Revisiting the AdS_3/CFT_2 correspondence

Farhang Loran

Isfahan University of Technology

Overview

1. AdS_3
   - Geometry
   - Asymptotic symmetries
   - Black hole solutions
   - Partition Function
2. CFT_2
   - Conformal symmetry
   - Modular invariance
3. AdS/CFT correspondence
4. Holomorphic factorization
   - Proposal
   - Summary

AdS$_3$

$\mathbb{R}^{(2,2)}$

$$ds^2 = dX_0^2 + dX_1^2 - dX_2^2 - dX_3^2 \tag{1}$$

AdS$_3$ geometry

$$X_0^2 + X_1^2 - \left(X_2^2 + X_3^2\right) = \ell^2 \tag{2}$$

$$ds^2 = r^2\left(-dt^2 + d\phi^2\right) + \frac{dr^2}{r^2} \tag{3}$$

Asymptotic symmetries

$$ds^2 = r^2\,(-dt^2 + d\phi^2) + \frac{dr^2}{r^2} \tag{4}$$

$$\delta\, g_{\mu\nu} = \nabla_\mu\, \xi_\nu + \nabla_\nu\, \xi_\mu \tag{5}$$

$$\begin{array}{ccc} \delta g_{tt} = \mathcal{O}(1) & \delta g_{\phi\phi} = \mathcal{O}(1) & \delta g_{rr} = \mathcal{O}(r^{-3}) \\ \delta g_{t\phi} = \mathcal{O}(1) & \delta g_{t\,r} = \mathcal{O}(r^{-1}) & \delta g_{\phi\, r} = \mathcal{O}(r^{-1}) \end{array} \tag{6}$$

Witt algebra

$$[\xi_n, \xi_m]_{\text{Lie}} = (n-m)\,\xi_{n+m}$$

Black hole solutions

$$ds^2 = -\left(\frac{r^2}{\ell^2} - 8GM\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell^2} - 8GM} + r^2 d\phi^2 \qquad (7)$$

Mass gap

$$M \in \left\{\frac{-1}{8G}\right\} \cup \{M | M \geq 0\} \qquad (8)$$

Free energy

$$S_{EH} = \frac{1}{16\pi G}\int\left(R+\frac{2}{\ell^2}\right) - \frac{1}{8\pi G}\oint\left(K+\frac{1}{\ell}\right) \quad (9)$$

Thermal AdS, saddle point approximation

$$Z = e^{\frac{\beta\ell}{8G}} \quad (10)$$

S. Carlip and C. Teitelboim, gr-qc/9405070.

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Conformal Field Theory

$$[L_n, L_m] = (n-m)\, L_{n+m} + \frac{c}{12}\,(n^3 - n)\,\delta_{n+m,0} \qquad (11)$$

$$L_n = \frac{1}{2\pi i} \oint dz\, z^{n+1}\, T(z) \qquad (12)$$

Spectrum

$$|n\rangle = L_{-n}|0\rangle\,, \qquad n \geq 2, \tag{13}$$

$$|h\rangle = \phi_h|0\rangle\,, \qquad h > 0. \tag{14}$$

Mass gap

$$Z(\beta) = e^{\frac{c\beta}{12}} + \mathcal{O}(1). \tag{15}$$

Modular invariance

$$Z(\beta) = Z\left(\frac{4\,\pi^2}{\beta}\right) \tag{16}$$

Entropy

$$Z(\beta) \simeq \exp\left(\frac{4\,\pi^2\,c}{12\beta}\right) \tag{17}$$

$$F = \beta^{-1}\ln Z(\beta), \qquad F = U - T\,S \tag{18}$$

Cardy formula

$$S = \frac{2\,\pi^2\,c\,T}{3} \tag{19}$$

J.L. Cardy, Nucl. Phys. B. 270, (1986) 186.

AdS/CFT correspondence

- Asymptotic symmetry algebra of the AdS geometry is the Virasoro algebra.
- Thermal AdS looks like the CFT vacuum state.
- BH-entropy is given by the Cardy formula.

central charge

$$c = \frac{3\ell}{2G} \gg 1 \tag{20}$$

J.D. Brown, M. Henneaux, Commun. Math. Phys. 104, (1986) 207.

Pure Gravity

$$S_{AdS} = I_+ + I_-, \qquad I_\pm \sim \int \left(A_\pm \wedge dA_\pm + \frac{2}{3} A_\pm^3 \right) \tag{21}$$

Holomorphic factorization

$$Z_{CFT} = |Z(\beta)|^2 , \tag{22}$$

$$Z(\beta) = f\left(J(q)\right)_{q=\exp(-\beta)}\,, \quad J(q) = q^{-1} + 196884q + \cdots \tag{23}$$

central charge

$$c \in 24\mathbb{N} \tag{24}$$

- $\mathcal{N} = 1$ SCFT with $c = \bar{c} = 12$. [E. Witten, hep-th/0706.3359].

$c = \bar{c} = 12$

$$J(q) = j^3, \qquad j = \sum_{i=1}^{3} x_i^2 \tag{25}$$

$$x_i = \left(\sqrt{\frac{\theta_i(q)}{\eta(q)}} \right)^8 \tag{26}$$

Non-holomorphically factorized Partition functions

• **Basis**

$$|j|^2 \, h \qquad\qquad h^3 \tag{27}$$

$$h = \sum_{i=1}^{3} |x_i|^2 \,, \qquad\qquad k = \sum_{i=1}^{3} (-1)^i x_i \, |x_i|^2 \,. \tag{28}$$

Example

$$Z^{(n)} = |j|^2 \, h + n \, |k|^2 \,, \qquad\qquad n \in \{-3, -2, -1, 0, 1\} \tag{29}$$

Summary

- For $c = 12$, there exist a *finite* set of modular invariant partition functions labeled by an integer n.