# Temperature in Calabi-Yau Throats

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Talk prepared for IPM String School and Workshop Feb 28- March 4, 2015

# Prelude

- In string theory, **horizons** and Hawking **temperatures** arise predominantly from **black holes** and **brane solutions** on the **background**.

- However, **rotating probe branes** admit **thermal horizons** with **temperatures** even if there is **no black hole in the bulk**. [Takayanagi, Das (2010); Russo, Townsend (2008),...].

- But this analysis has been **limited** to probes rotating in the $\underline{AdS_5 \otimes S^5}$, a noncompact throat and dual to $\underline{\mathcal{N} = 4}$ SYM.

- Our aim is to **extend** such analysis to **warped Calabi-Yau throats** contained in **compact** $\underline{\mathcal{N} = 1}$ **string solutions**.

# Calabi-Yau flux compactification of type IIB theory

The general $\mathcal{N} = 1$ string solution is the CY flux compactification of IIB theory [Giddings, Kachru, Polchinski (2002)].

- The type IIB action takes the form:

$$S_{\rm IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|g|} \left[ \mathcal{R} - \frac{|\,\partial\tau|^2}{2(\mathrm{Im}\tau)^2} - \frac{|\,G_3|^2}{12\,\mathrm{Im}\tau} - \frac{|\,\tilde{F}_5|^2}{4 \cdot 5\,!} \right] + \frac{1}{8i\kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge G_3^*}{\mathrm{Im}(\tau)} + S_{\rm loc}, \quad \text{with} \quad G_3 = F_3 - \tau H_3.$$

- The warped line element and self-dual 5-form read as

$$\begin{aligned} ds_{10}^2 &= h(y)^{1/2} \underbrace{g_{\mu\nu} dx^\mu dx^\nu}_{4D\ Mink.} + h(y)^{-1/2} (\underbrace{d\hat{r}^2 + \hat{r}^2 ds_{T^{1,1}}^2}_{6D\ CY-cone}), \\ \tilde{F}_5 &= (1 + \star_{10}) \Big[ d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \Big]. \end{aligned}$$

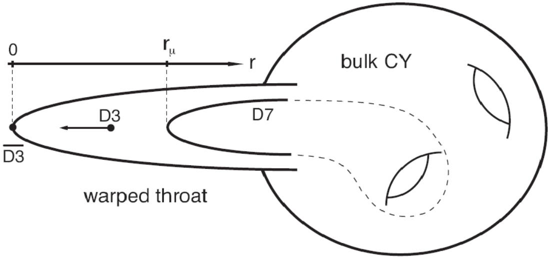


Figure : Standard flux compactification.

- We will consider a rotating probe brane in this throat background and study it's worldvolume horizon and temperature in the deep IR and far UV regions.

# Temperature in the Klebanov-Strassler Throat

## Induced metric on the worldvolume of probe D1-brane in KS

In the very deep IR region ($\eta \to 0$) [Klebanov; Strassler (2000)]:

- The warp factor is constant $h_0 = a_0(g_s M\alpha')^2\, 2^{2/3}$.
- Only the $S^3$ remains finite whereas the $S^2$ shrinks to zero.

The background metric then is:

$$
\begin{aligned}
ds_{10}^2 &\to \frac{\epsilon^{4/3}}{(2)^{1/3} a_0^{1/2} (g_s M\alpha')}(dx^2 - dt^2) \\
&\quad + (2^{-1}\, a_0^{1/2}\, 6^{-1/3})(g_s M\alpha')\left\{d\eta^2 + d\psi^2 + B(\eta) d\phi^2\right\}, \\
B(\eta) &= 1 + \eta^2/4.
\end{aligned}
$$

Also note: In the deep IR and along the above $S^3$ parametrization $F_3 = dC_2 = 0$, $C_2 =$ constant, locally.

The action of the probe D1-brane then is [K, Mosaffa (2015)]:

$$
\begin{aligned}
S_{\mathrm{D1}} &\equiv -g_s\, T_{D1} \int d^2\xi\, L, \\
L &= -\frac{\epsilon^{4/3}}{2(12)^{1/3}} \Big[ 1 + \frac{1}{2} B(\eta) (\phi')^2 - \frac{a_0 (g_s M \alpha')^2\, B(\eta)}{2(24\epsilon^4)^{1/3}} \dot{\phi}^2 \\
&\quad + \frac{1}{2} (\psi')^2 - \frac{a_0 (g_s M \alpha')^2}{2(24\epsilon^4)^{1/3}} \dot{\psi}^2 \Big].
\end{aligned}
$$

Note: We are considering slow rotations and so only the leading terms of the DBI action contribute.

The leading-order brane eqns. of motion are [K, Mosaffa (2015)]:

$$\begin{aligned} \frac{\partial}{\partial \eta}\left[B(\eta)\phi'(\eta,t)\right] &= \frac{a_0(g_s M\alpha')^2}{(24\epsilon^4)^{1/3}}\frac{\partial}{\partial t}\left[B(\eta)\dot{\phi}(\eta,t)\right], \\ \frac{\partial}{\partial \eta}\left[\psi'(\eta,t)\right] &= \frac{a_0(g_s M\alpha')^2}{(24\epsilon^4)^{1/3}}\frac{\partial}{\partial t}\left[\dot{\psi}(\eta,t)\right]. \end{aligned}$$

Now, consider solutions of the form [K, Mosaffa (2015)]:

$$\begin{aligned} \psi(\eta,t) &= \omega_1\, t + \xi_1(\eta) = \omega_1\, t + \eta + \psi_0, \\ \phi(\eta,t) &= \omega_2\, t + \xi_2(\eta) = \omega_2\, t - 2\tan^{-1}\left(\frac{\eta}{2}\right). \end{aligned}$$

Putting these solutions into the background metric, after a local coordinate transformation, we get the induced metric on the D1 [K, Mosaffa (2015)]:

$$\begin{aligned} ds^2_{ind} &= -\frac{a_0^{1/2}(g_s M\alpha')\Omega(\eta)}{2\cdot 6^{1/3}}d\tau^2 \\ &+\frac{a_0^{1/2}(g_s M\alpha')}{2\cdot 6^{1/3}B(\eta)\Omega(\eta)}\Bigg\{(1+2B(\eta))\left[\frac{a_0^{1/2}(g_s M\alpha')\Omega(\eta)}{2\cdot 6^{1/3}}\right] \\ &+B(\eta)\Delta\omega^2\Bigg\}d\eta^2, \end{aligned}$$

$$\Omega(\eta) = \frac{(24\,\epsilon^4)^{1/3}}{a_0(g_s M\alpha')^2} - \left(\omega_1^2 + B(\eta)\omega_2^2\right), \qquad \Delta\omega = \omega_2 - \omega_1.$$

To find the horizon, set in the induced metric $g^{\eta\eta} = 0$, which gives [K, Mosaffa (2015)]:

$$g^{\eta\eta}(\eta_0) = 0 \to \eta_0 = \left|\frac{2}{\omega_2}\right| \sqrt{\frac{(24\,\epsilon^4)^{1/3}}{a_0(g_s M\alpha')^2} - (\omega_1^2 + \omega_2^2)}$$

This relation tells us the following:

- First: If $\omega_1 = \omega_2 = 0$, this equation has no solutions.
- Next: If $\omega_2 \neq 0$ and $\omega_1 = 0$, then one can see that for $\omega_2 \to$ small, the horizon appears at very large values of $\eta$.
- As $\omega_2$ increases, the horizon moves towards smaller values of $\eta$ and in the limit of large $\omega_2$ it will hit $\eta \approx 0$.

This tells us that the worldvolume black hole nucleates at large values of $\eta$ with a horizon that grows by increasing $\omega$.

# Temperature in the Klebanov-Tseytlin Throat

## Induced metric on the worldvolume of probe D1-brane in KT

At large radii, the KS is well approximated by the KT solution [Klebanov, Tseytlin (2000)]. Validity rage of the UV solution:

$$\hat{r}_{\rm IR} \ll \hat{r} \ll \hat{r}_{\rm UV}.$$

Here one can show that: $\hat{r}_{\rm UV} \simeq 10^2\,\epsilon^{2/3}$ and $\hat{r}_{\rm IR} \simeq \epsilon^{2/3}$. The UV solution [Herzog,Klebanov,Ouyang]:

$$h(\hat{r}) = \frac{L^4}{\hat{r}^4}\ln(\hat{r}/\epsilon^{2/3}), \qquad L^4 = \frac{81(g_s M\alpha')^2}{8},$$

$$ds_{10}^2 = \frac{\hat{r}^2}{L^2\sqrt{\ln(\hat{r}/\epsilon^{2/3})}}(dx^2 - dt^2) + \frac{L^2\sqrt{\ln(\hat{r}/\epsilon^{2/3})}}{\hat{r}^2}d\hat{r}^2 + \frac{L^2}{\hat{r}^2}\sqrt{\ln(\hat{r}/\epsilon^{2/3})}ds^2_{T^{1,1}}.$$

Taking the same $S^3$ cycle as before, we get [K, Mosaffa (2015)]:

$$
\begin{aligned}
ds_{10}^2 &= \frac{\hat{r}^2}{L^2\sqrt{\ln(\hat{r}/\epsilon^{2/3})}}(dx^2 - dt^2) \\
&\quad + \frac{L^2\sqrt{\ln(\hat{r}/\epsilon^{2/3})}}{\hat{r}^2}\left(d\hat{r}^2 + \frac{\hat{r}^2}{6}d\phi^2 + \frac{\hat{r}^2}{9}d\psi^2\right).
\end{aligned}
$$

The action becomes [K, Mosaffa (2015)]:

$$
\begin{aligned}
S_{\mathrm{D1}} &= -g_s\, T_{D1}\int d^2\xi\, L, \\
L &= 1 + \frac{\hat{r}^2(\phi')^2}{12} - \frac{L^4}{12\,\hat{r}^2}\ln\left(\hat{r}/\epsilon^{2/3}\right)\dot{\phi}^2 \\
&\quad + \frac{\hat{r}^2(\psi')^2}{18} - \frac{L^4}{18\,\hat{r}^2}\ln\left(\hat{r}/\epsilon^{2/3}\right)\dot{\psi}^2.
\end{aligned}
$$

The equations of motion take the form [K, Mosaffa (2015)]:

$$\frac{\partial}{\partial \hat{r}}\left[\frac{\hat{r}^2\psi'(\hat{r},t)}{9}\right] = \frac{\partial}{\partial t}\left[\frac{L^4}{9\hat{r}^2}\ln\left(\hat{r}/\epsilon^{2/3}\right)\dot{\psi}(\hat{r},t)\right],$$

$$\frac{\partial}{\partial \hat{r}}\left[\frac{\eta^2\phi'(\hat{r},t)}{6}\right] = \frac{\partial}{\partial t}\left[\frac{L^4}{6\hat{r}^2}\ln\left(\hat{r}/\epsilon^{2/3}\right)\dot{\phi}(\hat{r},t)\right].$$

As before, consider solutions of the form [K, Mosaffa (2015)]:

$$\psi(\hat{r},t) = \omega_1\, t + g(\hat{r}) = \omega_1\, t - \frac{\omega_1}{\hat{r}} + \psi_0,$$

$$\phi(\hat{r},t) = \omega_2\, t + f(\hat{r}) = \omega_2\, t - \frac{\omega_2}{\hat{r}} + \phi_0.$$

Putting the above solutions into the background metric, and considering a coordinate transformation, gives the induced metric [K, Mosaffa (2015)]:

$$
\begin{aligned}
ds^2_{ind} &= -\frac{\left[\hat{r}^2 - L^4 \ln(\hat{r}/\epsilon^{2/3})\overline{\omega}^2\right]}{\sqrt{L^4 \ln\left(\hat{r}/\epsilon^{2/3}\right)}} d\tau^2 \\
&\quad + \sqrt{L^4 \ln\left(\hat{r}/\epsilon^{2/3}\right)} \left[\frac{\overline{\omega}^2 + \hat{r}^2 - L^4 \ln(\hat{r}/\epsilon^{2/3})\,\overline{\omega}^2}{\hat{r}^2(\hat{r}^2 - L^4 \ln(\hat{r}/\epsilon^{2/3})\,\overline{\omega}^2)}\right] d\hat{r}^2.
\end{aligned}
$$

To find the worldvolume horizon we set $g^{\hat{r}\hat{r}} = 0$!

## Horizon on the wolrdvolume of probe D1-brane in KT

The worldvolume horizon is described by [K, Mosaffa (2015)]:

$$g^{\hat{r}\hat{r}} = \hat{r}_H^2 - L^4\,\overline{\omega}^2\,\ln(\hat{r}_H/\epsilon^{2/3}) = 0.$$

- This equation can have at most two (real positive) zeros.
- The position and number of these zeros depends on the value of the conserved charge.
- Hence there can be two different situations, depending on the value of the conserved charge.

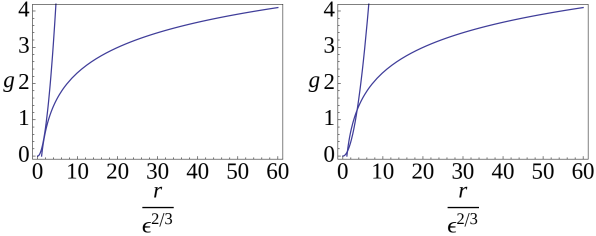


Figure : Plots of $g^{\hat{r}\hat{r}} = 0$ for $\overline{\omega}^2 = \epsilon^{4/3}/L^4$ (L), $\overline{\omega}^2 = 10\epsilon^{4/3}/L^4$ (R) [K, Mosaffa (2015)].

- These plots show $\hat{r}_H \simeq \epsilon^{2/3}$, by which the KT singularity is approached and the validity range of the UV solution is violated.

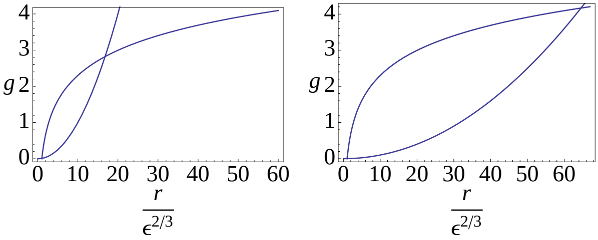


Figure : Plots of $g^{\hat{r}\hat{r}} = 0$ for $\overline{\omega}^2 = 50\epsilon^{4/3}/L^4$ (L), $\overline{\omega}^2 = 10^2\epsilon^{4/3}/L^4$ (R) [K, Mosaffa (2015)].

- These plots show $\hat{r}_H \to 10^2\epsilon^{2/3}$, by which the KT singularity is avoided and the validity range of the UV solution is maintained. Such horizons are of interest!

## Temperature on the wolrdvolume of the probe D1-brane in KT

To obtain the Hawking temperature, we Wick-rotate $\tau$ into a Euclidean time, and after a calculation we get [K, Mosaffa (2015)]:

$$T_{\rm H} = \frac{(g^{\hat{r}\hat{r}})'}{4\pi}\bigg|_{\hat{r}=\hat{r}_{\rm H}} \quad = \quad \frac{\hat{r}_{\rm H}(2\hat{r}_H^2 - L^4\,\overline{\omega}^2)}{4\pi(\overline{\omega}L)^2\sqrt{\ln\left(\hat{r}_{\rm H}/\epsilon^{2/3}\right)}}.$$

- Within the range of the UV solution $\epsilon^{2/3} \ll \hat{r} \ll 10^2\epsilon^{2/3}$ the worldvolume temperature, $T_H$, is finite and positive definite.
- Away from the mid throat region $T_H$ is more or less constant: $T_H \gtrsim L^2\,\epsilon^{2/3}$ for $\hat{r}_H \to 10^2\epsilon^{2/3}$, $T_H \lesssim L^2\,\epsilon^{2/3}$ for $\hat{r}_H \to \epsilon^{2/3}$.
- In the mid throat region the $T_H$ varies continuously with $\hat{r}_H$.

# Temperature in the Klebanov-Witten Throat

## The Klebanov-Witten solution

When $M = 0$, the KT solution joins the Klebanov-Witten (KW) solution [Klebanov, Witten (1998)]. The solution is:

$$h = \frac{L^4}{\hat{r}^4}, \quad \text{and} \quad L^4 \equiv \frac{27\pi}{4} g_s N (\alpha')^2,$$

$$ds^2 = \frac{\hat{r}^2}{L^2}(dx^2 - dt^2) + \frac{L^2}{\hat{r}^2} d\hat{r}^2 + \frac{L^2}{\hat{r}^2} ds^2_{T^{1,1}}.$$

## Induced metric on the worldvolume of probe D1-brane in KW

Taking the same $S^3$ cycle as before, we obtain the full background metric as:

$$ds_{10}^2 = \frac{\hat{r}^2}{L^2}\left(dx^2 - dt^2\right) + \frac{L^2}{\hat{r}^2}\left(d\hat{r}^2 + \frac{\hat{r}^2}{6}d\phi^2 + \frac{\hat{r}^2}{9}d\psi^2\right).$$

The action becomes [K, Mosaffa (2015)]:

$$\begin{aligned} S_{\rm D1} &= -g_s\, T_{D1}\int d^2\xi\, L, \\ L &= 1 + \frac{\hat{r}^2(\phi')^2}{12} - \frac{L^4}{12\,\hat{r}^2}\dot{\phi}^2 + \frac{\hat{r}^2(\psi')^2}{18} - \frac{L^4}{18\,\hat{r}^2}\dot{\psi}^2. \end{aligned}$$

The equations of motion take the form [K, Mosaffa (2015)]:

$$\frac{\partial}{\partial \hat{r}}\left[\frac{\hat{r}^2\psi'(\hat{r},t)}{9}\right] = \frac{\partial}{\partial t}\left[\frac{L^4}{9\hat{r}^2}\dot{\psi}(\hat{r},t)\right],$$
$$\frac{\partial}{\partial \hat{r}}\left[\frac{\eta^2\phi'(\hat{r},t)}{6}\right] = \frac{\partial}{\partial t}\left[\frac{L^4}{6\hat{r}^2}\dot{\phi}(\hat{r},t)\right].$$

As before, consider solutions of the form [K, Mosaffa (2015)]:

$$\psi(\hat{r},t) = \omega_1\, t + g(\hat{r}) = \omega_1\, t - \frac{\omega_1}{\hat{r}} + \psi_0,$$
$$\phi(\hat{r},t) = \omega_2\, t + f(\hat{r}) = \omega_2\, t - \frac{\omega_2}{\hat{r}} + \phi_0.$$

Putting the above solutions into the background metric, and considering a coordinate transformation, gives the induced metric [K, Mosaffa (2015)]:

$$ds^2_{ind} \;\; = \;\; -\frac{\left[\hat{r}^2 - L^4\,\overline{\omega}^2\right]}{\sqrt{L^4}}d\tau^2 + L^2\left[\frac{\overline{\omega}^2 + \hat{r}^2 - L^4\,\overline{\omega}^2}{\hat{r}^2(\hat{r}^2 - L^4\,\overline{\omega}^2)}\right]d\hat{r}^2.$$

As before, $\overline{\omega}^2 = \omega_1^2/9 + \omega_2^2/6$. To find the worldvolume horizon in KW, we set from this induced metric $g^{\hat{r}\hat{r}} = 0$!

## Horizon on the worldvolume of probe D1-brane in KW

The horizon in KW is described by [K, Mosaffa (2015)]:

$$g^{\hat{r}\hat{r}} = \hat{r}_H^2 - L^4\overline{\omega}^2 = \hat{r}_H^2 - L^4\left[\frac{\omega_1^2}{9} + \frac{\omega_1^2}{6}\right] = 0.$$

- This eq. has one (real positive) zero, forming a single horizon
- There is no double horizon as logarithmic warping is removed.
- It is also clear that $\hat{r}_H$ shrinks/expands linearly with $\overline{\omega}$, while suppressed by numerical prefactors $1/9, 1/6$.

# Temperature on the worldvolume of probe D1-brane in KW

The temperature on the worldvolume of probe D1-brane in KW is decribed by [K, Mosaffa (2015)]:

$$T_{\rm H} = \frac{(g^{\hat{r}\hat{r}})'}{4\pi}\bigg|_{\hat{r}=\hat{r}_{\rm H}} = \frac{\hat{r}_H}{2\pi} = \frac{L^2}{2\pi}\sqrt{\frac{\omega_1^2}{9} + \frac{\omega_2^2}{6}}.$$

- $T_H$ increases/decreases continuously with $\hat{r}_H$, similar to $T_H$ of rotating probes in $AdS_5 \otimes S^5$ throat.
- Note that $AdS_5 \otimes S^5$ extends from $r = 0$ to $r = \infty$, and so $\hat{r}_H$ and $T_H$ can increase to arbitrary large values.
- But in KW $T_H$ and $\hat{r}_H$ are constrained by the validity range of the UV solution $\epsilon^{2/3} \ll \hat{r} \ll 10^2\epsilon^{2/3}$ and therefore cannot increase to arbitrary large values; they remain always finite!

# Summary

- We found that worldvolume horizons and temperatures of expected features form at large radii, far from the bottom of the throat, where KS is approximated by KT & KW solutions.
- In both KW & KT we found worldvolume horizons with finite temperatures.
- In KT we found that the temperature is more or less constant.
- In KW we found horizons and temperatures similar to those of rotating probes in $AdS_5 \times S^5$, but relatively suppressed, and constrained by the UV/IR scales of the throat.

Thank you!