Collective Excitation of a Chiral Fluid from Kinetic Theory

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Outline of talk

- Motivation
- Chiral Kinetic Theory
- Energy & momentum modification
- How to obtain hydro modes
- Introduction of frame notion and its importance
- Conclusion

Motivation

- **Are we able to include underlying microscopic phenomena in a macroscopic approach?**

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Yes!!!

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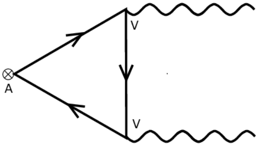
Yes!!!

Anomaly in QFT

- **Anomaly**: A symmetry that is not preserved at quantum level

$$J_V^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x), \quad J_A^\mu(x) = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x).$$



$$\partial_\mu J_V^\mu = 0,$$
$$\partial_\mu J_A^\mu = \frac{e^2}{4\pi^2}\mathbf{E}\cdot\mathbf{B}.$$

Michael E. Peskin, Dan V. Schroeder, An Introduction To Quantum Field Theory-Westview Press (1995).

History

- Weakly interacting system in presence of magnetic field:

  A. Vilenkin, Phys. Rev. D **20**, 1807 (1979),

  A. Vilenkin, Phys. Rev. D **22**, 3080 (1980).

- Strongly interacting system in presence of magnetic field:

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- **Chiral Kinetic Theory (CKT):**

  M. A. Stephanov and Y. Yin, Phys. Rev. Lett. **109**, 162001 (2012).

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- **Chiral Kinetic Theory (CKT)**:

  M. A. Stephanov and Y. Yin, Phys. Rev. Lett. **109**, 162001 (2012).

CKT

- **CKT**: A framework to study the kinetic theory of Weyl fermions. This approach is semi classic which involves first order quantum correction into the Boltzmann equation.
- Using path integral approach and then Diagonalize fermi-Dirac Hamiltonian in momentum space

$$S = \int_{t_i}^{t_f} dt \left([\mathbf{p} + e\mathbf{A}] \cdot \dot{\mathbf{x}} - \epsilon_p - \mathbf{a}_p \cdot \dot{\mathbf{p}} \right).$$

$\mathbf{a}_p$ here is Berry phase: $\mathbf{a}_p = iV_p^\dagger \vec{\nabla}_p V_p$

M. A. Stephanov and Y. Yin, Phys. Rev. Lett. **109**, 162001 (2012).

Equations of motion

- Berry flux contribution

$$\dot{\mathbf{x}} = \frac{\partial \epsilon_p}{\partial \mathbf{p}} + e\dot{\mathbf{p}} \times \vec{\Omega}_p,$$
$$\dot{\mathbf{p}} = e\vec{E} + e\,\dot{\mathbf{x}} \times \vec{B}.$$

$\vec{\Omega}_p$ is Berry flux: $\vec{\Omega}_p = \vec{\nabla}_p \times \mathbf{a}_p$

- Canonical forms of equations

$$\sqrt{G}\,\dot{\mathbf{x}} = \frac{\partial \epsilon_p}{\partial p_j} + e\,\vec{E} \times \vec{\Omega}_p + e\vec{B}(\hat{p} \cdot \vec{\Omega}_p),$$
$$\sqrt{G}\,\dot{\mathbf{p}} = e\vec{E} + e\,\hat{p} \times \vec{B} + e^2\vec{\Omega}_p(\vec{E} \cdot \vec{B}),$$
$$\sqrt{G} = 1 + e\vec{B} \cdot \vec{\Omega}_p.$$

- Phase space integrals

$$\int \frac{d^3x d^3p}{(2\pi)^6} \to \int \sqrt{G}\frac{d^3x d^3p}{(2\pi)^6}.$$

Anomaly and kinetic theory

- Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho \dot{\mathbf{x}})}{\partial \mathbf{x}} + \frac{\partial(\rho \dot{\mathbf{p}})}{\partial \mathbf{p}} = 2\pi e^2 \mathbf{E}\cdot\mathbf{B}\delta^3(p)f(p),$$

$$\Rightarrow \qquad \frac{\partial n}{\partial t} + \frac{\partial J_i}{\partial x_i} = \frac{e^2}{4\pi^2}\mathbf{E}\cdot\mathbf{B}, \quad (\text{with } \rho \equiv \sqrt{G}\, f).$$

- Chiral magnetic effect

$$\mathbf{J}_{CME} = \int \frac{d^3p}{(2\pi)^3}\sqrt{G}\,\dot{\mathbf{x}} f(p) = e\vec{B}\int \frac{d^3p}{(2\pi)^3}\hat{p}\cdot\vec{\Omega}_p f(p) = \frac{e\mu}{4\pi^2}\mathbf{B}.$$

- Chiral Vortical effect

$$\mathbf{J}_{CVE} = \int \frac{d^3p}{(2\pi)^3}\sqrt{G}\,\dot{\mathbf{x}} f(p) = 2\vec{\omega}\int \frac{d^3p}{(2\pi)^3} p\cdot\vec{\Omega}_p f(p) = \frac{\mu^2}{4\pi^2}\vec{\omega}.$$

M. A. Stephanov and Y. Yin, Phys. Rev. Lett. **109**, 162001 (2012).

Comparison with previous results

- Weakly interacting fermions in presence of arbitrary global rotation:

$$J_\omega(0) = \vec{\omega}\left(\frac{T^2}{12} + \frac{\mu^2}{4\pi^2}\right).$$

A. Vilenkin, Phys. Rev. D **20**, 1807 (1979)

- Weakly interacting fermions in presence of arbitrary magnetic field:

$$J_B = \vec{B}\frac{e\mu}{4\pi^2}.$$

A. Vilenkin, Phys. Rev. D **22**, 3080 (1980).

Current density

- Kinetic equation

$$(1 + eB \cdot \vec{\Omega}_p)\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_i}\left[\frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}_i} + e\,\mathbf{B}_i\left(\frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \vec{\Omega}_p\right)\right]$$
$$+e\frac{\partial f}{\partial p_i}(\frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \times \mathbf{B})_i = 0.$$

- Current density

$$\frac{\partial J_0}{\partial t} + \frac{\partial J_i}{\partial x_i} = 0, \;\; J_0 = \int \frac{d^3p}{(2\pi)^3}\sqrt{G}\, f(p),$$
$$J_i = -\int \frac{d^3p}{(2\pi)^3}\epsilon_{\mathbf{p}}\left[\frac{\partial f(p)}{\partial \mathbf{p}_j}\left(\delta_{ij} + e\,\mathbf{B}_i\vec{\Omega}_{p_j}\right) + (\vec{\Omega}_p \times \frac{\partial f(p)}{\partial x})_i\right],$$

D. T. Son and N. Yamamoto, Phys. Rev. D **87**, no. 8, 085016 (2013).

Energy-momentum density

- Energy density & energy flux

$$\frac{\partial T^{00}}{\partial t}+\frac{\partial T^{0i}}{\partial x_i}=0,\quad T^{00}=\int\frac{d^3p}{(2\pi)^3}\epsilon_{\mathbf{p}}\left(1+e\mathbf{B}\cdot\mathbf{b}_p\right)\,f(p)$$

$$T^{0i}=-\int\frac{d^3p}{(2\pi)^3}\frac{\epsilon_{\mathbf{p}}^2}{2}\left[\frac{\partial f(p)}{\partial \mathbf{p}_j}\left(\delta_{ij}+e\,\mathbf{B}_i\vec{\Omega}_{p_j}\right)+(\vec{\Omega}_p\times\frac{\partial f(p)}{\partial x})_i\right],$$

- Momentum density & momentum flux

$$\frac{\partial T^{i0}}{\partial t}+\frac{\partial T^{ij}}{\partial x_j}=(\vec{J}\times\vec{B})_i,\quad T^{i0}=\int\frac{d^3p}{(2\pi)^3}p^i\left(1+e\mathbf{B}\cdot\vec{\Omega}_p\right)\,f(p)$$

$$T^{ij}=\int\frac{d^3p}{(2\pi)^3}\mathbf{p}_i\left[\epsilon_{\mathbf{p}}\frac{\partial f(p)}{\partial \mathbf{p}_k}\left(\delta_{jk}+e\,\mathbf{B}_j\vec{\Omega}_{p_k}\right)+\epsilon_{jk\ell}\vec{\Omega}_{p_k}\epsilon_{\mathbf{p}}\frac{\partial f(p)}{\partial x_\ell}\right]$$
$$+\delta_{ij}T^{00}.$$

D. T. Son and N. Yamamoto, Phys. Rev. D **87**, no. 8, 085016 (2013).

Energy modification

- Lorentz invarance:

$$T^{0i} = T^{i0}$$

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- For homogeneous systems ($\frac{\partial f(p)}{\partial x_\ell} = 0$):

$$\epsilon_{\mathbf{p}} \frac{\partial \epsilon_p}{\partial p_j} \left(\delta_{ij} + e\,\mathbf{B}_i \vec{\Omega}_{p_j} \right) = p^i \left(1 + e\mathbf{B} \cdot \vec{\Omega}_p \right),$$

$$\epsilon_{\mathbf{p}} = \mathbf{p} \left(1 - \boxed{\hbar} \vec{B} \cdot \vec{\Omega}_p \right).$$

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Momentum modification

- For inhomogeneous systems ($\frac{\partial f(p)}{\partial x_\ell} \neq 0$):

$$\epsilon_{\mathbf{p}} \frac{\partial \epsilon_p}{\partial p_j} \left(\delta_{ij} + e\,\mathbf{B}_i \vec{\Omega}_{p_j} \right) - \frac{\epsilon_{\mathbf{p}}^2}{2} (\vec{\Omega}_p \times \partial)_i = p^i \left(1 + e\mathbf{B} \cdot \vec{\Omega}_p \right),$$

$$\epsilon_{\mathbf{p}} = \mathbf{p} \left(1 - \hbar \vec{B} \cdot \vec{\Omega}_p \right),$$

$$\tilde{p}_i = p_i - \frac{\epsilon_{\mathbf{p}}^2}{2} (\vec{\Omega}_p \times \partial)_i .$$

N. Abbasi, F. Taghinavaz and K. Naderi, JHEP **1803**, 191 (2018).

Similar works

- Hydro modes in chiral systems by setting $\delta\vec{u} = 0$ for **collision less particles.** Their fluctuations set is $(\delta\mu_R, \delta\mu_L, \delta T)$. Only currents and energy conservation equations are considered.

  D. Frenklakh, Phys. Rev. D **94**, no. 11, 116010 (2016).

- Hydro modes in chiral systems again by setting $\delta\vec{u} = 0$ for **collisioned particles modelled by RTA .**

  M. Stephanov, H. U. Yee and Y. Yin, Phys. Rev. D **91**, no. 12, 125014 (2015).

- **Both of them are incomplete due to neglect of momentum conservation equations.**

  N. Abbasi, F. Taghinavaz and K. Naderi, JHEP **1803**, 191 (2018).

How to obtain hydro modes in CKT

- CK equation

$$\sqrt{G}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial t} + \sqrt{G}\dot{\mathbf{x}}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial \mathbf{x}} + \sqrt{G}\dot{\mathbf{p}}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial \mathbf{p}} = 0.$$

- Fluctuations

$$\delta\phi_a = (\delta\mu_R, \delta\mu_L, \delta\epsilon, \delta u_i),$$

$$f^{(\lambda,e)}(\mathbf{p},x) \to f_{eq}^{(\lambda,e)}(\mathbf{p},x) + \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial \phi_a}\delta\phi_a,$$

$$\mu \to \mu_0 + \epsilon_F \delta\mu,\ \ \beta \to \beta_0 + \epsilon_F \delta\beta,\ \ u^i \to u^i_{eq} + \epsilon_F \delta u^i.$$

- Multiply both side of CK to collision invariants ($\mathbf{1}_R, \mathbf{1}_L$, modified energy, modified momentum).

N. Abbasi, F. Taghinavaz and K. Naderi, JHEP **1803**, 191 (2018).

Conservation equations

- Current conservation

$$\sum_{e=\pm 1}\int_{\mathbf{p}} e\left(\sqrt{G}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial t} + \sqrt{G}\dot{\mathbf{x}}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial \mathbf{x}} + \sqrt{G}\dot{\mathbf{p}}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial \mathbf{p}}\right) = 0.$$

- Energy conservation

$$\sum_{e,k=\pm 1}\int_{\mathbf{p}} \epsilon_{\mathbf{p}\chi}\left(\sqrt{G}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial t} + \sqrt{G}\dot{\mathbf{x}}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial \mathbf{x}} + \sqrt{G}\dot{\mathbf{p}}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial \mathbf{p}}\right) = 0.$$

- Momentum conservation

$$\sum_{e,k=\pm 1}\int_{\mathbf{p}} \tilde{p}^i_\chi\left(\sqrt{G}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial t} + \sqrt{G}\dot{\mathbf{x}}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial \mathbf{x}} + \sqrt{G}\dot{\mathbf{p}}_\chi \frac{\partial f^{(\lambda,e)}(\mathbf{p},x)}{\partial \mathbf{p}}\right) = 0.$$

$$\lambda_{R,L} = \pm\frac{1}{2}, \quad \int_{\mathbf{p}} = \int \frac{d^3p}{(2\pi)^3}.$$

N. Abbasi, F. Taghinavaz and K. Naderi, JHEP **1803**, 191 (2018).

Coefficient Matrix

$$\mathcal{M}_{ab}\delta\phi_b = 0.$$

- Collective modes

$$\mathcal{M}_{ab} = \begin{bmatrix} \bar{\chi}_R\,\omega - \frac{\mathrm{B}}{4\pi^2}\,k & 0 & T(\mu_R\bar{\chi}_R - 3n_R)\,\omega & 0 & 0 & \frac{\mu_R}{w}\left(\frac{\mathrm{B}}{4\pi^2}\,\omega - \frac{n_R}{\mu_R}\,k\right) \\ 0 & \bar{\chi}_L\,\omega + \frac{\mathrm{B}}{4\pi^2}\,k & T(\mu_L\bar{\chi}_L - 3n_L)\,\omega & 0 & 0 & -\frac{\mu_L}{w}\left(\frac{\mathrm{B}}{4\pi^2}\,\omega + \frac{n_L}{\mu_L}\,k\right) \\ 3n_R\,\omega - \frac{\mu_R\mathrm{B}}{4\pi^2}\,k & 3n_L\,\omega + \frac{\mu_L\mathrm{B}}{4\pi^2}\,k & -\omega C_v T^2 & 0 & 0 & \frac{(\chi_R-\chi_L)\,\mathrm{B}}{2w}\,\omega - k \\ 0 & 0 & 0 & \omega & -\frac{i}{w}\left((n_R+n_L)\mathrm{B} + \frac{1}{2}\,k\,\omega\,(n_R-n_L)\right) & 0 \\ 0 & 0 & 0 & \frac{i}{w}\left((n_R+n_L)\mathrm{B} + \frac{1}{2}\,k\,\omega\,(n_R-n_L)\right) & \omega & 0 \\ \frac{\mu_R\mathrm{B}}{4\pi^2}\,\omega - n_R\,k & -\frac{\mu_L\mathrm{B}}{4\pi^2}\,\omega - n_L\,k & -\frac{kC_vT^2}{3} & 0 & 0 & \omega - \frac{(\bar{\chi}_R-\bar{\chi}_L)\,\mathrm{B}}{2w}\,k \end{bmatrix}.$$

- Hydro modes

$$det(\mathcal{M}) = 0,$$
$$k \to \epsilon_f k, \;\; \omega \to \epsilon_f\omega^{(0)} + \epsilon_f^2\omega^{(1)}.$$

- Conservation equations are expanded up to first order in fluctuations (ϵ_F) and second order in gradients (ϵ_f^2).

Collective modes from CKT

- Chiral Magnetic Heat Wave

$$\omega_{1,2}(k) = -\left(\mathcal{A}_1 \pm \sqrt{\mathcal{A}_2^2 - 4\,\mathcal{A}_3\,\mathcal{E}}\right)\frac{1}{2\,\mathcal{E}}\,\mathrm{B}\,k.$$

- Sound Wave

$$\omega_{3,4}(k) = \pm\frac{1}{\sqrt{3}}\,k + \frac{\bar{\chi}_R - \bar{\chi}_L}{6\,w}\,\mathrm{B}\,k.$$

- Chiral Alfvén Wave

$$\omega_{5,6}(k) = \pm\frac{n_R + n_L}{w}\,\mathrm{B} + \frac{(n_R + n_L)(n_R - n_L)}{2w^2}\,\mathrm{B}\,k.$$

N. Abbasi, F. Taghinavaz and K. Naderi, JHEP **1803**, 191 (2018).

Collective modes in Landau frame

- Chiral Magnetic Heat Wave

$$\omega_{1,2}(k) = -\left(A_1 \pm \sqrt{A_2^2 - 4\,A_3\,\mathcal{E}}\right)\frac{1}{2\,\mathcal{E}}\,\mathrm{B}\,k.$$

- Sound Wave

$$\omega_{3,4}(k) = \pm\frac{1}{\sqrt{3}}\,k,$$

- Chiral Alfvén Wave

$$\omega_{5,6}(k) = \pm\frac{n_R + n_L}{w}\,\mathrm{B} + \left(\frac{(n_R + n_L)(n_R - n_L)}{2w^2} - \frac{\bar{\chi}_R - \bar{\chi}_L}{4w}\right)$$

N. Abbasi, D. Allahbakhshi, A. Davody and S. F. Taghavi, Phys. Rev. D **96**, no. 12, 126002 (2017).

Idea of frame

- Landau frame

$$T^{i0}_{LRF} = 0$$

- CKT frame

$$T^{i0}_{CKT} = \sum_\lambda \sum_e \int \frac{d^3\mathrm{p}}{(2\pi)^3} \sqrt{G}\, \mathrm{p}^i \left(\tilde{n}_{\mathbf{p}}^{(\lambda,e)} - \left(\frac{\partial \tilde{n}_{\mathbf{p}}^{(\lambda,e)}}{\partial \epsilon(\mathbf{p})} \right)_{eq.} \left(e\lambda \frac{\mathbf{B}\cdot\mathbf{p}}{\mathrm{p}^2} \right) \epsilon_f \right)$$

$$= \sum_\lambda \sum_e \int \frac{d^3\mathrm{p}}{(2\pi)^3}\ e\lambda \ \left(\tilde{n}_{\mathbf{p}}^{(\lambda,e)} - \left(\tilde{n}_{\mathbf{p}}^{(\lambda,e)} \right)^2 \right) \ \beta\, \mathrm{B}_j \frac{\mathrm{p}^i \mathrm{p}^j}{\mathrm{p}^2}$$

$$= \sum_\lambda \sum_e \int \frac{d\mathrm{p}}{2\pi^2}\ e\lambda \ \left(\tilde{n}_{\mathbf{p}}^{(\lambda,e)} - \left(\tilde{n}_{\mathbf{p}}^{(\lambda,e)} \right)^2 \right) \ \frac{1}{3}\beta\, \mathrm{B}_i$$

$$= \left(\frac{\mu_R^2 - \mu_L^2}{8\pi^2} \right) \mathrm{B}_i = \ \frac{\bar{\chi}_R - \bar{\chi}_L}{4}\, \mathrm{B}_i ???$$

N. Abbasi, F. Taghinavaz and K. Naderi, JHEP **1803**, 191 (2018).

Boost transformation

- **In the equilibrium, the rest frame of the fluid in the two above frames are related to each other by**

$$\beta = \frac{\bar{\chi}_R - \bar{\chi}_L}{4w}\,\mathbf{B} + \frac{n_R - n_L}{w}\boldsymbol{\Omega}.$$

Type of mode	Chiral Kinetic Theory	Landau-Lifshitz
CMHW	$v^{CKT}_{1,2}(k) = -\left(\mathcal{A}_1 \pm \sqrt{\mathcal{A}_2^2 - 4\,\mathcal{A}_3\,\mathcal{E}}\right)\frac{1}{2\,\mathcal{E}}\,B$	$v^{LL}_{1,2}(k) = -\left(\mathrm{A}_1 \pm \sqrt{\mathrm{A}_2^2 - 4\,\mathrm{A}_3\,\mathcal{E}}\right)\frac{1}{2\,\mathcal{E}}\,B$
Sound	$v^{CKT}_{3,4} = \pm\frac{1}{\sqrt{3}} + \frac{\bar{\chi}_R - \bar{\chi}_L}{6\,w}\,B$	$v^{LL}_{3,4} = \pm\frac{1}{\sqrt{3}}$
CAW	$v^{CKT}_{5,6} = \frac{(n_R+n_L)(n_R-n_L)}{2w^2}\,B$	$v^{LL}_{4,5} = \left(\frac{(n_R-n_L)(n_R+n_L)}{2w^2} - \frac{\bar{\chi}_R-\bar{\chi}_L}{4w}\right)\,B$

N. Abbasi, F. Taghinavaz and K. Naderi, JHEP **1803**, 191 (2018).

One evidence

$$v_i = \frac{\partial \omega}{\partial k_i},$$

$$v_i^{LL} = \frac{v_i^{CKT} - \beta_i}{1 - v_i^{CKT}\beta_i}.$$

- Sound mode

$$v_{3,4}^{CKT} \quad \rightarrow \quad \frac{\pm\frac{1}{\sqrt{3}} + \frac{\bar{\chi}_R - \bar{\chi}_L}{6\,w}\,\mathrm{B}\,\epsilon_f - \frac{\bar{\chi}_R - \bar{\chi}_L}{4w}\,\mathrm{B}\,\epsilon_f}{1 - \left(\pm\frac{1}{\sqrt{3}} + \frac{\bar{\chi}_R - \bar{\chi}_L}{6\,w}\,\mathrm{B}\,\epsilon_f\right)\frac{\bar{\chi}_R - \bar{\chi}_L}{4w}\,\mathrm{B}\,\epsilon_f}$$

$$= \pm\frac{1}{\sqrt{3}} + \left(\left(\pm\frac{1}{\sqrt{3}}\right)^2\left(\frac{1}{4}\right) + \frac{1}{6} - \frac{1}{4}\right)\frac{\bar{\chi}_R - \bar{\chi}_L}{w}\,\mathrm{B}\,\epsilon_f$$

$$= \pm\frac{1}{\sqrt{3}} = \boxed{v_{3,4}^{LL}}.$$

N. Abbasi, F. Taghinavaz and K. Naderi, JHEP **1803**, 191 (2018).

Conclusion

- CKT is a semi classical approach to study the Weyl particles in the regime of weak coupling for arbitrary momentum.
- It has been found that energy of particles is modified due to the Lorentz invariance. We found that momentum of particles as well as their energy is modified. **It is crucial to obtain correct hydro modes**.
- A mismatch is seen between hydro modes obtained from CKT and hydrodynamics one.

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