# Effective field theory in the open systems

# Schwinger-Keldysh formalism and its gravity dual

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# 0. Outline

- Closed and open systems of IR modes
- Schwinger-Keldysh formalism (CTP)
- Generating functional and effective action
- Hydrodynamic effective action
- thermo-field dynamics
- Eternal AdS black hole

# 1. Closed system of IR modes

Closed:   conserved energy of the IR modes

1) harmonic oscillator:

$$\ddot{X}(t) + \omega^2 X(t) = 0$$

$$S = \int dt \left( \frac{1}{2} m \dot{X}^2 - \frac{1}{2} m \omega^2 X^2 \right)$$

2) ideal fluid

$$\partial_\mu s^\mu = 0$$

[Dubovsky, Gregoire, Nicolis, and Rattazzi JHEP 0512260]

$$S_0 = \int d^4x \, F(b) \, , \qquad b \equiv \sqrt{\det \partial_\mu \phi^I \partial^\mu \phi^J}$$

$$J^\mu \equiv \tfrac{1}{6} \epsilon^{\mu\alpha\beta\gamma} \epsilon_{IJK} \partial_\alpha \phi^I \partial_\beta \phi^J \partial_\gamma \phi^K$$

$$b^2 = -J_\mu J^\mu \, , \qquad u^\mu = \frac{1}{b} J^\mu$$

# 2. Open system of IR modes: dissipation

Open system:

energy flow

$$\{\psi\} = \{\phi\}_{\rm IR} + \{\chi\}_{\rm UV}$$

1) damped harmonic oscillator

$$\ddot{X}(t) + \gamma \dot{X}(t) + \omega^2 X(t) = 0$$

?

2) viscous fluid

effective acti $\partial_\mu s^\mu > 0$

dissipative fluid

?

# 3. Non-equilibrium dynamics

Suppose the system at g $t_i \to -\infty$ ensity $\rho(t_i)$

If the system evolves
with a time independent. : $H$ $\rho(t) = e^{-iH(t-t_i)}\rho(t_i)e^{iH(t-t_i)}$

If $H$ me-dependent: $H(t)$ $\rho(t) = U(t,t_i)\,\rho(t_i)\,U(t_i,t)$

$$i\frac{\partial U(t,t')}{\partial t} = H(t)U(t,t')$$

$$U(t,t') = T\left(\mathrm{e}^{-i\int_{t'}^{t} dt''\,H(t'')}\right)$$

# 4. Schwinger-Keldysh contour (CTP)

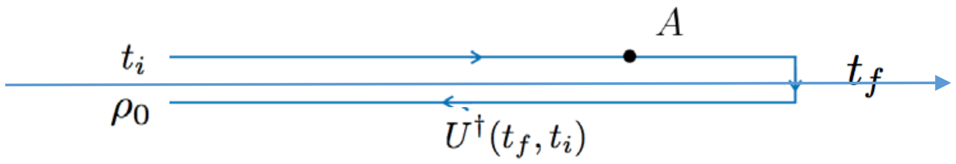
Time evolution of operators

$$\langle A\rangle(t) = \mathrm{Tr}\big(\rho(t)A\big)$$

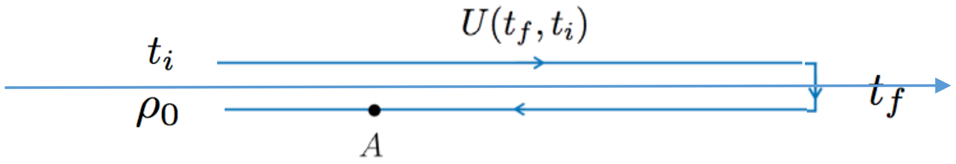
$$= \mathrm{Tr}\Big(U(t,t_i)\,\rho(t_i)\,U(t_i,t)\;A\Big)$$

$$= \mathrm{Tr}\Big(\rho(t_i)\,U(t_i,t)\;A\,U(t,t_i)\Big)$$

$$= \mathrm{Tr}\Big(\rho(t_i)\,U(t_i,t)\;A\,U(t,t_f)\,U(t_f,t_i)\Big) \qquad t_f \to +\infty$$



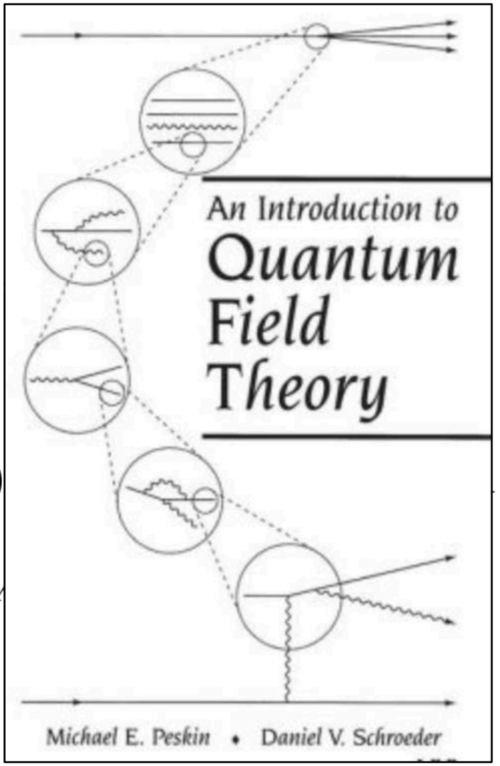

$$= \mathrm{Tr}\Big(\rho(t_i)\,U(t_i,t_f)\,U(t_f,t)\;A\,U(t,t_i)\Big)$$




**Closed time path**

# 5. Comparison with ?

Let us suppose the system initially




$$\rho(t_i) = |0_{t_i}\rangle\langle 0_{t_i}|$$

$$\langle A\rangle(t) = \mathrm{Tr}\big(\rho(t) A\big)$$

$$= \mathrm{Tr}\Big(U(t,t_i)\,\rho(t_i)\,U(t_i,t) \quad \cdots \quad U(t,t_i)\Big)$$

$$= \sum_n \langle n|0_{t_i}\rangle\langle 0_{t_i}|\,U(t_i,t)\ \cdots$$

$$= \langle 0_{t_i}|\,U(t_i,t_f)\,U(t_f,t)\ A\ \cdots$$

Adiabatic assumption $\quad U(t_f,t_i)|0_{t_i}\rangle = e^{iL}|0_{t_f}\rangle \quad \rightarrow \quad \langle 0_{t_f}|U(t_f,t_i)|0_{t_i}\rangle = e^{iL}$

$$= e^{-iL}\langle 0_{t_f}|\,U(t_f,t)\,A\,U(t,t_i)|0_{t_i}\rangle$$

$$= \frac{\langle 0_{t_f}|\,U(t_f,t)\,A\,U(t,t_i)|0_{t_i}\rangle}{\langle 0_{t_f}|U(t_f,t_i)|0_{t_i}\rangle}$$


$$\langle\Omega|T\{\phi(x)\phi(y)\}|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|T\Big\{\phi_I(x)\phi_I(y)\exp\big[-i\int_{-T}^{T}dt\,H_I(t)\big]\Big\}|0\rangle}{\langle 0|T\Big\{\exp\big[-i\int_{-T}^{T}dt\,H_I(t)\big]\Big\}|0\rangle}. \tag{4.31}$$

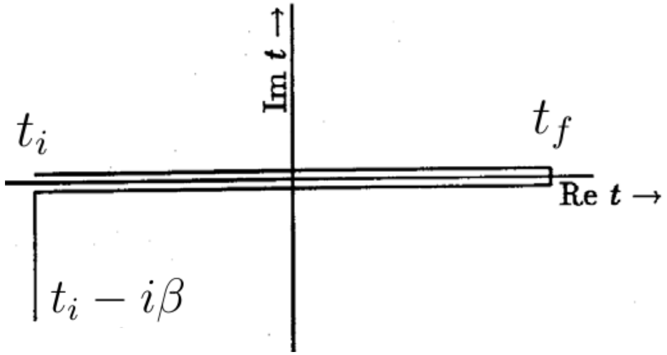
# 6. Dynamics around thermal equilibrium

Let us suppose the system initially in thermal equilibrium:

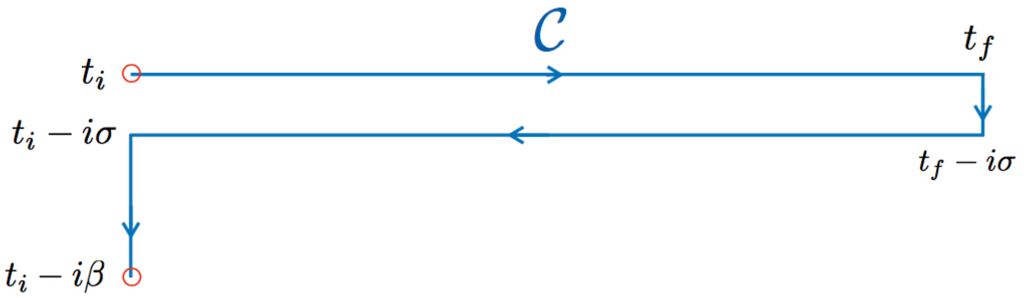
$$\rho(t_i) = \frac{e^{-\beta H_i}}{\mathrm{Tr}\, e^{-\beta H_i}} = \frac{U(T - i\beta, T)}{\mathrm{Tr}\, U(T - i\beta, T)}$$

$$H(t) = \begin{cases} H_i & \text{for} \quad \mathrm{Re}\, t \leq 0 \\ \mathcal{H}(t) & \text{for} \quad \mathrm{Re}\, t \geq 0 \end{cases}$$

$$\langle A \rangle(t) = \mathrm{Tr}\big(\rho(t) A\big) = \frac{\mathrm{Tr}\,\big(U(t_i - i\beta, t_i)\, U(t_i, t_f)\, U(t_f, t)\, A\, U(t, t_i)\big)}{\mathrm{Tr}\,\big(U(t_i - i\beta, t_i)\, U(t_i, t_f)\, U(t_f, t_i)\big)}$$



# 7. General SK contour around equilibrium




$$\sigma \in [0, \beta)$$

While the correlation functions are different compared to those of the $\sigma = 0$ case, they can be simply related to each other by analytic continuations

# 8. Generating functional

Rev of generating functional in zero temperature QFT

$$Z[J] = e^{iW[J]} = \int \mathcal{D}\phi \, e^{i\int d^4x(\mathcal{L}+J\phi)}$$


$$\langle \phi(x)\phi(y)\rangle_F \sim \frac{\delta^2 \ln Z[J]}{\delta J(x)\delta J(y)}$$

On the other hand

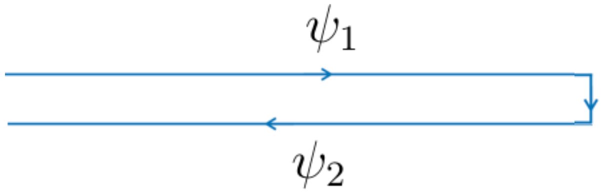
$$\langle \mathcal{O} \rangle = \int \mathcal{D}\phi \, e^{i\int d^4x \mathcal{L}} \, \mathcal{O} \; = \mathrm{Tr}\left(\rho(t_i)\mathcal{O}\right)$$

$$Z[J] = e^{iW[J]} = \int \mathcal{D}\phi \, e^{i\int d^4x \mathcal{L}} \; e^{i\int d^4x J\phi} \; = \langle \mathcal{T} e^{i\int d^4x J\phi} \rangle \; = \mathrm{Tr}\left(\rho(t_i)\, \mathcal{T} e^{i\int d^4x J\phi}\right)$$

$$\boxed{Z[J] = e^{iW[J]} = \mathrm{Tr}\left(\rho(t_i)\, \mathcal{T} e^{i\int d^4x J\phi}\right)}$$

# 9. SK formalism:
# generating functional and effective action on CTP

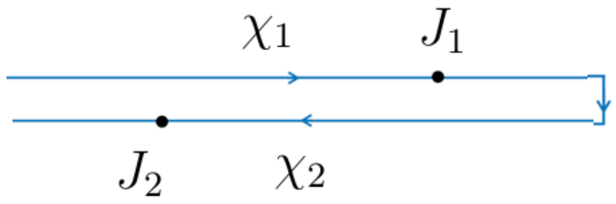
Doubling the dynamical fields at the micro level:




$$\langle \cdots \rangle = \mathrm{Tr}\left(\rho_0 \cdots\right) = \int_{\rho_0} \mathcal{D}\psi_1 \mathcal{D}\psi_2 \; e^{iS[\psi_1]-iS[\psi_2]+i\,\ln\cdots}$$
$$= \int \mathcal{D}\chi_1 \mathcal{D}\chi_2 \; e^{iS_{\mathrm{EF}}[\chi,\chi_2;\rho_0]+i\,\ln\cdots}$$

Connected correlation functions defined on a CTP contour:

$$Z[J] = e^{iW[J]} = \mathrm{Tr}\left(\rho_0\, \mathcal{P} e^{i\int d^4x (J_1(x)\chi_1(x)-J_2(x)\chi_2(x))}\right)$$
$$= \mathrm{Tr}\,\left(U_1(+\infty,-\infty;J_1)\,\rho_0\,U_2(-\infty,+\infty;J_2)\right)$$



# 10. r-a basis

It is more convenient to work with

$$\phi_r = \frac{\phi_1 + \phi_2}{2} \qquad \phi_a = \phi_1 - \phi_2$$

$$J_r = \frac{J_1 + J_2}{2} \qquad J_a = J_1 - J_2$$

$$\begin{aligned} e^{iW[J]} &= \mathrm{Tr}\left(\rho_0 \, \mathcal{P} e^{i\int d^4x (J_1(x)\chi_1(x) - J_2(x)\chi_2(x))}\right) \\ &= \mathrm{Tr}\left(\rho_0 \, \mathcal{P} e^{i\int d^4x (J_a(x)\chi_r(x) + J_r(x)\chi_a(x))}\right) \end{aligned}$$

$$\langle \chi_i(x)\chi_j(y)\rangle \sim \frac{\delta^2 W[J_1, J_2]}{\delta J_i(x)\delta J_j(y)}, \qquad i, j \in \{1, 2\} \qquad \langle \chi_{\alpha_1}(x)\chi_{\alpha_2}(y)\rangle \sim \frac{\delta^2 W[J_r, J_a]}{\delta J_{\bar{\alpha}_1}(x)\delta J_{\bar{\alpha}_2}(y)}, \qquad \alpha_1, \alpha_2 \in \{r, a\}$$

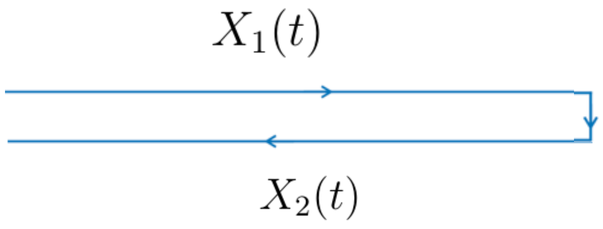
$$\begin{aligned} W[J_1, J_2] &= \frac{1}{2}\int d^4x d^4y \begin{pmatrix} J_1(x) & J_2(x) \end{pmatrix} \begin{pmatrix} G_F & -iG_- \\ -iG_+ & G_F \end{pmatrix} \begin{pmatrix} J_1(y) \\ J_2(y) \end{pmatrix} \\ &= \frac{1}{2}\int d^4x d^4y \begin{pmatrix} J_r(x) & J_a(x) \end{pmatrix} \begin{pmatrix} 0 & G_A \\ G_R & iG_S \end{pmatrix} \begin{pmatrix} J_r(y) \\ J_a(y) \end{pmatrix} \end{aligned}$$

# 11. Dissipation from EFT

Symmetries of the generating functional and effective action:

$$S_{\mathrm{EF;\ CTP}}[\chi_1, \chi_2] = -S^*_{\mathrm{EF;\ CTP}}[\chi_2, \chi_1]$$

e.g. damped Harmonic oscillator:




$$S_{\mathrm{EF;\ CTP}}[X_1, X_2] = S[X_1] - S[X_2] + S_{int}[X_1, X_2]$$

$$S[X] = \int dt \left(\frac{1}{2}m\dot{X}^2 - \frac{1}{2}m\omega^2 X^2\right) \qquad S_{int}[X_1, X_2] = \int dt \frac{1}{2}\gamma \left(\dot{X}_1 X_2 - X_1 \dot{X}_2\right)$$

$$\ddot{X}_{1,2} + \gamma \dot{X}_{2,1} + \omega^2 X_{1,2} = 0$$

$$X_1(t_f) = X_2(t_f) \qquad X_1(t) = X_2(t) = X(t)$$

$$\ddot{X} + \gamma \dot{X} + \omega^2 X = 0$$

# 12. Effective action for dissipative fluids

Symmetries:

[1511.03646, JHEP, Crossley, Glorioso and Liu]

1) KMS
2) Microscopic time reversal
3) Unitarity
4) Fluid world volume diff

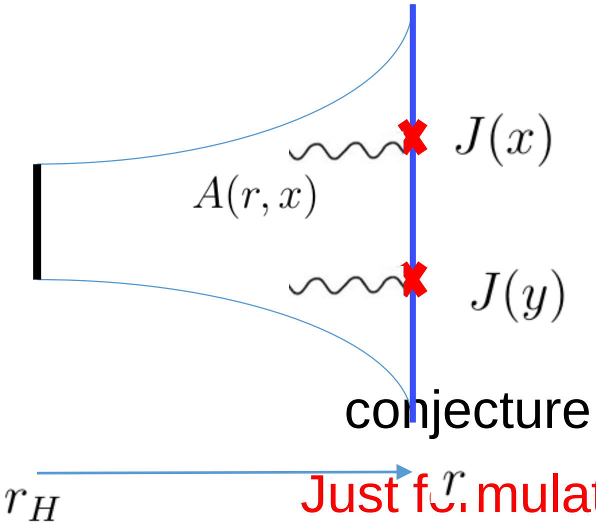
Fixe the form of the effective action

The EoMs read: $\partial_\mu T^{\mu\nu} = 0$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p\,\eta^{\mu\nu} - \eta P^{\mu\alpha} P^{\nu\beta} \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha\right) - \left(\zeta - \frac{2}{3}\eta\right) P^{\mu\nu} \partial . u$$

# 12+1. Dual gravity picture

Boundary thermal correlation functions from gravity




$$S = -\frac{1}{16\pi G_5}\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R - \frac{12}{L^2} + F^{MN}F_{MN}\right)$$

$$\Downarrow$$

$$\nabla_\mu F^{\mu\nu} = 0 \quad \Rightarrow \quad A^\mu_{sol}$$

conjecture:

$$Z_{grav.}[A^\mu_{sol}] = e^{iS[A^\mu_{sol}]} = Z_{CFT}[J]$$

Just formulated for Euclidean signature!

[Witten, 9802150]

Making analytic continuation to the Minkowski needs to knowing the $A^\mu_{sol}$ all Matsubara frequencies.

Practically impossible

# 14. Son-Starinets prescription [0205051, JHEP]

The on-shell action is formally written as

$$S[A^{\mu}_{sol}] = \int \frac{d^4p}{(2\pi)^4} A^{\mu}_0(-p)\mathcal{G}_{\mu\nu}(p,r)A^{\nu}_0(p)\Big|^{r\to\infty}_{r-r_H}$$

It derivative with respect to the boundary value of reads $A^{\mu}_{sol}$

$$G(p) = \mathcal{G}_{\mu\nu}(p,r)\Big|^{r\to\infty}_{r-r_H} - \mathcal{G}_{\mu\nu}(-p,r)\Big|^{r\to\infty}_{r-r_H}$$

In contrast to this c $G_R$ tity is real!

Recipe: $G_R(p) = -2\lim_{r\to\infty}\mathcal{G}_{\mu\nu}(p,r)$ ?, Son, Starinets]

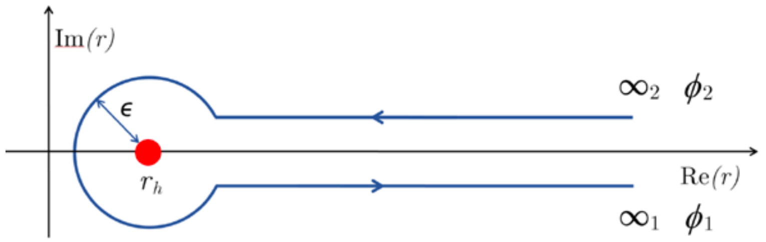
[0205052, JHEP, Policastro, Son, Starinets]

Towards the dual description of the SK contour in gravity

[0805.0150, PRL, Skenderis, Van Rees], [1504.07616, JHEP, de Boer, Heller, Pinzani-Fokeeva]

# 15. Very recent prescription:

Effective action. [1812.08785, Glorioso, Crossley, Liu]




$$ds^2 = -f(r,x^\mu)dv^2 + 2drdv + \lambda_{ij}(r,x^\mu)dx^i dx^j$$

$$f(r\to\infty)\to r^2, \qquad \lambda_{ij}(r\to\infty)\to\delta_{ij}r^2$$

$$S = -\frac{1}{2}\int d^{d+1}x\sqrt{-g}g^{MN}\partial_M\Phi\partial_N\Phi$$

$$\frac{1}{\sqrt{\lambda}}\partial_r\left(f\sqrt{\lambda}\partial_r\Phi\right) + \partial_0\partial_r\Phi + \frac{1}{\sqrt{\lambda}}\partial_r(\sqrt{\lambda}\partial_0\Phi) + \lambda^{ij}\partial_i\partial_j\Phi = 0$$

$$\Phi(r,x^\mu)\to\phi_1(x), \quad r\to\infty_1$$
$$\Phi(r,x^\mu)\to\phi_2(x), \quad r\to\infty_2$$

Derivative expansion

$$\Phi = \Phi^{(0)} + \Phi^{(1)} + \Phi^{(2)} + \cdots$$

$$\phi_r = \frac{1}{2}(\phi_1+\phi_2)$$

$$\phi_a = \phi_1 - \phi_2$$

# 16. Very recent prescription:

The on-shell action to 2nd order in boundary derivatives then reads

$$
\begin{aligned}
W &= W^{(0)} + W^{(1)} + W^{(2)} \\
&= \frac{i}{\beta}\int d^dx\,\sqrt{\lambda_h}\phi_a^2 - \int d^dx\,\sqrt{\lambda_h}\phi_a\partial_0\phi_r \\
&\qquad + \int d^dx\left\{Q_{ra}\partial_0\phi_a\partial_0\phi_r + Q_{ra}^{ij}\partial_i\phi_a\partial_j\phi_r + \frac{i}{2\beta}Q_{aa}(\partial_0\phi_a)^2 + \frac{i}{2\beta}Q_{aa}^{ij}\partial_i\phi_a\partial_j\phi_a\right\}
\end{aligned}
$$

$$
Q_{ra} = \frac{1}{2}r_h^2(\log 2 - 1), \quad Q_{ra}^{ij} = \frac{1}{2}r_h^2\delta_{ij}
$$

$$
= \frac{1}{2}\int d^4xd^4y\,\begin{pmatrix}J_r(x) & J_a(x)\end{pmatrix}\begin{pmatrix}0 & G_A \\ G_R & iG_S\end{pmatrix}\begin{pmatrix}J_r(y) \\ J_a(y)\end{pmatrix}
$$

$$
G_R(\omega, q) = r_h^3 i\omega + \frac{1}{2}r_h^2(\log 2 - 1)\omega^2 + \frac{1}{2}r_h^2 q^2
$$

[0205052, JHEP, Policastro, Son, Starinet]

# 17. Is the thermal state like a vacuum state?

Average in thermal state: $\langle A\rangle_\beta = Z^{-1}(\beta)\,\mathrm{Tr}\,e^{-\beta\mathcal{H}}A$

Could it be also written as , wi $\langle A\rangle_\beta = \langle 0,\beta|A|0,\beta\rangle$ acuum st $|0,\beta\rangle$

If yes, then

$$|0,\beta\rangle = \sum_n |n\rangle\langle n|0,\beta\rangle = \sum_n f_n(\beta)\,|n\rangle$$

so

$$\langle 0,\beta|A|0,\beta\rangle = \sum_{n,m} f_n^*(\beta) f_m(\beta)\langle n|A|m\rangle$$

and since

$$\langle A\rangle_\beta = Z^{-1}(\beta)\sum_n e^{-\beta E_n}\langle n|A|n\rangle$$

$$f_n^*(\beta) f_m(\beta) = Z^{-1}(\beta)\,e^{-\beta E_n}\delta_{nm}$$

With restricting our self to the original Hilbert space, we cannot define a thermal vacuum which gives

$$\langle A\rangle_\beta = Z^{-1}(\beta)\sum_n e^{-\beta E_n}\langle n|A|n\rangle$$

# 18. Thermo-field dynamics: thermal vacuum

Let us assume the product space of two systems

$$|n, \tilde{m}\rangle = |n\rangle \otimes |\tilde{m}\rangle$$

Now expand the thermal state as

$$|0, \beta\rangle = \sum_n f_n(\beta)|n, \tilde{n}\rangle = \sum_n f_n(\beta)|n\rangle \otimes |\tilde{n}\rangle$$

$$\begin{aligned}\langle 0, \beta|A|0, \beta\rangle &= \sum_{n,m} f_n^*(\beta) f_m(\beta)\langle n, \tilde{n}|A|m, \tilde{m}\rangle \\ &= \sum_{n,m} f_n^*(\beta) f_n(\beta)\langle n|A|n\rangle\, \delta_{nm} \\ &= \sum_n f_n^*(\beta) f_n(\beta)\langle n|A|n\rangle\end{aligned}$$

$$f_n^*(\beta) f_n(\beta) = Z^{-1}(\beta)\mathrm{e}^{-\beta E_n}$$

Compare it wit

$$\langle A\rangle_\beta = Z^{-1}(\beta)\sum_n \mathrm{e}^{-\beta E_n}\langle n|A|n\rangle$$

thermal vacuum state

$$|0, \beta\rangle = \frac{1}{\sqrt{Z(\beta)}}\sum_n e^{-\beta E_n/2}|n, \tilde{n}\rangle$$

# 19. Thermo-field dynamics: harmonic oscillator

Let us consider a fermionic harmonic oscillator:

$$H = \omega\, a^\dagger a \qquad [a, a^\dagger]_+ = 1 \qquad |1\rangle = a^\dagger|0\rangle$$

Make a copy:

$$\tilde{H} = \omega \tilde{a}^\dagger \tilde{a}$$

Space of states

$$|0,\tilde{0}\rangle;\ |0,\tilde{1}\rangle;\ |1,\tilde{0}\rangle;\ |1,\tilde{1}\rangle$$

Thermal vacuum

$$|0,\beta\rangle = \frac{1}{\sqrt{1+\mathrm{e}^{-\beta\omega}}}\left(|0,\tilde{0}\rangle + \mathrm{e}^{-\beta\omega/2}|1,\tilde{1}\rangle\right)$$

Thermal state can be constructed out of state: $|0,\tilde{0}\rangle$

$$U(\theta)|0,\tilde{0}\rangle = \cos\theta(\beta)\,|0,\tilde{0}\rangle + \sin\theta(\beta)\,|1,\tilde{1}\rangle = |0,\beta\rangle \qquad U(\theta) = \mathrm{e}^{-\theta(\beta)(\tilde{a}a - a^\dagger\tilde{a}^\dagger)}$$

We find the transformation of the ladder operators:

$$a(\beta) = a\cos\theta(\beta) - \tilde{a}^\dagger\sin\theta(\beta)$$

um state $a(\beta)|0,\beta\rangle = 0$

$$\tilde{a}(\beta) = \tilde{a}\cos\theta(\beta) + a^\dagger\sin\theta(\beta)$$

# 20. Thermo-field dynamics: Hamiltonian

Thermal Hilbert space of oscillator

$$|0,\beta\rangle;\ a^\dagger(\beta)|0,\beta\rangle;\ \tilde{a}^\dagger(\beta)|0,\beta\rangle;\ a^\dagger(\beta)\tilde{a}^\dagger(\beta)|0,\beta\rangle$$

These states are not eigen states of $H$ or $\tilde{H}$

But since

$$a^\dagger(\beta)a(\beta) - \tilde{a}^\dagger(\beta)\tilde{a}(\beta) = a^\dagger a - \tilde{a}^\dagger \tilde{a}$$

So the Hamiltonian of the combined system is

$$\hat{H} = H - \tilde{H}$$

Is there any relation between “thermo-field” and “CTP”?

# 21. Thermo-field dynamics: free KG theory

Let's consider

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2$$
$$\tilde{\mathcal{L}} = \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - \frac{m^2}{2}\tilde{\phi}^2$$

$$\hat{\mathcal{L}} = \mathcal{L} - \tilde{\mathcal{L}}$$

Green's function is given by a $2\times 2$ matrix

$$iG(x-y) = \langle 0,\tilde{0}|T(\Phi(x)\Phi(y))|0,\tilde{0}\rangle$$

$$G(k) = \begin{pmatrix} \frac{1}{k^2-m^2+i\epsilon} & 0 \\ 0 & -\frac{1}{k^2-m^2-i\epsilon} \end{pmatrix}$$

The Green's function in thermal vacuum

$$\begin{aligned} iG_\beta(x-y) &= \langle 0,\beta|T(\Phi(x)\Phi(y))|0,\beta\rangle \\ &= \langle 0,\tilde{0}|T(U^\dagger(\theta)\Phi(x)\Phi(y)U(\theta))|0,\tilde{0}\rangle \\ &= \overline{U}(-\theta)iG(x-y)\overline{U}^T(-\theta) \end{aligned}$$

$$G_\beta(k) = \begin{pmatrix} \frac{1}{k^2-m^2+i\epsilon} & 0 \\ 0 & -\frac{1}{k^2-m^2-i\epsilon} \end{pmatrix} - 2i\pi n_B(|k^0|)\delta(k^2-m^2)\begin{pmatrix} 1 & e^{\beta|k^0|/2} \\ e^{\beta|k^0|/2} & 1 \end{pmatrix}$$

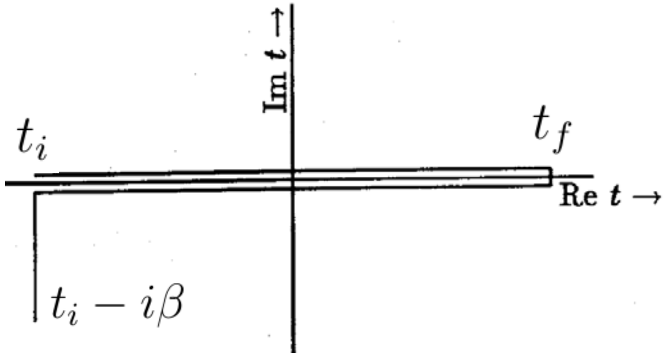
# 22. thermo-field vs. CPT

1.Propagator of KG field on CPT

$$G = \begin{pmatrix} G_{++} & G_{+-} \\ G_{-+} & G_{--} \end{pmatrix}$$

$$G_{++}(p) = \left(\frac{1}{p^2 - m^2 + i\epsilon} - 2i\pi n_B(|p^0|)\delta(p^2 - m^2)\right)$$

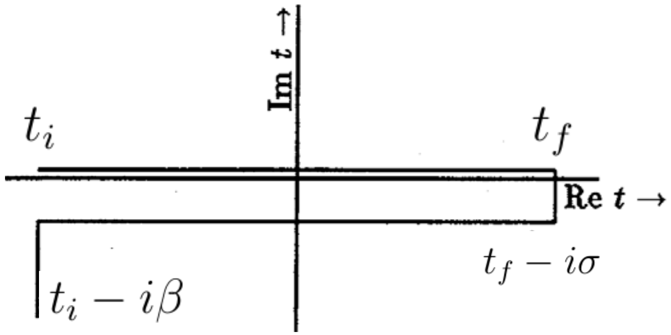
$$G_{+-}(p) = -2i\pi\left(\theta(-p^0) + n_B(|p^0|)\right)\delta(p^2 - m^2)$$




2.Propagator of KG field on Generalized CTP

$$G_{11}(p) = G_{++}(p)$$

$$G_{12}(p) = e^{\sigma p^0}\, G_{+-}(p)$$




3.Propagator of KG field on Generalized CTP

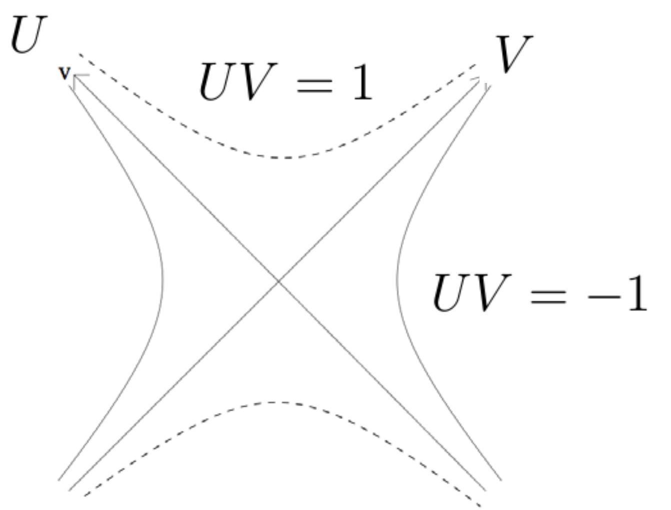
$$G_\beta(k) = \begin{pmatrix} \frac{1}{k^2 - m^2 + i\epsilon} & 0 \\ 0 & -\frac{1}{k^2 - m^2 - i\epsilon} \end{pmatrix} - 2i\pi n_B(|k^0|)\delta(k^2 - m^2)\begin{pmatrix} 1 & e^{\beta|k^0|/2} \\ e^{\beta|k^0|/2} & 1 \end{pmatrix}$$

**Thermo-field dynamics = CTP formalism with** $\sigma = \frac{\beta}{2}.$

# 23. Analogy with eternal AdS black hole I

Let's consider a "Schwartz-child" black-hole

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\vec{x}^2 \qquad f(r) = r^2\left(1 - \frac{r_H^4}{r^4}\right)$$




Kruskal coordinates:

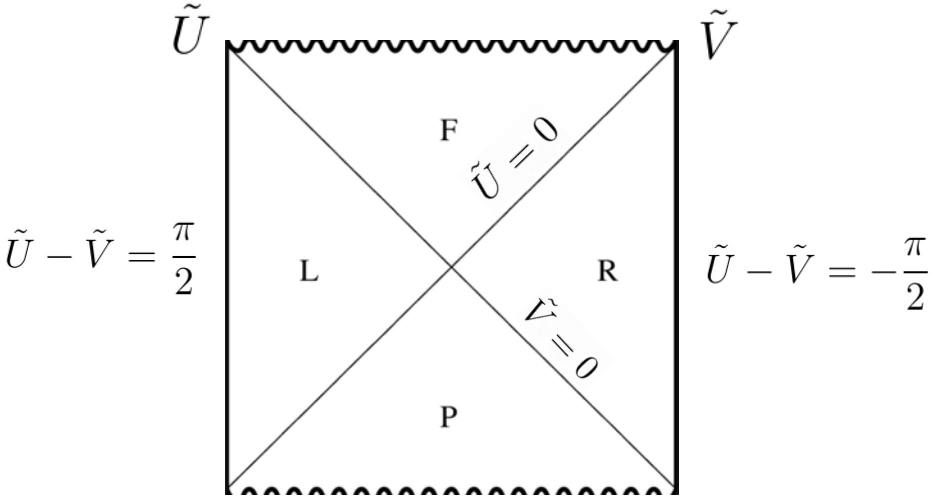
$$ds^2 = d\vec{x}^2 = 0 \quad \rightarrow \quad dt = \frac{1}{f(r)^2}dr \equiv dr^*$$

$$v = t + r_*$$
$$u = t - r_*$$

R:
$$U = -e^{-\frac{2\pi}{\beta}u}$$
$$V = e^{\frac{2\pi}{\beta}v}$$

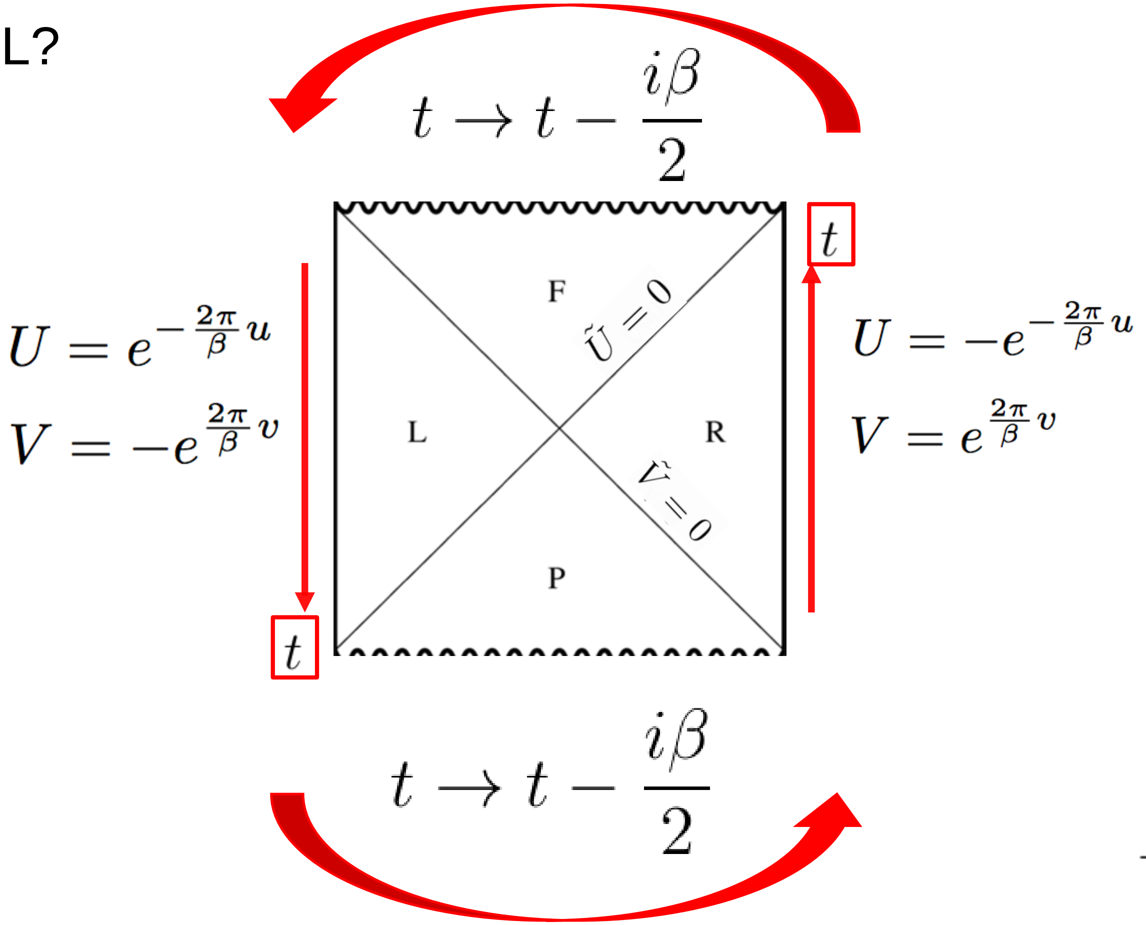
Penrose diagram:

R:
$$\tilde{U} = \tan^{-1} U$$
$$\tilde{V} = \tan^{-1} V$$

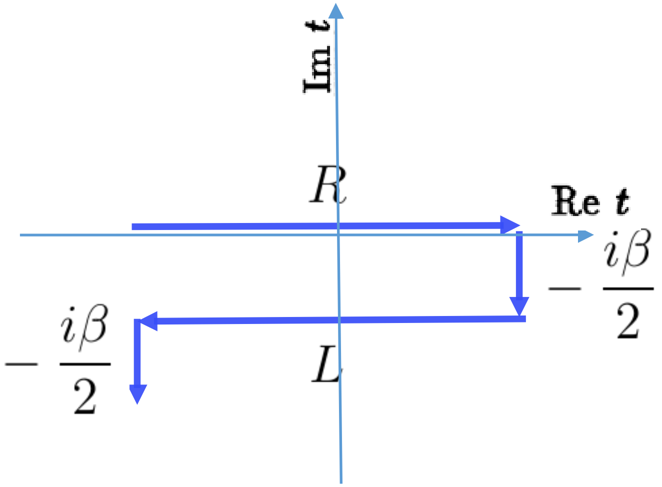


# 24. Analogy with eternal AdS black hole II

How to go from R to L?

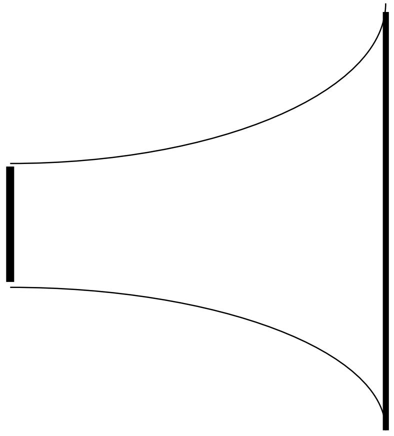



$$v = t + r_*$$
$$u = t - r_*$$

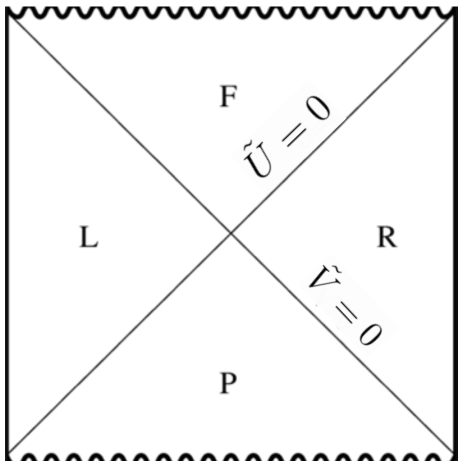


# 25. Summary

One-sided AdS black hole



eternal black-hole




Mixed thermal state

$$\rho = \mathrm{Tr}\, e^{-\beta H}$$

thermo-field state

$$|0, \beta\rangle$$

Thank you for your attention