Fluid-Gravity Correspondence and chiral transport

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Institute for Research in Fundamental Sciences

Outline of talk

- Motivation
- Fluid-Gravity correspondence
  - without charge
  - with charge
- Strongly interacting system with gauge anomaly
  - Thermodynamic properties
  - Magneto-transport
- Conclusion

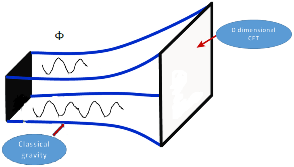
Motivation

- **Hair Theorem**: Every black hole is specified by some finite macroscopic parameter (Q, M, J).
- **Membrane paradigm**: For an external observer, the black holes seems to be as a fluid membrane. It has shear viscosity, diffusion coefficient, $\cdots$. Dynamics of this membrane is described by laws of fluid dynamics.
  T. Damour, Phys. Rev. D **18**, 3598 (1978).

  R. H. Price and K. S. Thorne, Phys. Rev. D **33**, 915 (1986).
- **Defect in membrane paradigm**: It gives negative bulk viscosity, $\xi = -\frac{1}{16\pi}$. (M. Parikh and F. Wilczek, Phys. Rev. D **58**, 064011 (1998)).

Gauge/Gravity duality conjecture

- Strongly interacting field theories on the boundary $\sim$ Classical gravity in the bulk



- Greens function

$$G(x_1, \cdots, x_n) = \frac{\delta \mathcal{Z}_{CFT}[J_1, \cdots, J_n]}{\delta J_1 \cdots \delta J_n},$$
$$\mathcal{Z}_{GR}[\Phi_1, \cdots, \Phi_n] = \mathcal{Z}_{CFT}[J_1, \cdots, J_n].$$

- Fields in the bulk are source on the boundary for perturbations.

Global hydro vs. gravity: rest frame

- Boundary thermodynamics: T
  Gravity:

$$
\begin{aligned}
ds^2 &= -2dvdr - r^2 f(br)dv^2 - r^2\delta_{ij}dx^i dx^j \\
f(r) &= 1 - \frac{1}{r^4}, \\
b &= \frac{1}{\pi T}, \\
E_{MN} &= R_{MN} - \frac{1}{2}g_{MN} - 6g_{MN} = 0.
\end{aligned}
$$

  M, N are bulk coordinates
- Ingoing Eddington-Finklestein(EF) coordinates

$$
v = t - \int_{\infty}^{r} dr' \frac{1}{r'^2 f(r')}.
$$

S. Bhattacharyya, et al, JHEP **0802**, 045 (2008).

Global hydro vs. gravity: boosted frame

- boundary thermodynamics : (T, u^μ)
  $\Rightarrow$ Gravity:

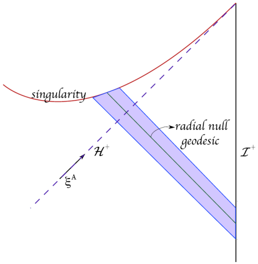
$$
\begin{aligned}
ds^2 &= -2u_\mu dx^\mu dr - r^2 f(br) u_\mu u_\nu dx^\mu dx^\nu - r^2 \mathcal{P}_{\mu\nu} dx^\mu dx^\nu, \\
\mathcal{P}_{\mu\nu} &= \eta_{\mu\nu} + u_\mu u_\nu, \\
E_{MN} &= R_{MN} - \frac{1}{2} g_{MN} - 6 g_{MN} = 0, \quad u_\mu u^\mu = -1.
\end{aligned}
$$

  (μ, ν) are boundary coordinates,
  It is a four-parameter solution of Einstein equation

S. Bhattacharyya, et al, JHEP **0802**, 045 (2008).

Advantages of using EF coordinates

- On the boundary $v = t$,
- hypersurface ($x^{\mu} = \{v, x^i\} = cte$) are null ingoing geodesics,



S. Bhattacharyya, et al, JHEP **0806**, 055 (2008).

- Congruence of ($v = cte$) regions yields a natural map from boundary of AdS to the horizon.

Conserved currents from AdS/CFT

- $T_{\mu\nu}$ of a conformal fluid on boundary:

$$T_{\mu\nu} = (\epsilon + \mathcal{P}) u_\mu u_\nu + \mathcal{P} \eta_{\mu\nu} = \mathcal{P} \left(\eta_{\mu\nu} + 4 u_\mu u_\nu \right).$$

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- $T_{\mu\nu}$ on the boundary:

$$T_{\mu\nu} = \lim_{r \to \infty} \frac{r^2}{8\pi G_5} \left(\mathcal{K}_{\mu\nu} - \mathcal{K} \gamma_{\mu\nu} \right) .$$

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V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208**, 413 (1999).

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Towards FG

- Localized version of boosted brane

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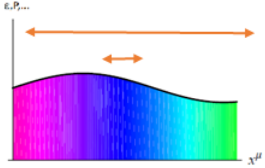
- However, it is a regular metric which approaches a solution in the limit of infinitely slow variation



$$
u_\mu(x) = u_\mu(x_0) + x^\nu\partial_\nu u_\mu(x) + \cdots,
$$

$$
\theta = x^\mu\partial_\mu \sim \frac{l_{Mic}}{L_{Mac}} \sim \frac{1}{TL} \ll 1.
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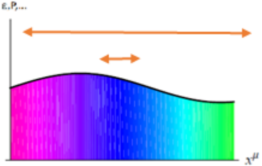

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- In a non-perturbative manner

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$$E_{MN}(g^{(0)}(r, x^\mu) + \underbrace{g^{(1)}(r, x^\mu)}_{\mathcal{O}(\theta)}) = 0 + \mathcal{O}(\theta^2),$$
$$, \cdots .$$

- Structure of unknown metric components

$$g^{(1)}_{MN}(r, x^\mu) = F(r) \underbrace{G(x^\alpha)}_{\mathcal{O}(\theta)}.$$

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Definition of problem

- Determine $g_{MN}(r, x^\alpha)$ in such a way that Einstein equation is satisfied up to desired order

$$\begin{aligned} ds^2 &= -2u_\mu(x^\alpha)dx^\mu dr - r^2 f(b(x^\alpha)r)u_\mu(x^\alpha)u_\nu(x^\alpha)dx^\mu dx^\nu \\ &\quad -r^2\mathcal{P}_{\mu\nu}(x^\alpha)dx^\mu dx^\nu + g_{MN}(r, x^\alpha)dx^M dx^N, \\ E_{MN} &= 0 + \mathcal{O}(\theta^N). \end{aligned}$$

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- To this end we have to replace

$$b(x^\alpha) = \sum_{n=0}^{\infty} \theta^n b^{(n)}(x^\alpha), \quad u^\mu(x^\alpha) = \sum_{n=0}^{\infty} \theta^n u^{\mu(n)}(x^\alpha),$$

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S. Bhattacharyya, et al, JHEP **0802**, 045 (2008).

How to solve

- Gauge fixing:

$$g_{rr} = 0, \quad g_{r\mu} = u_\mu, \quad Tr((g^0)^{-1} g) = 0.$$

$\Rightarrow$ 10 unknown components $g_{\mu\nu}$. We decompose them in terms of irreducible representations of local $SO(3)$

$$\mathbf{10} = \underbrace{\mathbf{2}}_{\text{scalar}} + \underbrace{\mathbf{3}}_{\text{vector}} + \underbrace{\mathbf{5}}_{\text{symmetric traceless tensor}}$$

- Number of Einstein equations

$$\mathbf{15} = \underbrace{\mathbf{1}}_{\text{trivial}} + \underbrace{\mathbf{4}}_{\text{constrained}} + \underbrace{\mathbf{10}}_{\text{have to be solved}}$$

S. Bhattacharyya, et al, JHEP **0802**, 045 (2008).

Constrained equations

- Are obtained from dotting Einstein equation to the normal vector of hypersurface ($r = r_c$)

$$S = r - r_c = 0,$$
$$n_\mu = \frac{\partial S}{\partial x^\mu} = (1, 0, 0, 0, 0)\,, \quad n^\nu = g^{\nu\mu} n_\mu = g^{\nu r}$$
$$E_{\mu\nu} n^\nu = E_{\mu\nu} g^{\nu r} = 0,$$

$$(\mathbf{1}) \qquad E_{rr} g^{rr} + E_{rv} g^{vr} = 0,$$
$$(\mathbf{3}) \qquad E_{ir} g^{rr} + E_{iv} g^{vr} = 0.$$

S. Bhattacharyya, et al, JHEP **0802**, 045 (2008).

Scalar channel

- Scalar components of metric

$$
\begin{aligned}
g^{(1)}_{ii}(r) &= 3\,r^2\,h_1(r), \quad g^{(1)}_{vv}(r) = \frac{k_1(r)}{r^2}, \\
g^{(1)}_{vr}(r) &= -\frac{3}{2}\,h_1(r).
\end{aligned}
$$

- leads to

$$
\begin{aligned}
&12\,r^3\,h_1(r) + (3r^4 - 1)\,h_1'(r) - k_1'(r) = -6\,r^2\,\frac{\partial_i \beta_i^{(0)}}{3}, \\
&5\,h_1'(r) + r\,h_1''(r) = 0.
\end{aligned}
$$

- By imposing appropriate boundary conditions we have

$$
h_1(r) = 0, \qquad k_1(r) = \frac{2\,r^3\,\partial_i \beta_i^{(0)}}{3}.
$$

S. Bhattacharyya, et al, JHEP **0802**, 045 (2008).

Other channels

- Vector channel

$$g^{(1)}_{vi}(r) = r^2\,(1 - f(r))\,j^{(1)}_i(r).$$

- Tensor channel

$$g^{(1)}_{ij}(r) = r^2 \alpha^{(1)}_{ij}(r).$$

- Constrained equations

$$\boxed{\partial_v b^0 = \frac{\partial_i \beta^{(0)}_i}{3}, \Leftrightarrow \partial_\mu T^{\mu 0}_{(0)} = 0}$$

$$\boxed{\partial_i b^0 = \partial_v \beta^{(0)}_i \;\Leftrightarrow \partial_\mu T^{\mu i}_{(0)} = 0.}$$

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Summary of the first order result

- Global solution to first order result

$$ds^2 = -2\,u_\mu\,dx^\mu dr - r^2\,f(b\,r)\,u_\mu u_\nu\,dx^\mu dx^\nu + r^2\,P_{\mu\nu}\,dx^\mu dx^\nu$$
$$+2\,r^2\,b\,F(b\,r)\,\sigma_{\mu\nu}\,dx^\mu dx^\nu + \frac{2}{3}\,r\,u_\mu u_\nu\,\partial_\lambda u^\lambda\,dx^\mu dx^\nu$$
$$-r\,u^\lambda \partial_\lambda\,(u_\nu u_\mu)\;dx^\mu dx^\nu\,,$$

- with

$$F(r) = \frac{1}{4}\left[\ln\left(\frac{(1+r)^2(1+r^2)}{r^4}\right) - 2\,\arctan(r) + \pi\right],$$
$$\sigma_{\mu\nu} = \partial_{(\mu}u_{\nu)} - \frac{2}{3}\eta_{\mu\nu}\partial_\lambda u^\lambda.$$

S. Bhattacharyya, et al, JHEP **0802**, 045 (2008).

Energy-momentum Tensor

- $T_{\mu\nu}$ of a dissipative fluid:

$$T_{\mu\nu} = \mathcal{P}\left(\eta_{\mu\nu} + 4u_\mu u_\nu\right) - 2\eta\sigma_{\mu\nu}.$$

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- $T_{\mu\nu}$ on the boundary from AdS/CFT conjecture

$$8\pi G_5 T_{\mu\nu} = \frac{1}{b^4}\left(4\, u_\mu u_\nu + \eta_{\mu\nu}\right) - \frac{2}{b^3}\,\sigma_{\mu\nu}.$$

- $\frac{\eta}{s}$ matches to the previously founded bound

$$\eta = \frac{\pi^3 T^3}{8\pi G_5},$$

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}.$$

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Charged system

- Bulk action correspond to boundary charged system:

$$S = \frac{1}{16\pi G_5}\int \sqrt{-g_5}\left[R + 12 - F_{AB}F^{AB} - \frac{4\kappa}{3}\epsilon^{LABCD}A_L F_{AB}F_{CD}\right].$$

- Equations of motion:

$$E: \qquad G_{AB} - 6g_{AB} + 2\left[F_{AC}F^{C}{}_{B} + \frac{1}{4}g_{AB}F_{CD}F^{CD}\right] = 0,$$

$$M: \qquad \nabla_B F^{AB} + \kappa\epsilon^{ABCDE}F_{BC}F_{DE} = 0.$$

N. Banerjee, et al, JHEP **1101**, 094 (2011),

J. Erdmenger, M. Haack, M. Kaminski and A. Yarom, JHEP **0901**, 055 (2009).

Solutions

- Boundary thermodynamics:(T, u^ν, μ)
  $\Rightarrow$ Gravity:

$$
\begin{aligned}
ds^2 &= -2u_\mu dx^\mu dr - r^2 V(r,m,q)\, u_\mu u_\nu dx^\mu dx^\nu \\
&\quad + r^2 P_{\mu\nu} dx^\mu dx^\nu , \\
A &= \frac{\sqrt{3}q}{2r^2} u_\mu dx^\mu + A_{\mu\,(bg)} dx^\mu , \\
V(r,m,q) &\equiv 1 - \frac{m}{r^4} + \frac{q^2}{r^6} .
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Currents on the boundary

- Current from hydro on the boundary:

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Localized solution

- Localized version is not the solution to EoM:

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\begin{aligned}
&ds^2 = -2u_\mu(x^\alpha)dx^\mu dr + r^2 P_{\mu\nu}(x^\alpha)dx^\mu dx^\nu \\
&-r^2 V(r, m(x^\alpha), q(x^\alpha))\ u_\mu(x^\alpha)u_\nu(x^\alpha)dx^\mu dx^\nu \\
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- Boundary parameters are slowly varying and we could solve for EoM perturbatively in orders of θ.

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Anomalous current!!!

- Current after solving equations

$$J^\mu = J^\mu_{(0)} + J^\mu_{(1)} = \lim_{r\to\infty} \frac{r^2 A^\mu}{2\pi G_5} = nu^\mu + \xi_B B^\mu + \xi_\omega \omega^\mu,$$

$$\xi_\omega = -\frac{3q^2\kappa}{2\pi G_5 m} = C\mu^2\left(1 - \frac{2}{3}\frac{n\mu}{\epsilon + \mathcal{P}}\right),$$

$$\xi_B = -\sqrt{3}\frac{(m + 3r_+^4)q\kappa}{4\pi G_5 m r_+^2} = C\mu\left(1 - \frac{1}{2}\frac{n\mu}{\epsilon + \mathcal{P}}\right),$$

- Identification is as follows

$$-\frac{\kappa}{\pi G_5} = \frac{C}{2}, \quad \sqrt{3}q = \mu r_+^2,$$

$$\epsilon = 3\mathcal{P} = \frac{3m}{16\pi G_5}, \quad n = \frac{\sqrt{3}q}{8\pi G_5}.$$

D. T. Son and P. Surowka, Phys. Rev. Lett. **103**, 191601 (2009).

So far review

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- Einstein equations in the limit of long-wavelength correspond to the boundary Navier-Stokes equation. FG correspondence provides a map from solution space of fluid dynamics to solution space of Einstein space

Gravity set up

- Bulk action

$$S = -\frac{1}{16\pi G_5}\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R - \frac{12}{L^2} + F^{MN}F_{MN}\right) + S_{CS} + S_{bdy},$$
$$S_{CS} = \frac{k}{12\pi G_5}\int A\wedge F\wedge F = \frac{k}{192\pi G_5}\int d^5x\sqrt{-g}\epsilon^{MNPQE}A_M F_{NP}F_{QR},$$
$$S_{bdy} = -\frac{1}{8\pi G_5}\int_{\partial\mathcal{M}} d^4x\,\sqrt{-\gamma}\left(\mathcal{K} - \frac{3}{L} + \frac{L}{4}R(\gamma) + \frac{L}{2}\ln\left(\frac{r}{L}\right)F^{\mu\nu}F_{\mu\nu}\right).$$

- Equations of motion

$$\begin{aligned} 0 &= d * F + k\,F\wedge F \\ R_{MN} &= 4g_{MN} + \frac{1}{3}g_{MN}F^{AB}F_{AB} - 2F_{MP}F_N^{\ P} \end{aligned}$$

E. D'Hoker and P. Kraus, JHEP **1003**, 095 (2010).

Solutions

- Solution ansatz

$$ds^2 = \frac{dr^2}{U(r)^2} - U(r)dt^2 + e^{2V(r)}(dx_1^2 + dx_2^2) + e^{2W(r)}(dx_3 + C(r)dt)^2,$$

$$F = E(r)dr \wedge dt + Bdx_1 \wedge dx_2 + P(r)dx_3 \wedge dr$$

- Structure of functions in weak B

$$U = U_0 + B^2 U_2 \qquad E = E_0 + B^2 E_2$$

$$W = W_0 + B^2 W_2 \qquad C = C_0 + BC_1$$

$$V = V_0 + B^2 V_2 \qquad P = P_0 + BP_1$$

E. D'Hoker and P. Kraus, JHEP **1003**, 095 (2010).

Important points

- We have solved these unknown functions in the limit of small "B" and small μ in such a way that $B \ll \mu^2 \ll T^2$.
- Horizon's radius is modified due to the presence of "B".
- Due to presence of "B", T_0 and μ_0 are not the physical quantities of the boundary theory. They are modified and give rise the true ones.
- We introduce an energy scale Δ on the boundary in which our results make sense.

N. Abbasi, A. Ghazi, F. Taghinavaz and O. Tavakol, arXiv:1812.11310 [hep-th].

Thermodynamic properties

- Conserved currents

$$T_{00} = \frac{N_c^2}{8\pi^2}\left(3(\pi T)^4 + 12(\pi T)^2\mu^2 + 8\mu^4\right) + \frac{N_c^2 B^2}{4\pi^2}\left(\left(1 - \ln\frac{\pi T}{\Delta}\right) - \frac{2}{3}\frac{\mu^2}{\pi T^2}(8\ln 2 - 3)\right)$$

$$T_{0z} = \frac{c}{2}\mu^2 B,$$

$$T_{ii} = \frac{N_c^2}{24\pi^2}\left(3(\pi T)^4 + 12(\pi T)^2\mu^2 + 8\mu^4\right) + \frac{N_c^2 B^2}{4\pi^2}\left(\ln\frac{\pi T}{\Delta} + \frac{2}{3}\frac{\mu^2}{\pi T^2}(8\ln 2 - 3)\right)$$

$$J^0 = \frac{N_c^2}{3\pi^2}\left(3(\pi T)^2\mu + 4\mu^3\right) + \frac{N_c^2 B^2}{3\pi^2}\frac{\mu}{(\pi T)^2}\left(8\ln 2 - 3\right),$$

$$J^z = c\mu B.$$

- These currents satisfy Gibbs Duhem relation. Also they pass the entropy and number density check.

N. Abbasi, A. Ghazi, F. Taghinavaz and O. Tavakol, arXiv:1812.11310 [hep-th].

Response to external sources

- We read the response of system to the weak external electric field and temperature gradient by using general hydro formula

$$\sigma = \frac{2N_c^2\,\mu^2\,\tau}{\pi^2}\left(1+\frac{2B^2}{3\pi^2T^2\mu^2}\right),$$

$$T\alpha = N_c^2T^2\,\mu\,\tau\left(1-\frac{2\mu^2}{3\pi^2T^2}+\frac{B^2}{3\pi^4T^4}\left(8\ln 2-7\right)\right),$$

$$T\kappa = \frac{\pi^2N_c^2\,T^4\,\tau}{2}\left(1+\frac{4\mu^4}{3\pi^4T^4}+\frac{B^2}{2\pi^4T^4}\left(1-\frac{8\mu^2}{3\pi^2T^2}\left(8\ln 2-5\right)\right)\right).$$

N. Abbasi, A. Ghazi, F. Taghinavaz and O. Tavakol, arXiv:1812.11310 [hep-th].

Conclusion

- Fluid-gravity correspondence has to do with the long-wavelength regime of Einstein equation. The concept of slow variation has great importance in this topic.
- We have study the thermodynamic properties of strongly interacting fermions in the regime $B \ll \mu^2 \ll T^2$.
- By proposing the notion of horizon's radius as well as temperature and chemical potential modification, we could show that all the thermodynamic relations are satisfied.
- We also derived the magneto-transport of the above system and show that they satisfy the Ward identities.
- We are working to generalize the above mentioned system in the case of mixed gauge-gravitational anomaly.