Entanglement in AdS/CFT

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School of Particles & Accelerators

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Outline

Holographic EE

Geometric Entanglement in QFTs

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- Universal information in geometric EE

$$S_A = g_{d-1}(\partial A)\epsilon^{-(d-1)} + \cdots + S_0(\partial A)$$

Entanglement in Even-dim CFTs

- Trace anomaly for even dimsional CFT$_d$

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- Example 2: CFT_4

$$S_{\text{Sphere}} = \#\frac{\ell^2}{\epsilon^2} - 4a\log\frac{\ell}{\epsilon}$$

An Example of Holography: AdS/CFT

Holographic Principle

['t Hooft 93, Susskind 94,...]

(d+2) dim.	Equivalent ⟷	(d+1) dim.
Quantum gravity		Non-gravitational theory

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AdS/CFT
[Maldacena 97]

IIB string theory on $AdS_5 \times S^5$ $\cong$ $\mathcal{N} = 4$ SU(N) SYM on 4 dim. Minkowski space

↓ Classical limit

General relativity ($\Lambda < 0$)

↓ Large N limit, Strong coupling limit

Strongly interacting CFT

AdS/CFT correspondence

- To make it work [Gubser-Klebanov-Polyakov '98, Witten '98]

$$Z_{\text{CFT}}\left[\phi(x)\right] = Z_{\text{Q.G.}}\left[\Phi(z \to 0, x) = \phi(x)\right]$$

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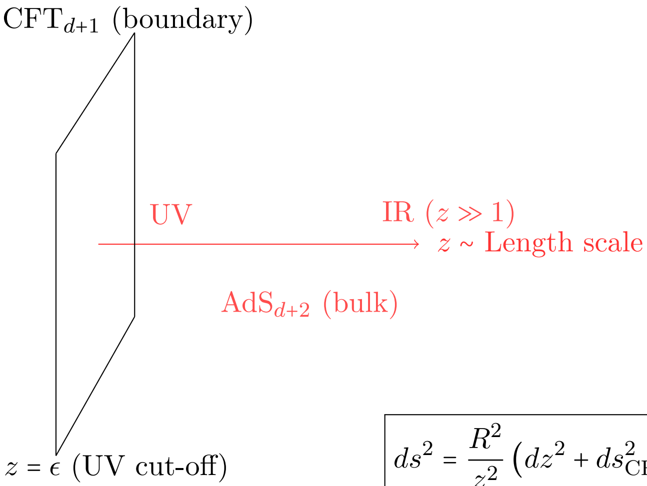
- CFT n-point functions:

$$\langle\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle = \left.\frac{\delta^n S^{\text{on-shell}}_{\text{grav.}}}{\delta\phi(x_1)\cdots\delta\phi(x_n)}\right|_{\phi=0}$$

AdS/CFT (RG Geometrization)



$$ds^2 = \frac{R^2}{z^2}\left(dz^2 + ds^2_{\text{CFT}}\right)$$

(Parts of) AdS/CFT Dictionary

- Symmetries SO(2,4)

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\vec{x}_3^2\right) + R^2 \frac{dr^2}{r^2}$$

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Entanglement entropy on gravity side?

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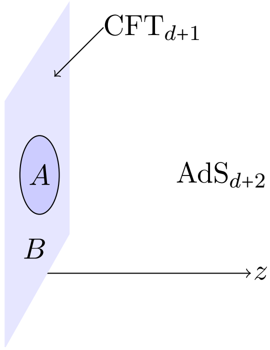

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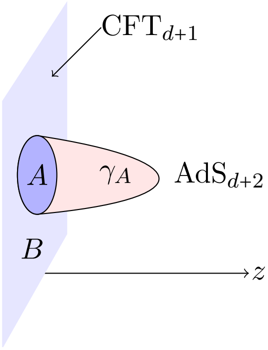

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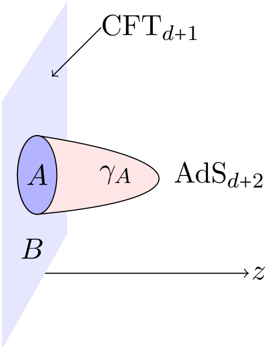
$$S_A=\frac{1}{4G_N^{(d+2)}}\min_{\partial A=\partial\gamma_A}[\mathrm{Area}(\gamma_A)]$$

[Ryu-Takayanagi '06]



Proofs for RT Proposal

1. Spherical Regions
   - Core idea: map EE to thermal entropy
   - CHM map → TBH entropy via AdS/CFT

   [Casini-Huerta-Myers '11]

2. Generic Regions
   - Generalized Gravitational Entropy

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   - For Einstein gravity $\mathcal{A}$ is a minimal surface ($\mathcal{K} = 0$)
     [Lewkowycz-Maldacena '13]

Example: AdS_3/CFT_2

- $AAdS_3$ geometry

$$ds^2 = \frac{R^2}{z^2}\left[-f(z)dt^2 + dx^2 + \frac{dz^2}{f(z)}\right], \qquad f(z) = 1 - mz^2, \qquad T = \frac{\sqrt{m}}{2\pi}$$

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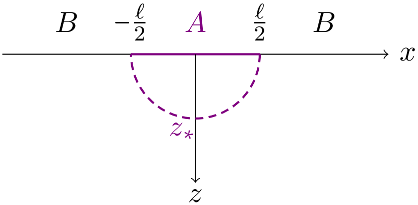

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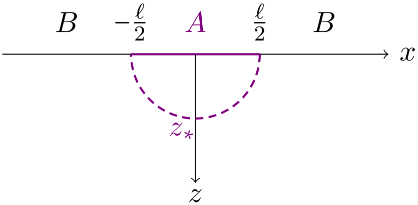
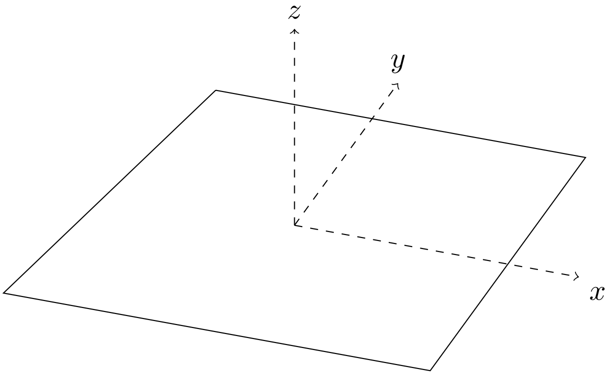


- EE of 2-d CFT at finite T

$$S_A = \frac{c}{3}\log\left[\frac{\sinh(\ell\pi T)}{\epsilon\pi T}\right] \xrightarrow{T\to 0} \frac{c}{3}\log\frac{\ell}{\epsilon}$$

Example: Ball Entangling Surface



For CFT_4

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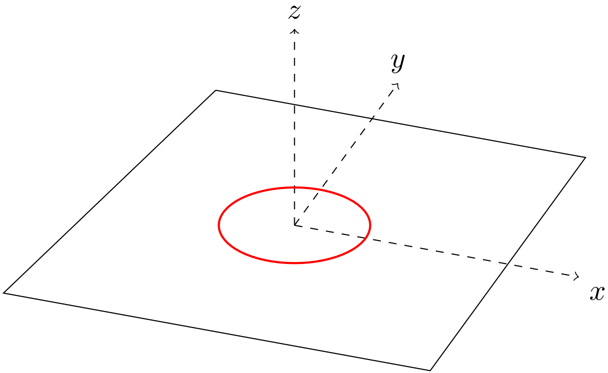

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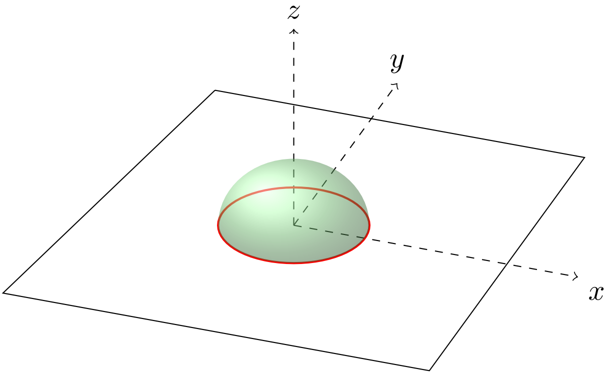

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Some Holographic Results

- Infinite Strip Entangling Region

$$A_{\text{Strip}} = \{x_i | x_1 \in [-\frac{\ell}{2}, \frac{\ell}{2}], x_{2,3,\ldots,d-1} \in [-H, H]\}$$

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- Spherical Entangling Region ($r = \sqrt{\sum_{i=1}^{d-1} x_i^2}$)

$$A_{\text{Shere}} = \{x_i | r < \ell\}$$

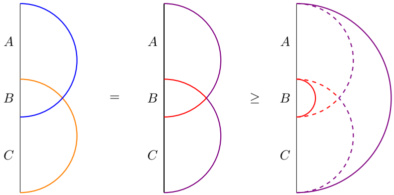
Entanglement Entropy

$$S_A = p_1\left(\frac{\ell}{\epsilon}\right)^{d-2} + p_3\left(\frac{\ell}{\epsilon}\right)^{d-4} + \cdots + \begin{cases} p_{d-1}(\ell/\epsilon) + p_d, & d : Odd, \\ p_{d-2}(\ell/\epsilon)^2 + 1\log(\ell/\epsilon), & d : Even, \end{cases}$$

Some Properties of Holographic EE

- Holographic Strong Subadditivity [Headrick-Takayanagi '07]

$$S_{A\cup B} + S_{B\cup C} \le S_{A\cup B\cup C} + S_B$$



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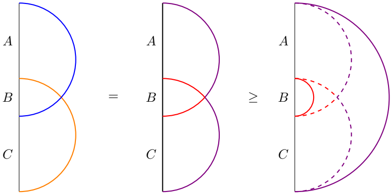

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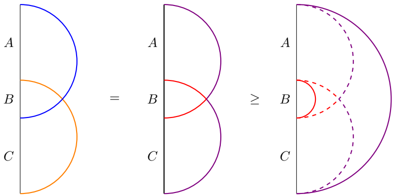

- Mutual information

- A similar argument → mutual information is monogamous [Hayden-Headrick-Maloney '11]

Holographic Entanglement Beyond RT

Covariant Holographic Entanglement Entropy

- HEE in non-static cases?

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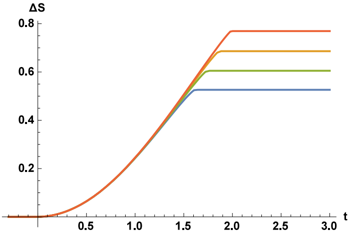
- Local extrema of area functional with minimum length gives EE
- Recently proved [Dong-Lewkowycz-Rangamani '16]

Example: Holographic "Quantum Quench"

- Global quantum quench modelled by AdS_3-Vaidya geometry [Abajo-Arrastia -Apartcio-Ltopez '10]

$$ds^2 = -\left(r^2 - m(v)\right) dv^2 + 2drdv + r^2 dx^2$$



- Quadratic, linear and saturation regimes [Liu-Suh '13]

Higher Derivative Theories

- For Example

$$S = \int \left[\mathcal{R} - 2\Lambda + \alpha_1 \mathcal{R}^2 + \alpha_2 \left(\mathcal{R}_{\mu\nu} \right)^2 + \alpha_3 \left(\mathcal{R}_{\mu\nu\alpha\beta} \right)^2 \right]$$

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  Curvature Squared Theories [Fursaev-Patrushev-Solodukhin '13]

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- In 4d CFT dual to Gauss-Bonnet Theory $a \neq c$

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- For Example

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- In hand proposals:

  Curvature Squared Theories [Fursaev-Patrushev-Solodukhin '13]

  Lovelock Theories [de Boer-Kulaxizi-Parnachev '11, Hung-Myers-Smolkin '11]

  (Any Order) Contraction of Riemann [Dong '13, Camps '13]

- In 4d CFT dual to Gauss-Bonnet Theory $a \neq c$

- Holographic c-theorems
  1. Even dim: the coefficient of a-type flows (Cardy's conjecture)
  2. Odd dim: the universal part of EE flows

  [Myers-Sinha '10]

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- GHEE: EE of A is given by

$$S_{\mathrm{ent}} = \frac{\mathrm{Area}(\gamma)}{4G_N},$$

γ: the minimal surface in $\mathrm{AdS}_p \times \mathrm{X}_q$ ($\partial\gamma = \partial A$)

[Mollabashi-Shiba-Takayanagi '14]

Other Important Progresses

- Linearized Gravity from Entanglement in CFTs
  [Van Raamsdonk, Myers, Lashkari, McDermott, Faulkner, Guica, Hartman]
- F-Theorem in CFT_3 [Casini, Huerta, Myers]
- AdS/MERA [Swingle, Takayanagi, Ryu, ···]
- Renyi Entropy [Myers, Dong, Hung, Smolkin, Mosaffa, ···]
- Surface/State correspondence [Takayanagi, Miyaji, Shiba, Ryu, ···]
- HEE & Causality in CFT and Gravity [Rangamani, Hubeny, Headrick, ···]
- Holographic Entropy Cone [Ooguri, Bao, ···]
- Singular Entangling Regions [Takayanagi, Myers, Boeno, ···]
- Differential Entropy [Balasubramanian, de Boer, Myers, ···]
- Higher Dimensional Twist Operators [Hung-Myers-Smolkin '14]
- HEE as a Probe for QPT in CFTs [Takayanagi, Klebanov, ···]