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A Geometric Approach to Phase Transitions

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Scaling Limit of Lattice Models:

- Lattice shrinks (continuum limit)
- Site variables become local fields
- Lattice models become field theories

- Lattice models at the critical point become **conformal field theories (CFT)**

Parametrized by the central charge: c

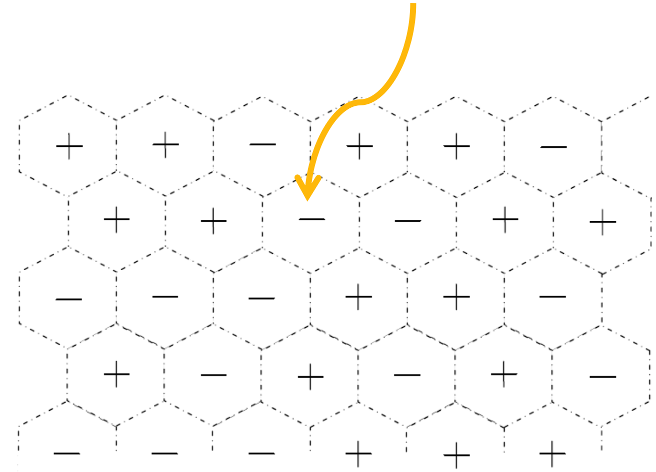
A *Geometric* way of thinking about CFT

Schramm-**L**oewner **E**volution

(**SLE**$_\kappa$)

$$c_\kappa = \frac{(8-3\kappa)(\kappa-6)}{2\kappa} = 1 - 3\,\frac{(\kappa-4)^2}{2\kappa}$$

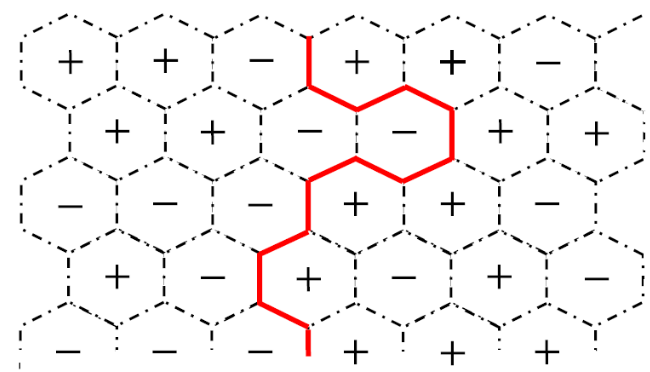
S_i *:Lattice variable*



e.g., 2D Ising model in the scaling limit

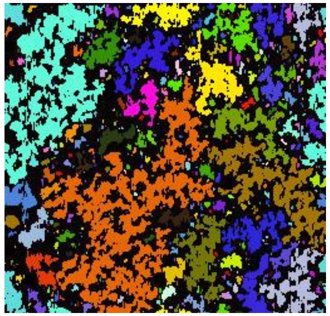
$c = 1/2$

$\kappa = 3$



SLE$_\kappa$

Two important notions in theory of **critical phenomena**:



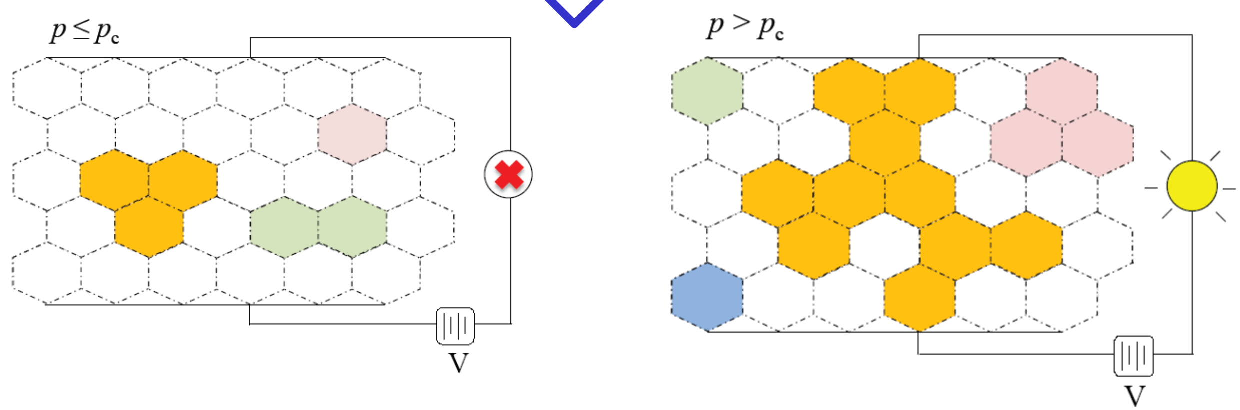
*A.A.S., et.al., PRL **100**, 044504 (2008)*

- scale invariance at the critical point
- critical exponents

A *Geometric* translation

The properties of a **fractal**:
- self-similarity
- a fractal (*non integer*) dimension
  different from the topological dimension

The Percolation theory

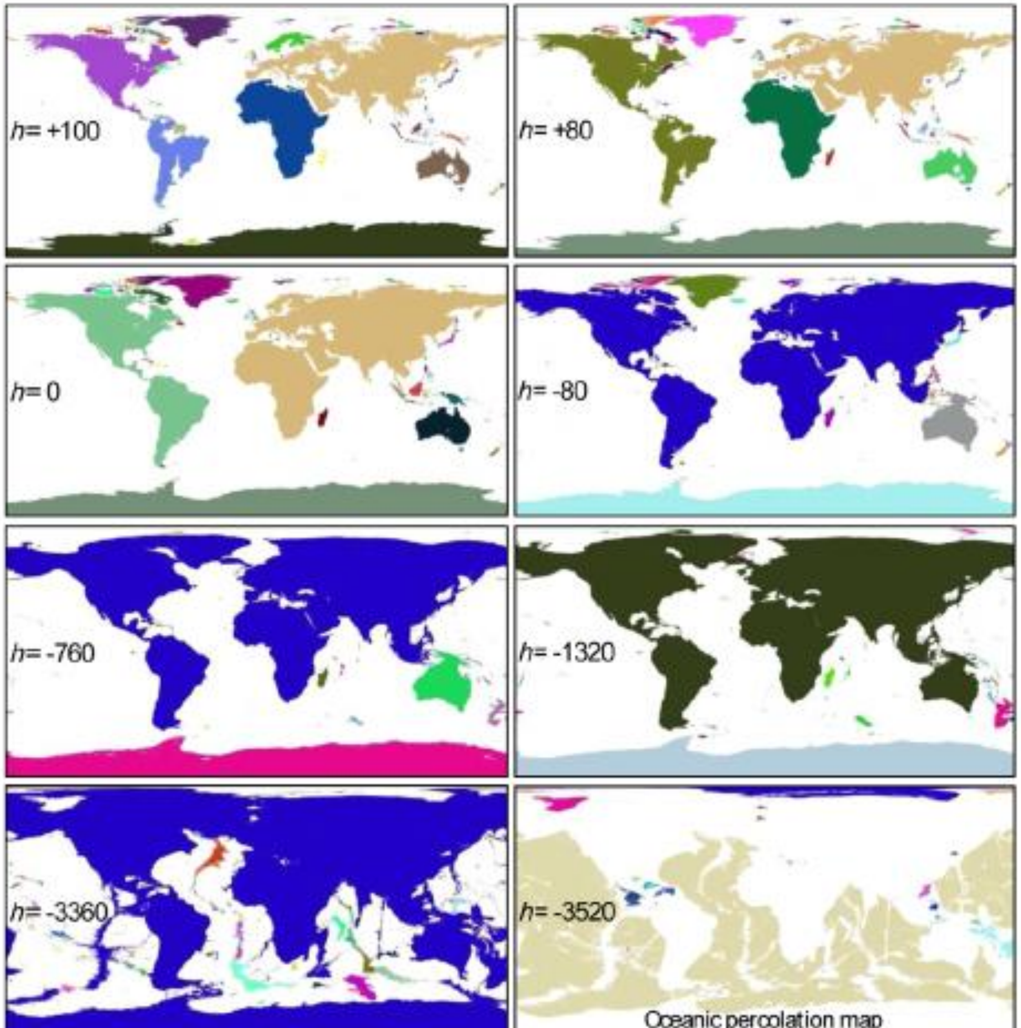


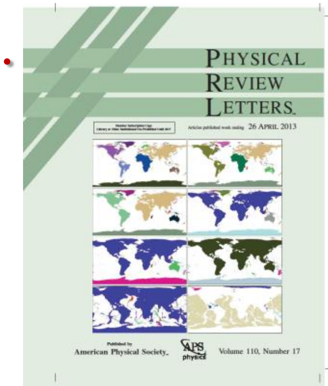
A.A.S., Recent advances in percolation theory and its applications, Physics Reports (2015)

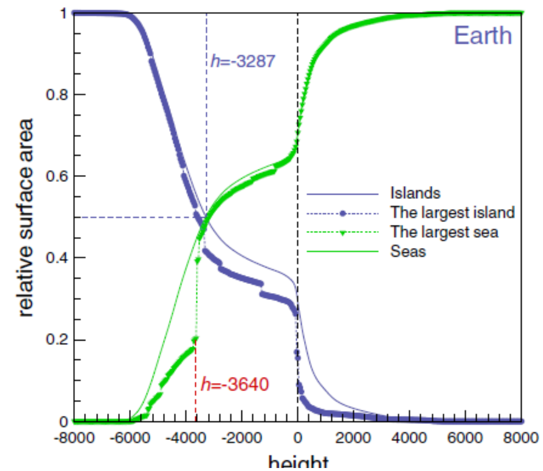
1

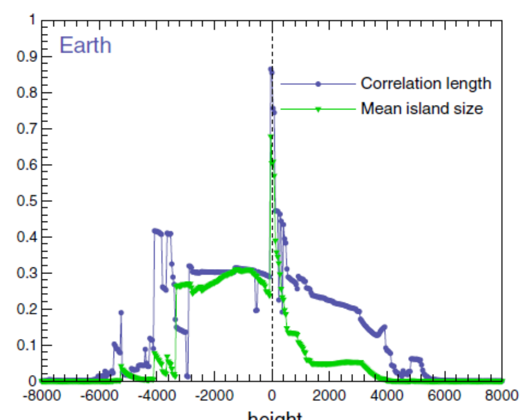
Percolation description of the global topography of Earth…

A.A.S, PRL 110, 178501 (2013)









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Percolation description of a phase transition in QCD

PHYSICAL REVIEW D 89, 054509 (2014)

Fractality and other properties of center domains at finite temperature: SU(3) lattice gauge theory

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See also: Annals of Physics 348 (2014): 341-361

low temperatures — *high temperatures*

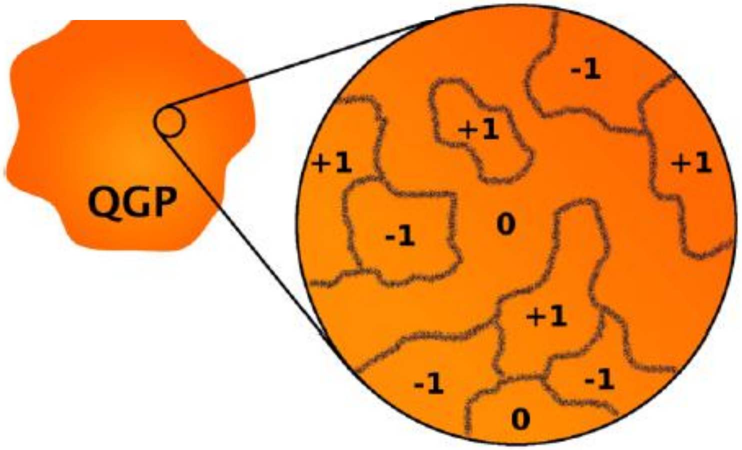
hadronic confined phase $\xrightarrow{T=T_c}$ deconfined phase (*Quark-gluon Plasma*)

$\langle L(\boldsymbol{x})\rangle = 0$ — $\langle L(\boldsymbol{x})\rangle \neq 0$

The order parameter: Polyakov Loop: $L(x) = \mathrm{Tr}\mathcal{P}\exp\left(\int_0^{1/T} A_4(x,t)dt\right) = \rho(x)\exp(i\theta(x))$

center sector number

$$n(x) = \begin{cases} -1 & \text{for } \theta(x) \in [-\pi+\delta, -\pi/3-\delta], \\ 0 & \text{for } \theta(x) \in [-\pi/3+\delta, \pi/3-\delta], \\ +1 & \text{for } \theta(x) \in [\pi/3+\delta, \pi-\delta]. \end{cases}$$



Schematic illustration of center clusters slightly above T_c PRL, 110, 202301 (2013)

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2D and 3D Ising Model

$$\mathcal{H} = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j - h\sum_i \sigma_i$$

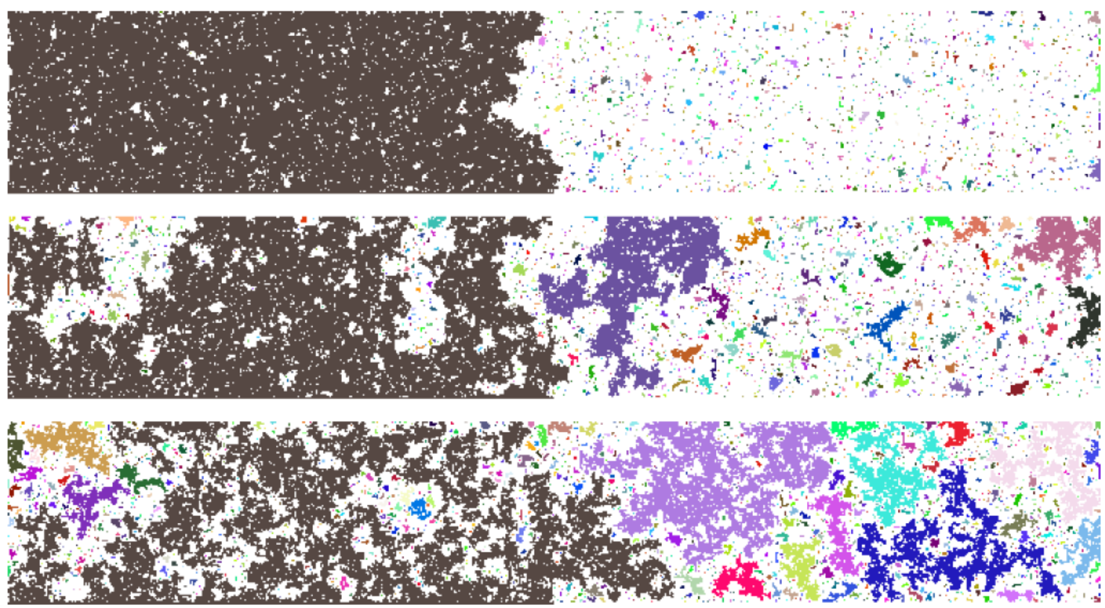
Nucl. Phys. B, 388 (1992) 648-670

Random Walk representation on a square lattice

$$Z'[\beta] = \sum_{\{admissible\ \gamma\}} (-1)^{n(\gamma)} e^{-2\beta L[\gamma]}$$

$$= \exp\left[\sum_{\left\{\substack{connected\\ admissible\ \gamma}\right\}} (-1)^{n(\gamma)} e^{-2\beta L[\gamma]}\right]$$

$n(\gamma)$: intersection number of the curve γ



$T < T_c$

$T = T_c$

$T > T_c$

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Random Surface Representation for 3D Ising Model

$$Z[\beta] \sim \sum_{\{\Sigma\}} (-1)^{l(\Sigma)} \frac{1}{C(\Sigma)} e^{-\mu(\beta) A[\Sigma]}$$

$$\sim \exp\left[\sum_{\left\{ \substack{connected \\ \Sigma} \right\}} (-1)^{l(\Sigma)} \frac{1}{C(\Sigma)} e^{-\mu(\beta) A[\Sigma]} \right]$$

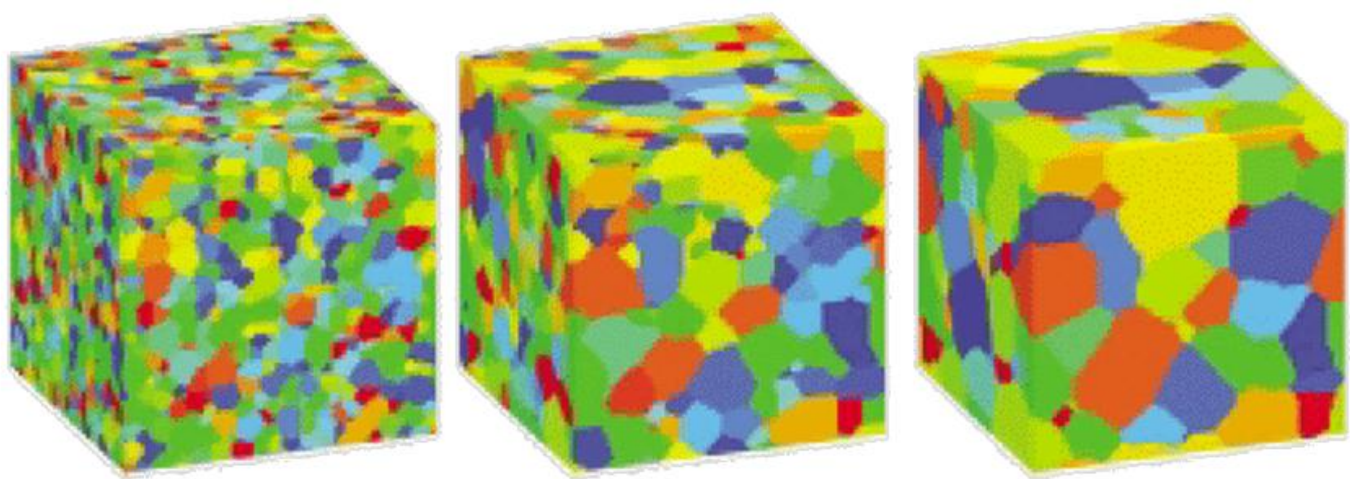
$$\mu(\beta) = -\ln \tanh \beta$$

Σ: a closed lattice surface

$l(\Sigma)$: number of linkes where the surface intersects itself

$A[\Sigma]$: the lattice area of Σ

$C(\Sigma)$: a symmetry factor for the surface



Spin clusters of the 3d Ising model on a cubic lattice, at different temperatures

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The continuum limit:

☼ It is by no means clear how to take the continuum limit of this lattice surface theory...!

It is because that the sign factor $(-1)^{I(\Sigma)}$ **oscillates** very rapidly in the length scale of the lattice spacing .

☼ For a closed path in 2d it is possible to obtain a simple geometrical expression of the sign factor

$$(-1)^{n(\gamma)} = \exp\left(\frac{1}{2} i \oint e_i \, de_i\right)$$

where $e_i(\xi)$ is the tangent vector on the path γ.
But, an analogous expression is not known for $(-1)^{I(\Sigma)}$ for a surface Σ Immersed in R^3.

☼ String theory notions ⟶ tachyon divergencies

J. Ambjorn, *et al*, Phys. Lett. B, 303 (1993) 327-333;
E. Fradkin, *et al*, Phys. Rev. D, 21 (1980) 2885;
V. Dotsenko, *Thesis*, Landau Institute (1981);
A. Polyakov, *Gauge Fields and Strings*, Contemporary Concepts in Physics, Vol. 3.

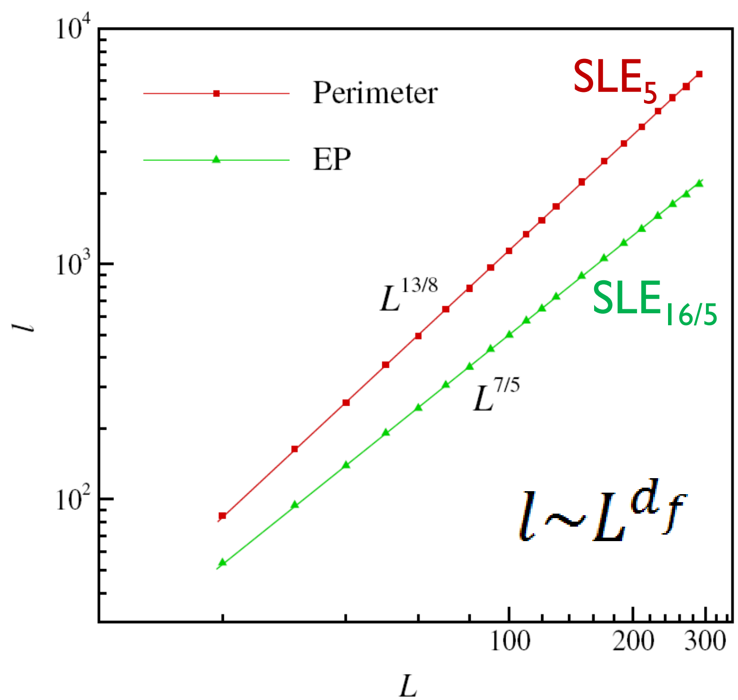
3

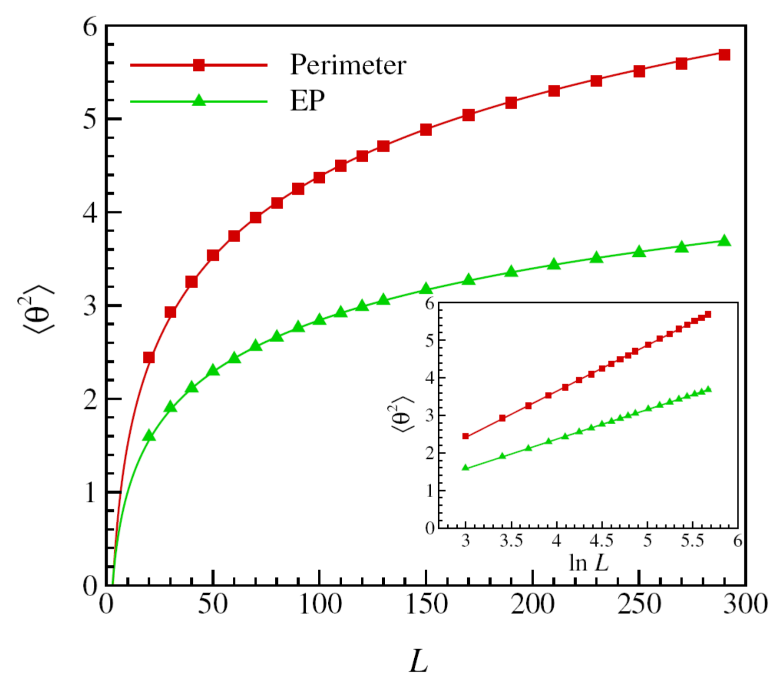
Our observation for 3D Ising Model

Percolation threshold of the 2d spin clusters on a boundary of a 3d model, coincides with the Curie point

A. A. S., H. Dashti, Euro. Phys. Lett. 92 (2010) 67005

Random Surface Representation —*can be reduced to the*→ *Random Walk Representation*





Thank you….