Collective Excitations in QCD Plasma

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1- From Short to Large Distances:

Short distances: microscopic local QFT

e.g.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

Large distances: Derivative expansion of QFT

$$O\left(\frac{E}{\Lambda}\right) \sim O\left(\frac{p}{\Lambda}\right) \sim O\left(\frac{l_{ch}}{L}\right) \sim O(l_{ch}\,\partial)$$

e.g. Low energy chiral theory in QCD

$$U = \exp\left(\frac{i}{f}\phi^a\lambda^a\right) \qquad \mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}(U, \partial U, \partial^2 U, \ldots)$$

2- Energy-Momentum Tensor

Λ

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

$$t^{\mu}{}_{\nu}(x) = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\partial_\nu \phi - \mathcal{L}\delta^{\mu}{}_{\nu}$$

$$j^\mu(x) = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\Delta\phi$$

Thermal field theory

E

$$\hat{\rho} = \frac{1}{Z}e^{\beta u_\mu P^\mu + \beta\mu N}$$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p\eta^{\mu\nu}$$

$$J^\mu = n u^\mu$$

3- Derivative Expansion

In the **long wave-length limit**:

$$\frac{\ell_{mfp}}{L} \sim \ell_{mfp}\partial \ll 1$$

$$T^{\mu\nu}(x) = \underbrace{T^{\mu\nu}_{(0)}}_{O(\partial^0)} + \underbrace{T^{\mu\nu}_{(1)}}_{O(\partial)} + \dots$$

$$J^{\mu}(x) = \underbrace{J^{\mu}_{(0)}}_{O(\partial^0)} + \underbrace{J^{\mu}_{(1)}}_{O(\partial)} + \dots$$

To first order in derivative expansion:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p\eta^{\mu\nu} - \eta P^{\mu\alpha}P^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha\right) - \left(\zeta - \frac{2}{3}\eta\right)P^{\mu\nu}\partial.u$$

$$J^\mu = nu^\mu - \sigma T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu$$

4- Hydrodynamic Equations

Conservation eqs:

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu$$
$$\partial_\mu J^\mu = 0$$

hydro fields:

$$T(x),\ \mu(x),\ u^\mu(x)$$

or equivalently:

$$\epsilon(x),\ \ n(x),\ \ \boldsymbol{\pi}(x)$$

5- Hydrodynamic Fluctuations in Presence of Magnetic Filed:

Equilibrium state:

$$\bar{u}^\mu = (1, \mathbf{0}), \ \bar{T} = \text{const.}, \ \bar{\mu} = 0$$

External Field:

$A_\mu \sim O(\partial^0)$ and $F_{\mu\nu} \sim O(\partial)$

$$E^\mu = F^{\mu\nu} u_\nu = 0$$

$$B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta} = (0,0,0,B)$$

6- Hydrodynamics Modes

EoM linearized: $\phi_a(t,\mathbf{k}) = (n,\ \pi^i,\ \epsilon)$

$v_s^2 = \partial p/\partial \epsilon$

$\bar{w} = \bar{\epsilon} + \bar{p}$

Collective excitations:

N.A, A.Davody (Phys.Lett. B (2016))

Hydrodynamic modes in presence of magnetic field	Hydrodynamic modes at $\mathbf{B}=0$
$\omega_{1,2}(\mathbf{k}) = \pm v_s k - \frac{i}{2}\left(\mathbf{k}^2\gamma_s + \frac{\sigma}{\bar{w}}\mathbf{B}^2\sin^2\theta\right)$	$\omega_{1,2}(\mathbf{k}) = \pm v_s k - \frac{i}{2}\mathbf{k}^2\gamma_s$
$\omega_{3,4}(\mathbf{k}) = -\frac{i}{2}\left(\mathbf{k}^2(D+\gamma_\eta) + \frac{\sigma}{\bar{w}}\mathbf{B}^2 \pm \sqrt{(\mathbf{k}^2(D-\gamma_\eta) - \frac{\sigma}{\bar{w}}\mathbf{B}^2)^2 + \frac{4D\sigma}{\bar{w}}\mathbf{B}^2\mathbf{k}^2\sin^2\theta}\right)$	$\omega_3(\mathbf{k}) = -iD\mathbf{k}^2$, $\omega_4(\mathbf{k}) = -i\gamma_\eta\mathbf{k}^2$
$\omega_5(\mathbf{k}) = -i\left(\mathbf{k}^2\gamma_\eta + \frac{\sigma}{\bar{w}}\mathbf{B}^2\cos^2\theta\right)$	$\omega_5(\mathbf{k}) = -i\gamma_\eta\mathbf{k}^2$

Different From Standard **Magneto-hydrodynamics** Results

“No Alfven Wave”

$$\omega \sim Bk$$

7- Parity Violating Currents

VILENKIN (PHYS.LETT. B80 (1978)):

$$\gamma^{\mu}(\partial_{\mu} - \Gamma_{\mu})\psi = 0, \quad (1 - \gamma^{5})\psi = 0$$

$$f(\omega, m) = (e^{(\omega - m\Omega)/T} + 1)^{-1}$$

→

CVE

$$J = -\frac{1}{12}\,\mathbf{\Omega}\,T^{2}$$

Early universe

$T \gtrsim 1$ MeV

VILENKIN (PHYS.REV. D22 (1980))

$$\gamma^{\mu}(i\partial_{\mu} - eA_{\mu})\psi = 0$$

$$(1 + \gamma^{5})\psi = 0$$

→

CME

$$\vec{j} = -(e^{2}\chi/2\pi^{2})\vec{B}$$

8- Physical Reason

Vilenkin(1980):

PHYSICAL REVIEW D VOLUME 22, NUMBER 12 15 DECEMBER 1980

Equilibrium parity-violating current in a magnetic field

Alexander Vilenkin
Physics Department, Tufts University, Medford, Massachusetts 02155
(Received 1 August 1980)

It is argued that if the Hamiltonian of a system of charged fermions does not conserve parity, then an equilibrium electric current parallel to $\vec{B}$ can develop in such a system in an external magnetic field $\vec{B}$. The equilibrium current is calculated (i) for noninteracting left-handed massless fermions and (ii) for a system of massive particles with a Fermi-type parity-violating interaction. In the first case a nonzero current is found, while in the second case the current vanishes in the lowest order of perturbation theory. The physical reason for the cancellation of the current in the second case is not clear and one cannot rule out the possibility that a nonzero current appears in other models.

H.B. NIELSEN, M. NINOMIYA (PHYS.LETT. B130 (1983)):

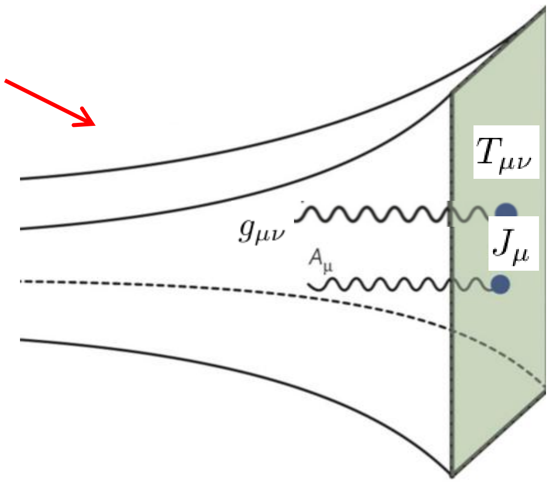
Chiral Anomaly

9- Gauge-Gravity Duality

MALDACENA, 97

AdS5

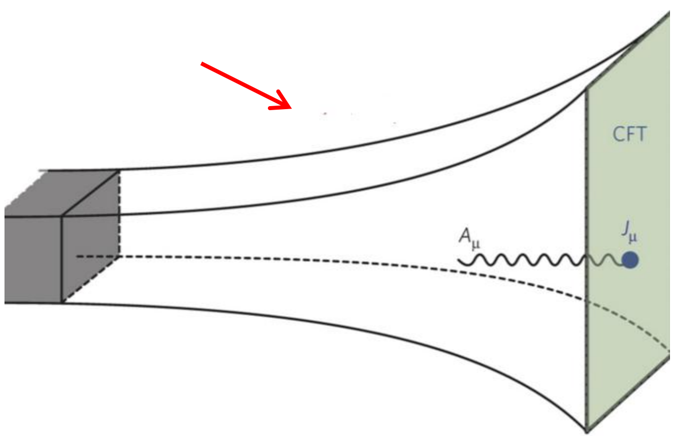
CFT4



$$ds^2 = \frac{dr^2}{r^2} + r^2\left(dx^\mu dx_\mu\right)$$

AdS5 Black-Brain

Thermal CFT



$$ds^2 = \frac{dr^2}{r^2 f(br)} - r^2 f(br) dt^2 + r^2 d\vec{x}^2$$

$$f(r) = 1 - \frac{1}{r^4}, \qquad b = \frac{1}{\pi T}$$

10- Fluid-Gravity Duality

MINWALLA, ET. AL JHEP,2007

Metric:

$$ds^2 = -2\,u_\mu\,dx^\mu dr - r^2\,f(b\,r)\,u_\mu u_\nu\,dx^\mu dx^\nu + r^2\,P_{\mu\nu}\,dx^\mu dx^\nu$$
$$+\,2\,r^2\,b\,F(b\,r)\,\sigma_{\mu\nu}\,dx^\mu dx^\nu + \frac{2}{3}\,r\,u_\mu u_\nu\,\partial_\lambda u^\lambda\,dx^\mu dx^\nu - r\,u^\lambda \partial_\lambda\,(u_\nu u_\mu)\;dx^\mu dx^\nu$$

Constraint Eqs:

$$\partial_\mu T^{\mu\nu} = 0 \quad \longleftrightarrow \quad T^{\mu\nu} = \frac{1}{b^4}\,(4\,u^\mu u^\nu + \eta^{\mu\nu})$$

Energy-Momentum Tensor on the boundary

$$T^\mu_\nu = -2\,\lim_{r\to\infty} r^4\,(K^\mu_\nu - \delta^\mu_\nu) \quad \Rightarrow \quad T^{\mu\nu} = \frac{1}{b^4}\,(4\,u^\mu u^\nu + \eta^{\mu\nu}) - \frac{2}{b^3}\,\sigma^{\mu\nu}$$

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

11- Generalizations

1. Forced Fluid — MINWALLA, ET. AL JHEP,2009
2. Non-Relativistic Fluid — MINWALLA, ET. AL JHEP,2009
3. **(Chirally)Charged Fluid** — BHATTACHARYYA, ET.AL JHEP 2009

$$S = \frac{1}{16\pi G_5} \int \sqrt{-g_5} \left[R + 12 - F_{AB}F^{AB} - \frac{4\kappa}{3} \epsilon^{LABCD} A_L F_{AB} F_{CD} \right]$$

$$T_{\mu\nu} = p(\eta_{\mu\nu} + 4u_\mu u_\nu) - 2\eta\sigma_{\mu\nu} + \dots$$

$$J_\mu = n\, u_\mu - \mathfrak{D}\, P_\mu^\nu \mathcal{D}_\nu n + \xi\, l_\mu + \dots$$

vorticity

$$l^\mu \equiv \epsilon^{\nu\lambda\sigma\mu} u_\nu \partial_\lambda u_\sigma$$

Rmemeber Vilenkin

$$J_\mu \sim \Omega_\mu$$

12- Parity Violating Terms

Motivated by Fluid/Gravity

$$J^\mu \;=\; nu^\mu - \sigma T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu \boxed{+\,\xi\omega^\mu + \xi_B B^\mu}$$

Contradiction with second law!

Hydro equations modified:

Son, Surowka(Phys.Rev.Lett. 103 (2009))

$$\partial_\mu j^\mu = C E^\mu B_\mu,$$
$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

$$S^\mu = s u^\mu \boxed{+\,D\xi^\mu + D_B B^\mu}$$

13- Equality Constraints; for the First Time:

$$
\begin{aligned}
T\mathcal{D}_\mu s^\mu &= 2\eta\sigma_{\mu\nu}\sigma^{\mu\nu} + \sigma\left(T\Delta^{\mu\nu}\mathcal{D}_\nu\bar{\mu} - E^\mu\right)\left(T\Delta_{\mu\alpha}\mathcal{D}^\alpha\bar{\mu} - E_\mu\right) \\
&+ \left(-T\xi\mathcal{D}_\mu\bar{\mu} + T\mathcal{D}_\mu D + \left(\frac{\xi}{n} - \frac{2TD_B}{n}\right)\mathcal{D}_\mu p\right)\omega^\mu \\
&+ \left(-T\xi_B\mathcal{D}_\mu\bar{\mu} + T\mathcal{D}_\mu D_B + \left(\frac{\xi_B}{n} - \kappa\frac{\mu}{n}\right)\mathcal{D}_\mu p\right)B^\mu
\end{aligned}
$$

$$\eta \geq 0, \qquad \sigma \geq 0$$

LANDAU-LIFSHITZ

Variational Principle

$$\xi = \mathcal{C}\mu^2\left(1 - \frac{2}{3}\frac{\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right) + \mathcal{D}T^2\left(1 - \frac{2\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right)$$

$$\xi_B = \mathcal{C}\mu\left(1 - \frac{1}{2}\frac{\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right) - \frac{\mathcal{D}}{2}\frac{\bar{n}T^2}{\bar{\epsilon}+\bar{p}}$$

SON, SUROWKA(PHYS.REV.LETT. 103 (2009))
KHARZEEV, YEE, PHYS.REV. D84 (2011)
NEIMAN, OZ (JHEP (2011))

14- Symmetry Considerations

Time Reversal Sym.

Ohm Law: $\boldsymbol{J} = \sigma \boldsymbol{E}$ ➡ **DISSIPATIVE**

(-) (-)(+)

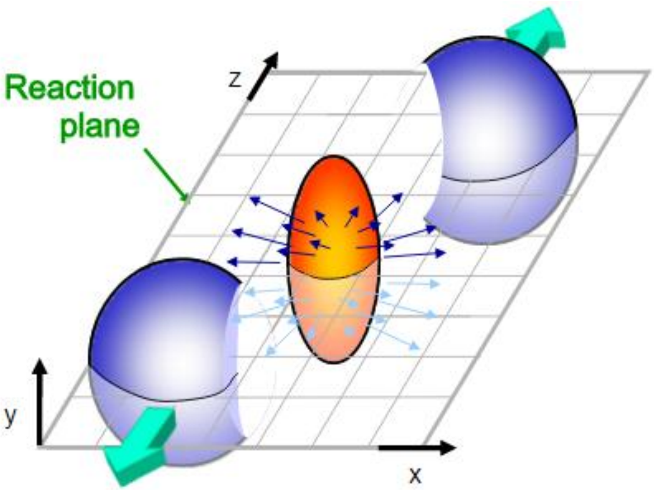
London 2nd eq.: $\frac{\partial \boldsymbol{J}_s}{\partial t} = \sigma_s \boldsymbol{E}$ ➡ **NON DISSIPATIVE**

(+) (+)(+)

Anomalous transport: $\boldsymbol{J} = \xi_B \boldsymbol{B}$ ➡ **GUESS WHAT?**

(-) (+)(-)

15- Quark-Gluon-Plasma Experiment



STEFFAN BASS: PROBING THE QGP AT RHIC

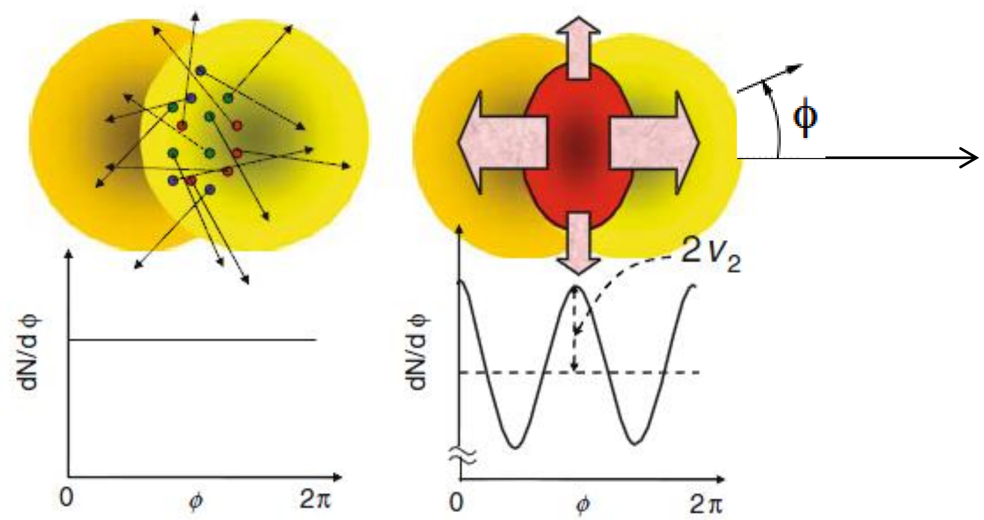
1- Initial State

2- Evolution

3- Particle Spectrum in detectors

16- Observables in QGP

Elliptic Flow:



(a) Large mean free path (b) Small mean free path

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_t\,\mathrm{d}y} = \frac{1}{2\pi p_T}\frac{\mathrm{d}N}{\mathrm{d}p_T\,\mathrm{d}y}\left[1 + 2v_1\cos(\phi - \Phi_R) + 2v_2\cos 2(\phi - \Phi_R) + \cdots\right]$$

17- Why Hydrodynamically Evolution?

initial spatial anisotropy ➡ anisotropy in pressure gradients

⬇

HYDRODYNAMICS ⬅ a non-zero v_2

$$\frac{d(\epsilon\vec{v})}{dt} = -\vec{\nabla} P$$

Data : $\frac{\eta}{s} \sim 0.1$ Nearly Perfect fluid !!!

SHURYAK (2008)

From AdS/CFT: $\frac{\eta}{s} = \frac{1}{4\pi} = 0.08$

SON , STARINETS , KOVTUN, (2004)

➡ STRONGLY COUPLED PLASMA

18- Axial & Vector Currents in QGP:

Microscopic:

vector current: $J^\mu = \bar{\psi}\gamma^\mu\psi$

axial current: $J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J_5^\mu = \mathcal{C} E_\mu B^\mu$$ axial anoamly

Macroscopic: (QCD Plasma):

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p\,\eta^{\mu\nu}$$

$$J^\mu = n u^\mu + \xi\,\omega^\mu + \xi_B B^\mu$$

$$J_5^\mu = n_5 u^\mu + \xi_5\,\omega^\mu + \xi_{B5} B^\mu.$$

19- CMW in QGP:

CME:

$$\delta \boldsymbol{j} = \frac{\boldsymbol{B}}{2\pi^2\chi}\delta n_5$$

CSE:

$$\delta \boldsymbol{j_5} = \frac{\boldsymbol{B}}{2\pi^2\chi}\delta n$$

Chiral magnetic wave:

$$\left(\partial_t^2 - v_{\rm CMW}^2\partial_z^2\right)\delta n(x) = 0$$

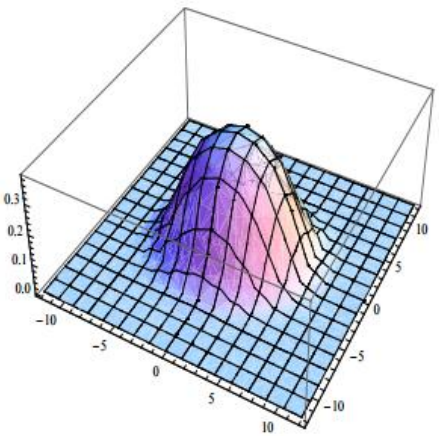
$$\left(\partial_t^2 - v_{\rm CMW}^2\partial_z^2\right)\delta n_5(x) = 0$$

$$v_{\rm CMW} = \frac{B}{2\pi^2\chi}$$

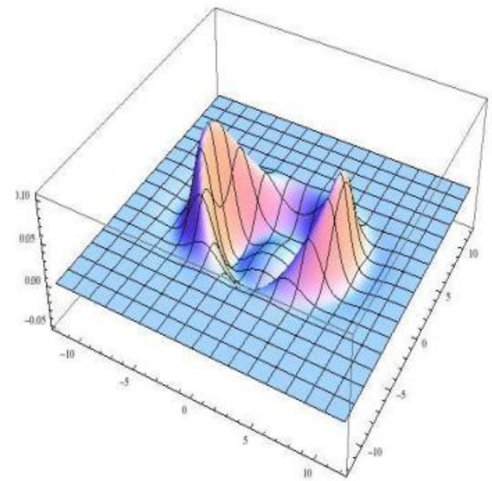
Kharzeev, Yee (2011)

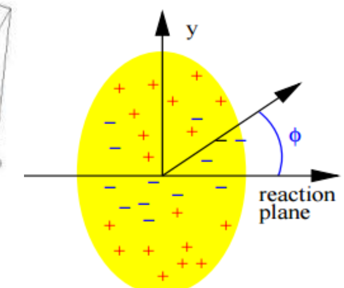
20- From the CMW to the charge dependence of v2

Burnier, Kharzeev, Liao, Yee, (2012)



CMW evolution →





$$A = \frac{\bar{N}_+ - \bar{N}_-}{\bar{N}_+ + \bar{N}_-}$$

$$\frac{dN_\pm}{d\phi} = \bar{N}_\pm \left[1 + (2v_2 \mp r_e\, A)\cos(2\phi)\right]$$

$$\boxed{v_2^- - v_2^+ = r_e\, A}$$

21- Signature in Experiment

PRL **114**, 252302 (2015) PHYSICAL REVIEW LETTERS week ending 26 JUNE 2015

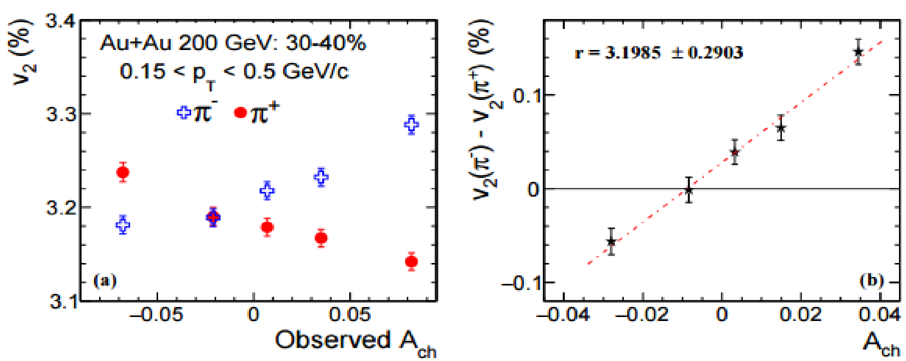
Observation of Charge Asymmetry Dependence of Pion Elliptic Flow and the Possible Chiral Magnetic Wave in Heavy-Ion Collisions

L. Adamczyk,[1] J. K. Adkins,[20] G. Agakishiev,[18] M. M. Aggarwal,[30] Z. Ahammed,[47] I. Alekseev,[16] J. Alford,[19] A. Aparin,[18] D. Arkhipkin,[3] E. C. Aschenauer,[3] G. S. Averichev,[18] A. Banerjee,[47] R. Bellwied,[43] A. Bhasin,[17] A. K. Bhati,[30] P. Bhattarai,[42] J. Bielcik,[10] J. Bielcikova,[11] L. C. Bland,[3] I. G. Bordyuzhin,[16] J. Bouchet,[19] A. V. Brandin,[26] I. Bunzarov,[18]

…

(STAR Collaboration)

We present measurements of π^- and π^+ elliptic flow, v_2, at midrapidity in Au + Au collisions at $\sqrt{s_{NN}} = 200$, 62.4, 39, 27, 19.6, 11.5, and 7.7 GeV, as a function of event-by-event charge asymmetry, A_{ch}, based on data from the STAR experiment at RHIC. We find that π^- (π^+) elliptic flow linearly increases (decreases) with charge asymmetry for most centrality bins at $\sqrt{s_{NN}} = 27$ GeV and higher. At $\sqrt{s_{NN}} = 200$ GeV, the slope of the difference of v_2 between π^- and π^+ as a function of A_{ch} exhibits a centrality dependence, which is qualitatively similar to calculations that incorporate a chiral magnetic wave effect. Similar centrality dependence is also observed at lower energies.



22- CVW in QGP

Jiang, Huang, Liao, (2015)

vector and axial charge fluctuations

at finite density

$$\delta \boldsymbol{j} = \frac{\mu \boldsymbol{\Omega}}{2\pi^2 \chi} \delta n_5(x)$$

$$\delta \boldsymbol{j}_5 = \frac{\mu \boldsymbol{\Omega}}{2\pi^2 \chi} \delta n(x)$$

Chiral vortical wave

$$v_{CVW} = \frac{\mu \Omega}{2\pi^2 \chi}$$

23- Full Hydro Computations

N.A,, D,AllahBakhshi, A.Davody, F.Taghavi (2016)

$$
\begin{bmatrix}
-i\alpha_1\omega & ik_j & -i\alpha_2\omega & -i\alpha_3\omega \\
i\alpha_1 v_s^2 k^i & \begin{matrix} -i\omega\delta^i_j - \epsilon^i{}_{jl}\Omega^l - \frac{\bar{n}}{\bar{w}}\epsilon^i{}_{jl}B^l \\ -i\frac{\xi}{2\bar{w}}\left(\boldsymbol{B}\cdot\boldsymbol{k}\delta^i_j - B_j k^i\right)\end{matrix} & \begin{matrix} i\alpha_2 v_s^2 k^i \\ +\left(\frac{\partial\xi}{\partial\mu}\right)\underline{(\boldsymbol{B}\times\boldsymbol{\Omega})^i}\end{matrix} & \begin{matrix} i\alpha_3 v_s^2 k^i \\ +\left(\frac{\partial\xi}{\partial\mu_5}\right)\underline{(\boldsymbol{B}\times\boldsymbol{\Omega})^i}\end{matrix} \\
\begin{matrix} -i\beta_1\omega + \left(\frac{\partial\xi}{\partial T}\right)i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_B}{\partial T}\right)i\boldsymbol{B}\cdot\boldsymbol{k}\end{matrix} & \begin{matrix} \frac{\bar{n}}{\bar{w}}ik_j - \frac{2\xi}{\bar{w}}i\omega\Omega_j \\ -\frac{\xi_B}{\bar{w}}i\omega B_j - \frac{\xi_B}{\bar{w}}\underline{(\boldsymbol{B}\times\boldsymbol{\Omega})_j}\end{matrix} & \begin{matrix} -i\beta_2\omega + \left(\frac{\partial\xi}{\partial\mu}\right)i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_B}{\partial\mu}\right)i\boldsymbol{B}\cdot\boldsymbol{k}\end{matrix} & \begin{matrix} -i\beta_3\omega + \left(\frac{\partial\xi}{\partial\mu_5}\right)i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_B}{\partial\mu_5}\right)i\boldsymbol{B}\cdot\boldsymbol{k}\end{matrix} \\
\begin{matrix} -i\gamma_1\omega + \left(\frac{\partial\xi_5}{\partial T}\right)i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_{5B}}{\partial T}\right)i\boldsymbol{B}\cdot\boldsymbol{k}\end{matrix} & \begin{matrix} \frac{\bar{n}_5}{\bar{w}}ik_j - \frac{2\xi_5}{\bar{w}}i\omega\Omega_j \\ -\frac{\xi_{5B}}{\bar{w}}i\omega B_j - \frac{\xi_{5B}}{\bar{w}}\underline{(\boldsymbol{B}\times\boldsymbol{\Omega})_j}\end{matrix} & \begin{matrix} -i\gamma_2\omega + \left(\frac{\partial\xi_5}{\partial\mu}\right)i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_{5B}}{\partial\mu}\right)i\boldsymbol{B}\cdot\boldsymbol{k}\end{matrix} & \begin{matrix} -i\gamma_3\omega + \left(\frac{\partial\xi_5}{\partial\mu_5}\right)i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_{5B}}{\partial\mu_5}\right)i\boldsymbol{B}\cdot\boldsymbol{k}\end{matrix}
\end{bmatrix}
\begin{bmatrix} \delta T \\ \delta\pi_i \\ \delta\mu \\ \delta\mu_5 \end{bmatrix} = 0
$$

two sectors: 1) scalar 2) scalar-vector

24- Mixed CMVW in Scalar Sector

Kharzeev-Yee

$$v_{CMW} = \frac{\mathcal{C}\,B}{\chi}\frac{\left(1-\frac{n\mu}{w}\right)}{\mathcal{K}},$$

$$v_{CMVW} = \pm\left(v_{CVW} + v_{CMW}\right)$$

$$v_{CVW} = \frac{\mathcal{C}\,\mu\,\Omega}{\chi}\frac{\left(1-\frac{n\mu}{w}-\frac{\mathcal{D}}{\mathcal{C}}\frac{nT^2}{\mu w}\right)}{\mathcal{K}}$$

Jiang-Huang-Liao

$$\omega_{1,2} = \pm(v_{CMW} + v_{CVW})k$$

$$= \pm\frac{Bk}{2\pi^2\chi}\frac{1-\frac{\mu n}{w}}{\sqrt{1-\frac{\mu n}{w}-\frac{n}{\chi w}\left(\frac{n}{c_s^2}-\chi\mu\right)}} \pm \frac{\Omega\mu k}{2\pi^2\chi}\frac{1-\frac{\mu n}{w}-\frac{\pi^2 nT^2}{3w}}{\sqrt{1-\frac{\mu n}{w}-\frac{n}{\chi w}\left(\frac{n}{c_s^2}-\chi\mu\right)}}$$

Gravitational Anomaly: New obsevation

25- Outlook

1. The full Spectrum **in Presence of Dissipation**
2. The full Spectrum **to Second Order**
3. The full spectrum **from Chiral Kinetic Theory**
4. The full spectrum **from AdS/CFT**
5. The full spectrum **at Large D**
6. Anomalous Shock waves

Thank You
For Your Attention