Hydrodynamic Fluctuations in a Hot Medium

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0. Outline

First Part

- Relativistic hydro
- Motivation for hydrodynamic chiral transport
- Non_dissipative feature of chiral transport
- Spectrum of chiral hydro modes

Second Part

- Chiral hydro modes from Kinetic theory
- Frame choice and hydro modes

1. Hydrodynamics

Response of the system to perturbations

at low energy and Long wave-length limit

Hydro equations:

local conservation equations

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu$$
$$\partial_\mu J^\mu = 0$$

variables → $T(x),\ \mu(x),\ u^\mu(x)$

$(u^\mu u_\mu = -1)$

2. Constitutive Relations

Structure of $T^{\mu\nu}$ and J^{μ} ?

The main idea:

14 unknown fields in terms of **5** fields and their derivatives

$$T^{\mu\nu} = (\epsilon + p)u^{\mu}u^{\nu} + p\,\eta^{\mu\nu} \; - \eta P^{\mu\alpha}P^{\nu\beta}\left(\partial_{\alpha}u_{\beta} + \partial_{\beta}u_{\alpha}\right) - \left(\zeta - \frac{2}{3}\eta\right)P^{\mu\nu}\partial.u$$

$$J^{\mu} = nu^{\mu} \; - \sigma T P^{\mu\nu}\partial_{\nu}\left(\frac{\mu}{T}\right) + \sigma E^{\mu}$$

3. Dual gravity picture?

Early studies:

Membrane Paradigm: Fluid on the horizon

[Damur; Thorne Price Macdonald 1970's]

New viewpoint:

Fluid–Gravity Duality: Fluid on the boundary

[Bhattacharyya Hubeny Minwalla Rangamani 2007]

4. Extensions of Fluid–Gravity:

1. Forced Fluid [Bhattacharyya Loganayagam Minwalla Nampuri Trividi 2007]
2. Non-Relativistic Fluid [Bhattacharyya Minwalla Wadia 2008]
3. **(Chirally) Charged Fluid** [Erdmenger Haack Kaminski Yarom 2008]

[Banerjee Bhattacharyya Bhattacharyya Dutta Loganayagam Surowka 2008]

$$S = \frac{1}{16\pi G_5}\int \sqrt{-g_5}\left[R + 12 - F_{AB}F^{AB} - \underline{\frac{4\kappa}{3}\epsilon^{LABCD}A_L F_{AB} F_{CD}}\right]$$

anomaly

$$T_{\mu\nu} = p(\eta_{\mu\nu} + 4u_\mu u_\nu) - 2\eta\sigma_{\mu\nu} + \dots$$

$$J_\mu = n\, u_\mu - \mathfrak{D}\, P_\mu^\nu \mathcal{D}_\nu n + \xi\, l_\mu + \dots$$

vorticity: $l^\mu \equiv \epsilon^{\nu\lambda\sigma\mu} u_\nu \partial_\lambda u_\sigma$

5. Macroscopic Manifestation of Anomalies

Motivated by Fluid / Gravity:

adding parity violating terms to hydro:

$$J^\mu = nu^\mu - \sigma T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu \boxed{+\xi\omega^\mu + \xi_B B^\mu}$$

$$\partial_\mu j^\mu = C E^\mu B_\mu, \qquad \partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

[Son Surowka 2009]

[Kharzeev Yee ; **Neiman Oz** 2011]

[Jensen Loganayagam Yarom 2012]

In Landau–Lifhitz frame

$$\xi = \mathcal{C}\mu^2\left(1 - \frac{2}{3}\frac{\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right) + \mathcal{D}T^2\left(1 - \frac{2\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right)$$

$$\xi_B = \mathcal{C}\mu\left(1 - \frac{1}{2}\frac{\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right) - \frac{\mathcal{D}}{2}\frac{\bar{n}T^2}{\bar{\epsilon}+\bar{p}}$$

6. Chiral transport is non–dissipative:

Ohm Law: $\boldsymbol{J} = \sigma \boldsymbol{E}$ dissipative

(-) (-)(+)

London 2nd eq.: $\frac{\partial \boldsymbol{J}_s}{\partial t} = \sigma_s \boldsymbol{E}$ non-dissipative

(+) (+)(+)

Transport in system of
Single right-handed fermions: $\boldsymbol{J} = \xi_B \boldsymbol{B}$?

$(-)$ $(+)(-)$

non-dissipative

7. A more realistic model :

Chiral hydrodynamics with both vector and axial currents:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$
$$\partial_\mu J^\mu = 0$$
$$\partial_\mu J_5^\mu = \mathcal{C} E_\mu B^\mu$$

with

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p\,\eta^{\mu\nu}$$
$$J^\mu = n u^\mu + \xi\,\omega^\mu + \xi_B B^\mu$$
$$J_5^\mu = n_5 u^\mu + \xi_5\,\omega^\mu + \xi_{B5} B^\mu$$

[Gao Liang Pu Wang Wang 2012]

[Landsteiner Megias Pena-Benitez 2013]

$$\xi = 2\mathcal{C}\left(\mu\mu_5 - \frac{n\mu_5}{3w}\left(3\mu^2 + \mu_5^2\right)\right) - 2\mathcal{D}\,\frac{n\mu_5}{w} T^2$$
$$\xi_5 = \mathcal{C}\left(\mu^2 + \mu_5^2 - \frac{2n_5\mu_5}{3w}\left(3\mu^2 + \mu_5^2\right)\right) + \mathcal{D}\left(1 - \frac{2n_5\mu_5}{w}\right) T^2$$
$$\xi_B = \mathcal{C}\,\mu_5\left(1 - \frac{n\mu}{w}\right)$$
$$\xi_{5B} = \mathcal{C}\mu\left(1 - \frac{n_5\mu_5}{w}\right)$$

8. Hydro excitations in a chiral system:

Long-wavelength excitations around equilibrium:

$$u^\mu = \Big(1,\ \boldsymbol{\Omega}\times\boldsymbol{x}\Big) \qquad \Omega r \ll 1,$$

$$T = Const.,\ \ \mu = Const.,\ \ \mu_5 = Const.$$

$$\boldsymbol{B} = Const.$$

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J_5^\mu = \mathcal{C} E_\mu B^\mu$$

perturbing the hydro fields: →

$$\phi_a = (T,\ \boldsymbol{\pi_i},\ \mu,\ \mu_5), \qquad a = 1, 2, ..., 6$$

$$M_{ab}^{B\Omega}(\boldsymbol{k},\omega)\delta\phi_a(\boldsymbol{k},\omega) = 0,$$

9. Fluid coupled to weak magnetic field

$$
\begin{bmatrix}
-i\alpha_1\omega & ik_j & -i\alpha_2\omega & -i\alpha_3\omega \\
i\alpha_1 v_s^2 k^i & -i\omega\delta^i_j - i\frac{\xi}{2\bar{w}}\left(\boldsymbol{B}\cdot\boldsymbol{k}\delta^i_j - B_j k^i\right) - \frac{\bar{n}}{\bar{w}}\epsilon^i{}_{jl}B^l & i\alpha_2 v_s^2 k^i & i\alpha_3 v_s^2 k^i \\
-i\beta_1\omega + \left(\frac{\partial\xi_B}{\partial T}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & \frac{\bar{n}}{\bar{w}} ik_j - \frac{\xi_B}{\bar{w}} i\omega B_j & -i\beta_2\omega + \left(\frac{\partial\xi_B}{\partial\mu}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & -i\beta_3\omega + \left(\frac{\partial\xi_B}{\partial\mu_5}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \\
-i\gamma_1\omega + \left(\frac{\partial\xi_{5B}}{\partial T}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & \frac{\bar{n}_5}{\bar{w}} ik_j - \frac{\xi_{5B}}{\bar{w}} i\omega B_j & -i\gamma_2\omega + \left(\frac{\partial\xi_{5B}}{\partial\mu}\right) i\boldsymbol{B}\cdot\boldsymbol{k} & -i\gamma_3\omega + \left(\frac{\partial\xi_{5B}}{\partial\mu_5}\right) i\boldsymbol{B}\cdot\boldsymbol{k}
\end{bmatrix}
$$

6 linear coupled equations give **6** hydro modes:

$\boldsymbol{k} \parallel \boldsymbol{B}$

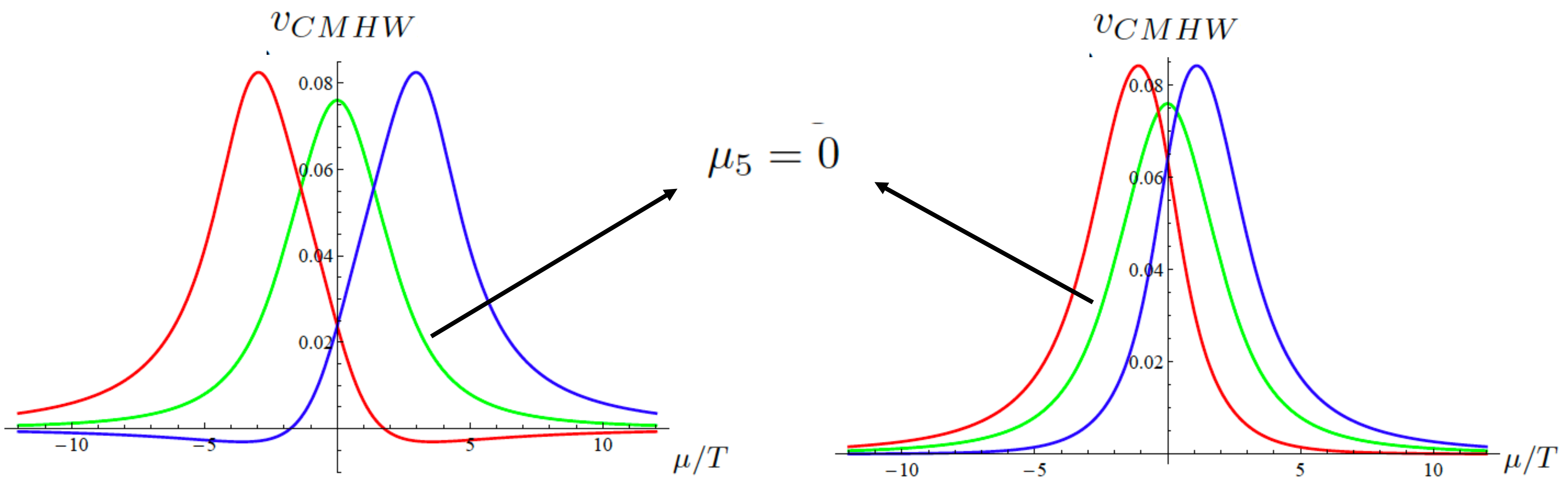
1,2: Chiral-Magnetic-Heat wave: $\omega_{1,2}(k) = -\frac{\mathcal{A}_1 \pm \sqrt{\mathcal{A}_1^2 - \mathcal{A}_2\mathcal{E}}}{\mathcal{E}}\boldsymbol{B}.\boldsymbol{k}$

3,4: Ordinary sound wave: $\omega_{3,4}(k) = \pm c_s k$

5,6: Chiral Alfven wave: $\omega_{5,6}(k) = \pm\frac{\xi}{2w}Bk$

10. Chiral –Magnetic–Heat Wave

Equation of state $$\epsilon = 3p = \frac{7\pi^2}{60}T^4 + \frac{1}{2}(\mu^2 + \mu_5^2)T^2 + \frac{1}{4\pi^2}(\mu^4 + 6\mu^2\mu_5^2 + \mu_5^4)$$



(a) $\frac{\mu_5}{T} = 3$ and $\frac{B}{T^2} = 0.5$

(b) $\frac{\mu_5}{T} = 1$ and $\frac{B}{T^2} = 0.5$

[N.A Allahbakhshi Davody Taghavi 2016]

11. CMHW in QGP:

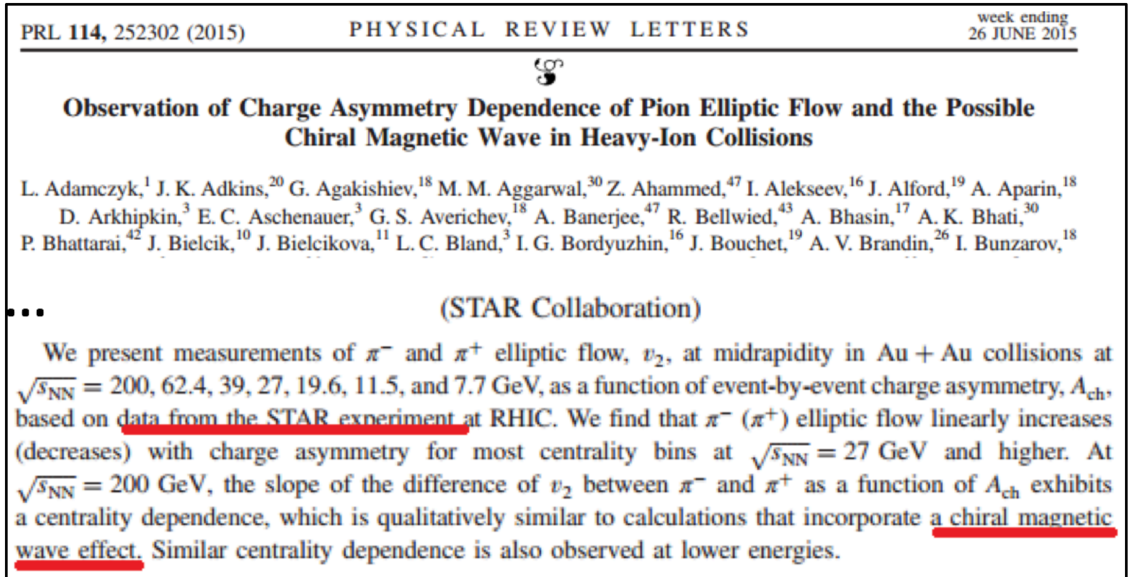
It is well–understood that the QGP produced in HIC in initially non–chiral: $\mu_5 = 0$

$$\omega_{1,2}(k) = -\frac{\mathcal{A}_1 \pm \sqrt{\mathcal{A}_1^2 - \mathcal{A}_2 \mathcal{E}}}{\mathcal{E}} \boldsymbol{B}.\boldsymbol{k}$$

$\mu_5 = 0$

CMW: [Kharzeev, Yee. 2012]

$$\omega_{1,2} = \pm \frac{Bk}{2\pi^2 \chi} \frac{1 - \frac{\mu n}{w}}{\sqrt{1 - \frac{\mu n}{w} - \frac{n}{\chi w}\left(\frac{n}{c_s^2} - \chi\mu\right)}}$$



PRL **114**, 252302 (2015) PHYSICAL REVIEW LETTERS week ending 26 JUNE 2015

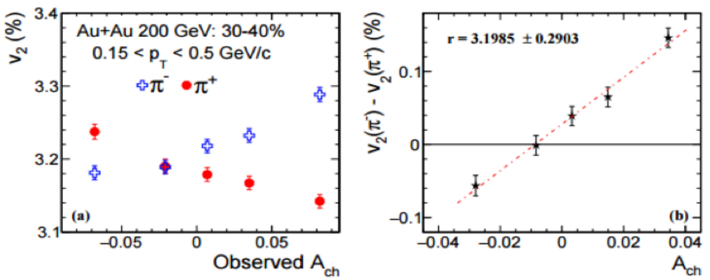
Observation of Charge Asymmetry Dependence of Pion Elliptic Flow and the Possible Chiral Magnetic Wave in Heavy-Ion Collisions

L. Adamczyk,[1] J. K. Adkins,[20] G. Agakishiev,[18] M. M. Aggarwal,[30] Z. Ahammed,[47] I. Alekseev,[16] J. Alford,[19] A. Aparin,[18] D. Arkhipkin,[3] E. C. Aschenauer,[3] G. S. Averichev,[18] A. Banerjee,[47] R. Bellwied,[43] A. Bhasin,[17] A. K. Bhati,[30] P. Bhattarai,[42] J. Bielcik,[10] J. Bielcikova,[11] L. C. Bland,[3] I. G. Bordyuzhin,[16] J. Bouchet,[19] A. V. Brandin,[26] I. Bunzarov,[18]

...

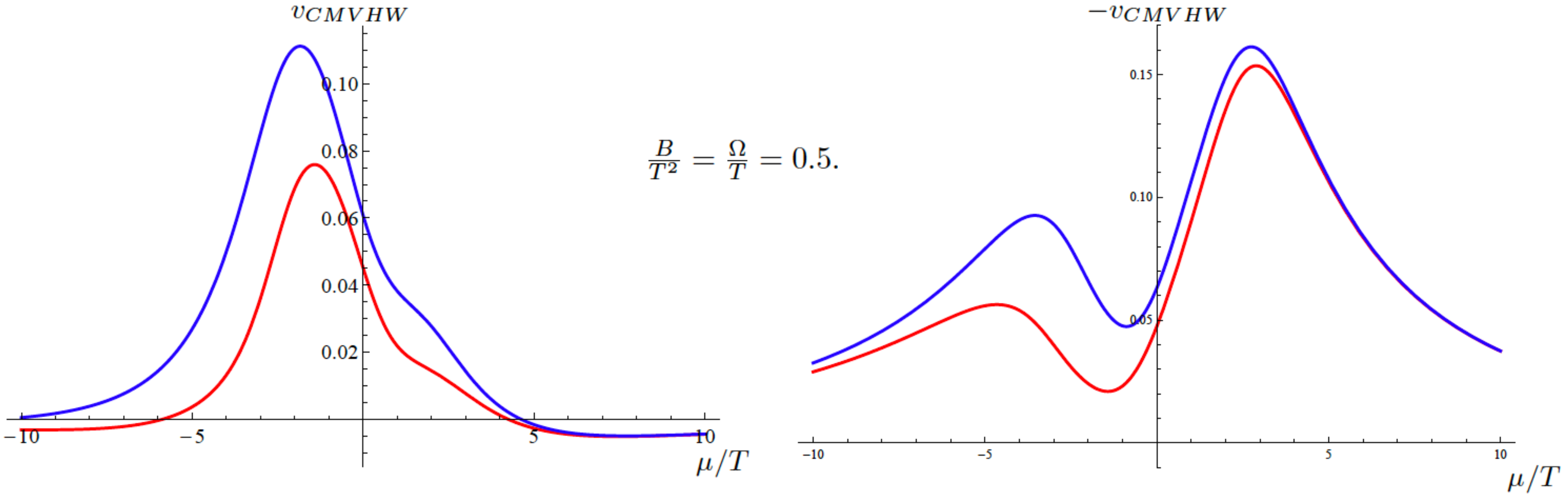
(STAR Collaboration)

We present measurements of π^- and π^+ elliptic flow, v_2, at midrapidity in Au + Au collisions at $\sqrt{s_{NN}} = 200$, 62.4, 39, 27, 19.6, 11.5, and 7.7 GeV, as a function of event-by-event charge asymmetry, A_{ch}, based on data from the STAR experiment at RHIC. We find that π^- (π^+) elliptic flow linearly increases (decreases) with charge asymmetry for most centrality bins at $\sqrt{s_{NN}} = 27$ GeV and higher. At $\sqrt{s_{NN}} = 200$ GeV, the slope of the difference of v_2 between π^- and π^+ as a function of A_{ch} exhibits a centrality dependence, which is qualitatively similar to calculations that incorporate a chiral magnetic wave effect. Similar centrality dependence is also observed at lower energies.



12. **Rotating** Fluid coupled to weak magnetic field

Scalar sector: mixed chiral-magnetic-vortical-heat wave



blue curve:

$$v_{sum} := v_{CMHW} + v_{CVHW} = v_{CMVHW}|_{\Omega=0} + v_{CMVHW}|_{B=0}$$

$$v_{CMVHW} \neq v_{sum}.$$

13. Out of equilibrium dynamics from **Kinetic theory**

In a system of **classical particles** with **rare collisions**

the dynamics of distribution function is given by:

Kinetic equation:

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \dot{\mathbf{x}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} = I_{coll}\{n_{\mathbf{p}}\}$$

Hamilton equations:

$$\begin{aligned} \dot{\mathbf{x}} &= \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}}, \\ \dot{\mathbf{p}} &= e\mathbf{E} + e\,\dot{\mathbf{x}} \times \mathbf{B}, \end{aligned}$$

14. In a system of Weyl Fermions

Particles interact with the Berry monopole located at the origin of momentum space

[Stephanov Yee 2012]

[Son Yamamoto 2012]

$$\dot{\mathbf{x}} = \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} + \dot{\mathbf{p}} \times \mathbf{\Omega}_{\mathbf{p}}$$

$$\dot{\mathbf{p}} = e\mathbf{E} + e\,\dot{\mathbf{x}} \times \mathbf{B},$$

$$\Omega_{\mathbf{p}} = \mathbf{\nabla} \times \mathbf{A}_{\mathbf{p}} = \lambda\, e \frac{\mathbf{p}}{\mathrm{p}^3}$$

CKT

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \dot{\mathbf{x}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{x}} + \dot{\mathbf{p}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} = I_{coll}\{n_{\mathbf{p}}\}$$

15. Hydro modes from CKT

For slowly-varying macroscopic fields $\beta(x), \boldsymbol{u}(x), \mu_{R,L}(x)$
we expand the distribution function around the thermo distribution:

$$\tilde{n}_{\mathbf{p}}^{(\lambda,e)} = \frac{1}{e^{\beta(\epsilon(\mathbf{p})-e\mu_\chi)}+1}.$$

to find the linearized equations:

$$M_{ab}\ \delta\phi_a(\omega, \boldsymbol{k}) = 0$$

with: $\phi_a(x) = \big(\beta(x), \boldsymbol{\pi}(x), \mu_R(x), \mu_L(x)\big)$

16. Comparison between **Hydro in LL** and **CKT**

For a fluid of single right-handed fermions at $\mu = 0$:

kinetic theory

relativistic hydro

[N.A Taghinavaz Naderi ?]

Type of mode	Chiral Kinetic Theory	Landau-lifshitz
CMHW	$v^{CKT}_{1,2}(k) = -\left(\mathcal{A}_1 \pm \sqrt{\mathcal{A}_2^2 - 4\,\mathcal{A}_3\,\mathcal{E}}\right)\frac{1}{2\,\mathcal{E}}\,B$	$v^{LL}_{1,2}(k) = -\left(\mathrm{A}_1 \pm \sqrt{\mathrm{A}_2^2 - 4\,\mathrm{A}_3\,\mathcal{E}}\right)\frac{1}{2\,\mathcal{E}}\,B$
Custard Sound	$v^{CKT}_{3,4} = \pm\frac{1}{\sqrt{3}} + \frac{\chi_R - \chi_L}{6\,w}\,B\,k,$	$v^{LL}_{3,4} = \pm\frac{1}{\sqrt{3}}$
CAW	$v^{CKT}_{5,6} = \frac{(n_R+n_L)(n_R-n_L)}{2w^2}\,B$	$v^{LL}_{4,5} = \left(\frac{(n_R-n_L)(\mu_R-\mu_L)}{2w^2} - \frac{\chi_R-\chi_L}{4w}\right)B$

17. Resolution of the differnce

Energy flow in the RF of the fluid in **Landau-Lifshitz** Frame

$$T^{i0}_{LL} = 0.$$

Energy flow computed in the RF of the fluid in **CKT**

$$T^{i0}_{CKT} = \frac{\bar{\chi}_R - \bar{\chi}_L}{4} \mathrm{B}_i$$

Perhaps these frames are Lorentz frames related to each other with:

[N.A Taghinavaz Naderi ?]

$$\boldsymbol{v}_{boost} = \frac{\bar{\chi}_R - \bar{\chi}_L}{4w} \mathbf{B}$$

18. The idea is true

By making the boost:

$$v_i^{CKT} \rightarrow \frac{v_i^{CKT} - v_{boost}}{1 - v_i^{CKT} v_{boost}} = v_i^{LL}$$

This means that:

in the absence of dissipation

Hydrodynamic modes, are **not** frame **invarinat**!

↓ hydro frame

They are in fact frame **covariant**!

19. In the absence of dissipation

Landau-Lifshitz

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p\,\eta^{\mu\nu}$$
$$J^\mu = n u^\mu + \xi \omega^\mu + \xi_B B^\mu$$

CKT

$$T^{\mu\nu} = w u^\mu u^\nu + p g^{\mu\nu} + \sigma^{\mathcal{B}}_\epsilon u^{(\mu} B^{\nu)} + \sigma^{\mathcal{V}}_\epsilon u^{(\mu} \omega^{\nu)}$$
$$j^\mu = n u^\mu + \sigma^{\mathcal{V}} \omega^\mu + \sigma^{\mathcal{B}} B^\mu$$

$$\delta u^\mu = -\frac{1}{\epsilon + p}\left(\sigma^{\mathcal{B}}_\epsilon B^\mu + \sigma^{\mathcal{V}}_\epsilon \omega^\mu\right)$$

20. How to get the same result from these two frames?

Thermodynamic (Lab)	Landau-Lifshitz.	Boost
$u^\mu = (1, \mathbf{0})$	$u^\mu = (1, \mathbf{0})$	$v_{LL} = \frac{v_{Lab} - \tilde{v}_{rel}}{1 - v_{Lab}\,\tilde{v}_{rel}}$ $\tilde{v}_{rel} = -\frac{\sigma_\epsilon^{\mathcal{B}} B + \sigma_\epsilon^{\mathcal{V}} \omega}{w}$
$u^\mu = (1, \mathbf{0})$	$u^\mu = (1, \tilde{\boldsymbol{v}}_{rel}), \quad u^\mu u_\mu = -1 + O(\tilde{\boldsymbol{v}}^2_{rel})$	$v_{LL} = v_{Lab}$

Physical frame, consistent with **Vilenkin 89** results.

"No-drag frame: [Stephanov Yee 2015] "

21. Consequences:

1- In single chirality fluid in the Lab frame CAW does not exist!

[Yamamoto 2105, **PRL**]

[N.A Davody Rezaei Hejazi 2016]

2- In QCD type fluid: CAW does exist:

$$v_{CAW} = \left(\mathcal{C}\mu\mu_5 - \frac{2n}{3w}\mathcal{C}\mu_5(3\mu^2 + \mu_5^2) - \frac{2n}{w}\mathcal{D}T^2 \right) B$$

$$\mathcal{C} = \frac{1}{2\pi^2}, \qquad \mathcal{D} = \frac{1}{6}$$

Gauge-gravitational anomaly in QGP.

22. Open questions

1. Non-hydro modes from chiral realtivistic hydrodynamic?

[Romatschke 2017]

2. Non-hydro modes from CKT?

[Romatschke 2015]

3. The full spectrum of chiral magneto hydrodynamics?

Thank you