Splitting symmetry and its implications for the effective action

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Introduction and motivation

- *QFT and the Background Field Method* [Abbott 1981]
- *Functional Renormalization Group* [Wetterich 1993, Morris 1994]

Introduction and motivation

- *QFT and the Background Field Method*:

The total field is split into a background field and a fluctuation field $\phi = \varphi + \xi$ or more generally $\phi = \phi(\varphi, \xi)$

Introduction and motivation

- *QFT and the Background Field Method*: Advantages

Linear splitting $\phi = \varphi + \xi$

- No need to compute diagrams with external quantum lines
- In gauge theories, gauge invariance is maintained.

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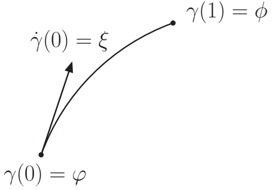
Exponential splitting $\phi = Exp_{\varphi}\xi$

$$\phi^i = \varphi^i + \xi^i - \frac{1}{2}\Gamma^i_{mn}\xi^m\xi^n + \cdots$$



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For BFM with Exp parametrization

Introduction and motivation

- *Functional Renormalization Group*

  ◇ Regards the scale (k) dependent *1PI effective action* Γ_k

  ◇ Wilson's idea realized by: $S \to S + \frac{1}{2}\xi^i (R_k)_{ij}\, \xi^j$

    ◦ $R_k(p^2)$ monotonically decreasing function of p^2

    ◦ $R_k(p^2) \to 0$ for $p^2/k^2 \to \infty$

    ◦ $R_k(p^2) \to \infty$ for $k \to \infty$

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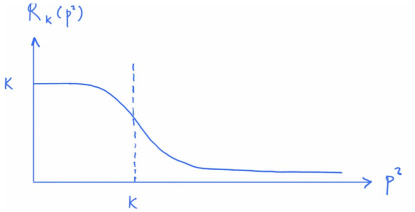

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$$e^{-W_k} = \int D\phi\, \mu[\phi]\; e^{-S[\phi] - \frac{1}{2}\,\xi \cdot R_k \cdot \xi - J \cdot \xi}$$

$$\Gamma_k = W_k - J \cdot \bar{\xi} - \frac{1}{2}\,\bar{\xi} \cdot R_k \cdot \bar{\xi}$$

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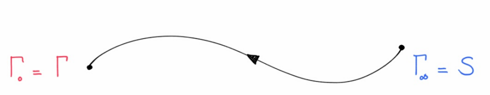

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$$\partial_t \Gamma_k = \frac{1}{2} G^{mn} (\partial_t R_k)_{nm} \qquad t = \log k \qquad G_{mn} = \Gamma_{;mn} + R_{mn}$$

Introduction and motivation

- Applying FRG to QFT's with BFM, usually not implemented correctly
- Main question: What is the most general form (background-quantum dependence) of the 1PI effective action?
- Single-field dependence of the UV action brings additional constraints

Splitting symmetry & its Ward identity

$$S[\phi] = S[\varphi + \xi]$$

$$\begin{aligned} \varphi &\to \varphi + \delta\varphi \\ \xi &\to \xi - \delta\varphi \end{aligned} \quad \Rightarrow \quad \phi \to \phi \quad \Rightarrow \quad S[\phi] \to S[\phi]$$

$$S[\phi] = S[\phi(\varphi, \xi)]$$

$$\begin{aligned} \varphi &\to \varphi + \delta\varphi \\ \xi &\to \xi + \delta\xi \end{aligned} \quad \Rightarrow \quad \phi \to \phi \quad \Rightarrow \quad S[\phi] \to S[\phi]$$

$$\delta\xi = F[\varphi, \xi]\delta\varphi$$

Splitting symmetry & its Ward identity

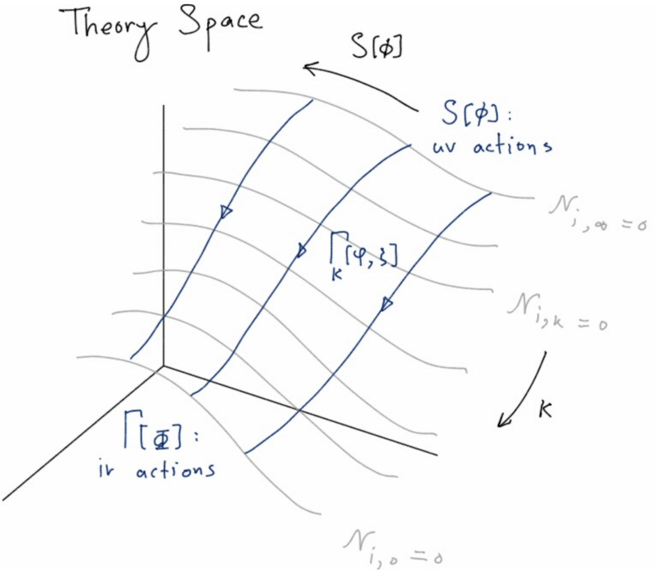
$$Q[\varphi, \xi], \qquad Q_{,i} \equiv \delta Q / \delta \varphi^i, \qquad Q_{;i} \equiv \delta Q / \delta \xi^i$$

Splitting Ward identity

$$\mathcal{N}_i \equiv \Gamma_{,i} + \Gamma_{;j} \langle \xi^j_{,i} \rangle - \frac{1}{2} G^{mn} (R_{nm})_{,i} - G^{np} R_{pm} \langle \xi^m_{,i} \rangle_{;n} = 0$$

$$R_{mn} = 0, \quad \phi^i = \varphi^i + \xi^i, \qquad \Rightarrow \qquad \Gamma_{,i} - \Gamma_{;i} = 0 \quad \Rightarrow \quad \Gamma = \Gamma[\varphi + \xi]$$

Splitting symmetry & its Ward identity



Splitting symmetry & its Ward identity

$$\partial_t \mathcal{N}_i = -\frac{1}{2}\,(G\dot{R}G)^{qp}(\mathcal{N}_i)_{;pq}$$

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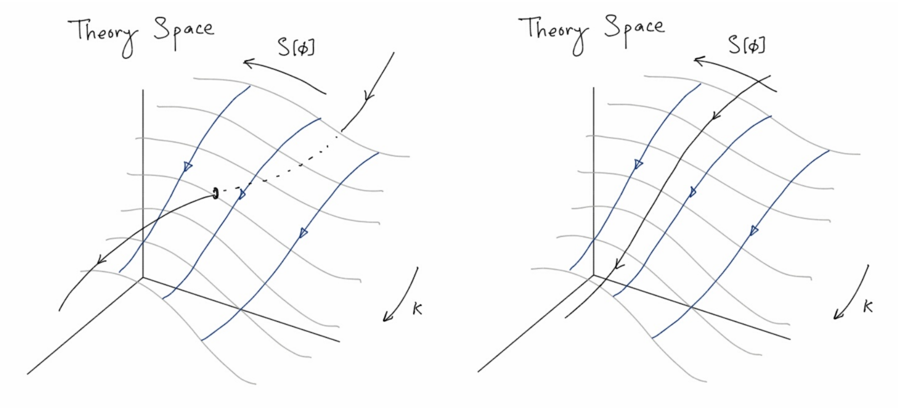


Splitting symmetry & its Ward identity

For the exponential splitting the Ward identity

$$\Gamma_{,i} + \Gamma_{;j} \langle \xi^{j}_{,i} \rangle - \frac{1}{2} G^{mn} (R_{nm})_{,i} - G^{np} R_{pm} \langle \xi^{m}_{,i} \rangle_{;n} = 0$$

is *covariant*

Splitting symmetry & its Ward identity

- But this identity is divergent!

- This is overcome by following the BRS idea:
  $S \to S + I_j\, c^i \xi^j_{,i} \equiv \Sigma$

$$c^i \Gamma_{,i} + \Gamma^j \Gamma_{;j} - \tfrac{1}{2}\, G^{mn}\, c^i (R_{nm})_{,i} - G^{np} R_{pm} \Gamma^m_{;n} = 0$$

$$\Gamma^j \equiv \delta\Gamma / \delta I_j = \langle c^i \xi^j_{,i} \rangle$$

- One can renormalize this equation:
  $\Sigma \to \Sigma_r = \Sigma -$ counter-terms.

$$c^i \Gamma_{r,i} + \Gamma^j_r \Gamma_{r,j} - \tfrac{1}{2}\, G^{mn}_r\, c^i (R_{nm})_{,i} - G^{np}_r R_{pm} \Gamma^m_{r;n} = 0$$

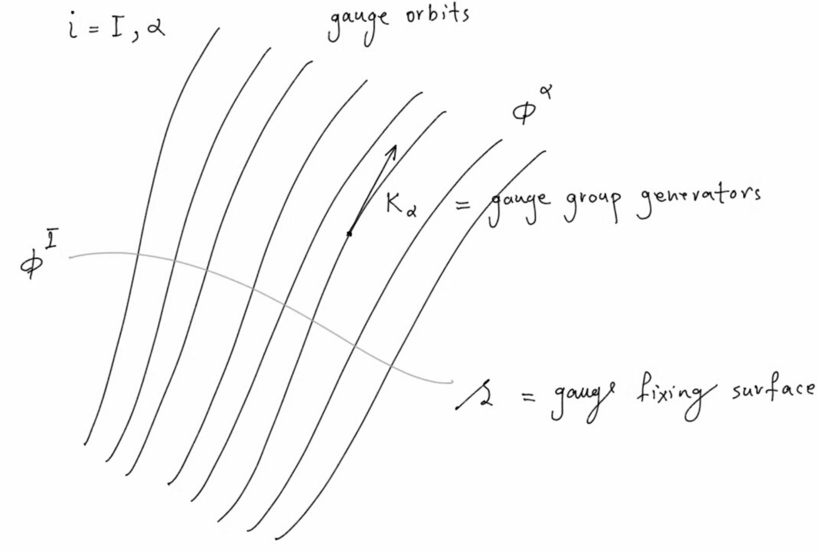
Gauge theories

We use the covariant approach with the Vilkovisky connection:

$$\nabla^V_k \, g^{\perp}_{ij} = 0$$

$$g^{\perp}_{ij} = P^m_i P^n_j g_{mn}, \quad P^i_j \equiv \delta^i_j - K^i_\alpha \gamma^{\alpha\beta} K^k_\beta g_{kj}, \quad \gamma_{\alpha\beta} = g_{ij} K^i_\alpha K^j_\beta$$

Gauge theories



Gauge theories

In the adapted coordinates, Vilkovisky connection is given by

$$(\Gamma_V)^K_{IJ} = \frac{1}{2}h^{KL}(\partial_I h_{LJ} + \partial_J h_{LI} - \partial_L h_{IJ}), \quad (\Gamma_V)^K_{\alpha j} = 0, \quad \partial_\alpha h_{IJ} = 0$$

h_{IJ} is $g^{\perp}_{ij}$ induced on $\mathscr{S}$

Gauge theories

$$Q[\varphi,\bar{\xi}] = \tilde{Q}[\varphi,\bar{\phi}] \qquad \bar{\phi} \equiv Exp_{\varphi}\bar{\xi}$$

$$\tilde{Q}_{,i} \equiv \delta\tilde{Q}/\delta\varphi^{i}, \qquad \tilde{Q}_{;i} \equiv \delta\tilde{Q}/\delta\bar{\phi}^{i}$$

- Ultraviolet action is gauge invariant

$$K^{i}_{\alpha}[\varphi]\tilde{S}_{,i} = 0, \qquad K^{i}_{\alpha}[\phi]\tilde{S}_{;i} = 0$$

- As a consequence, the 1PI effective action satisfies:

$$K^{i}_{\alpha}[\varphi](\tilde{\Gamma}_{,i} - \tfrac{1}{2}G\,\nabla^{V}_{i}R) = 0, \qquad K^{i}_{\alpha}[\phi]\tilde{\Gamma}_{;i} = 0$$

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Gauge theories

Assuming gauge invariance of the ultraviolet action:

$$K^i_\alpha[\varphi]\nabla^V_i R = 0$$

- Background gauge invariance $K^i_\alpha[\varphi]\tilde{\Gamma}_{,i} = 0$
- Gauge fixing independence

Gauge theories

$$
\begin{aligned}
d\phi^i &= d_{\parallel}\phi^i + d_{\perp}\phi^i \\
&= P^i_j \phi^i + K^i_\alpha d\epsilon^\alpha \qquad d\epsilon^\alpha = \gamma^{\alpha\beta} K^i_\beta \, g_{ij} \, d\phi^j
\end{aligned}
$$

$$
\begin{aligned}
\text{measure} &= \left(\prod_i d\phi^i\right) \sqrt{\det g_{ij}} \\
&= \left(\prod_\alpha d\epsilon^\alpha\right) \left(\prod_i d_{\perp}\phi^i\right) \sqrt{\det{}_{\perp} g^{\perp}_{ij}} \sqrt{\det \gamma_{\alpha\beta}} \\
&= \left(\prod_\alpha d\epsilon^\alpha\right) \left(\prod_i d\phi^I\right) \sqrt{\det h_{IJ}(\phi^I)} \sqrt{\det \gamma_{\alpha\beta}(\phi^I)}
\end{aligned}
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can be dropped if the integrand is gauge invariant

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&\to \left(\prod_i d\phi^I\right)\sqrt{\det h_{IJ}(\phi^I)}\sqrt{\det\gamma_{\alpha\beta}(\phi^I)}
\end{aligned}
$$

Gauge theories

$$K^i_\alpha[\varphi]R_{ij} = 0$$

$$\Gamma_{,I} - \frac{1}{2}G^{MN}(R_{NM})_{,I} + \Gamma_{;J}\langle \xi^J{}_{,I} \rangle - G^{NP}R_{PM}\langle \xi^M{}_{,I} \rangle_{;N} = 0$$

Summary and Conclusions!

- We have introduced the “Splitting Ward identity” in the presence of an infrared regulator, for general quantum-background split, and for gauge and non-gauge theories

- This is expected to prove important in FRG applications to QFT’s with the background field method.

Thank You