}} \mathrm{B}^2 \sin^2\theta\right)$
$\omega_{3,4}(\mathbf{k}) = -\frac{i}{2}\left(\mathbf{k}^2(D+\gamma_\eta) + \frac{\sigma}{\bar{w}} \mathrm{B}^2 \pm \sqrt{(\mathbf{k}^2(D-\gamma_\eta) - \frac{\sigma}{\bar{w}} \mathrm{B}^2)^2 + \frac{4D\sigma}{\bar{w}} \mathrm{B}^2 \mathbf{k}^2 \sin^2\theta}\right)$
$\omega_5(\mathbf{k}) = -i\left(\mathbf{k}^2 \gamma_\eta + \frac{\sigma}{\bar{w}} \mathrm{B}^2 \cos^2\theta\right)$

Chiral Fluid

Navid Abbasi,[1] Ali Davody,[1] and Z. Rezaei[2]
arXiv:1509.08878

Type of mode	Dispersion relation	$\sigma=\eta=\zeta=0$
sound	$\omega_{1,2}(\boldsymbol{k}) = \pm v_s k - \frac{i}{2}\left(k^2\gamma_s + \frac{\sigma}{\bar{w}}\boldsymbol{B}^2\sin^2\theta\right)$	$\omega^{\rm nd}_{1,2}(\boldsymbol{k})$
Alfvén Type-M	$\omega_3(\boldsymbol{k}) = -\frac{\mathcal{D}}{2}\frac{T^2}{\bar{w}}\boldsymbol{B}.\boldsymbol{k} - i\left(k^2\gamma_\eta + \frac{\sigma}{\bar{w}}\boldsymbol{B}^2\cos^2\theta\right)$	$\omega^{\rm nd}_3(\boldsymbol{k})$
Alfvén mixed M-D	$\omega_{4,5}(\boldsymbol{k}) = \left(\frac{\mathcal{C}}{2\chi} - \frac{\mathcal{D}}{2}\frac{T^2}{\bar{w}}\right)\boldsymbol{B}.\boldsymbol{k} - \frac{i}{2}\left(k^2(D+\gamma_\eta) + \frac{\sigma}{\bar{w}}\boldsymbol{B}^2\right)$	$\omega^{\rm nd}_3(\boldsymbol{k})$
	$\pm\frac{1}{2}\sqrt{\left(ik^2(D-\gamma_\eta) - i\frac{\sigma}{\bar{w}}\boldsymbol{B}^2 - \frac{\mathcal{C}}{\chi}\boldsymbol{B}.\boldsymbol{k}\right)^2 - \frac{4D\sigma}{\bar{w}}\boldsymbol{B}^2 k^2\sin^2\theta}$	$\omega^{\rm nd}_4(\boldsymbol{k})$

Naoki Yamamoto
Phys.Rev.Lett.115:141601,2015

Type of mode	Dispersion relation
sound	$\omega^{\rm nd}_{1,2}(\boldsymbol{k}) = \pm v_s k$
Alfvén Type-M	$\omega^{\rm nd}_3(\boldsymbol{k}) = -\frac{\mathcal{D}}{2}\frac{T^2}{\bar{w}}\boldsymbol{B}.\boldsymbol{k}$
Alfvén Type-D	$\omega^{\rm nd}_4(\boldsymbol{k}) = \left(\frac{\mathcal{C}}{\chi} - \frac{\mathcal{D}}{2}\frac{T^2}{\bar{w}}\right)\boldsymbol{B}.\boldsymbol{k}$

Outlook

1) Adding another U(1) conserved current: **CMW**

K. Fukushima, D. E. Kharzeev, and H. J. Warringa
Phys. Rev. D **78**, 074033 (2008).

2) Rotating chiral fluid: **Chiral heat waves**

M. N. Chernodub, arXiv:1509.01245 [hep-th].

Thank You