Magneto-Transport in a Chiral Fluid from Kinetic Theory

Navid Abbasi, Farid Taghinavaz, O.T

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Motivation form Condensed matter

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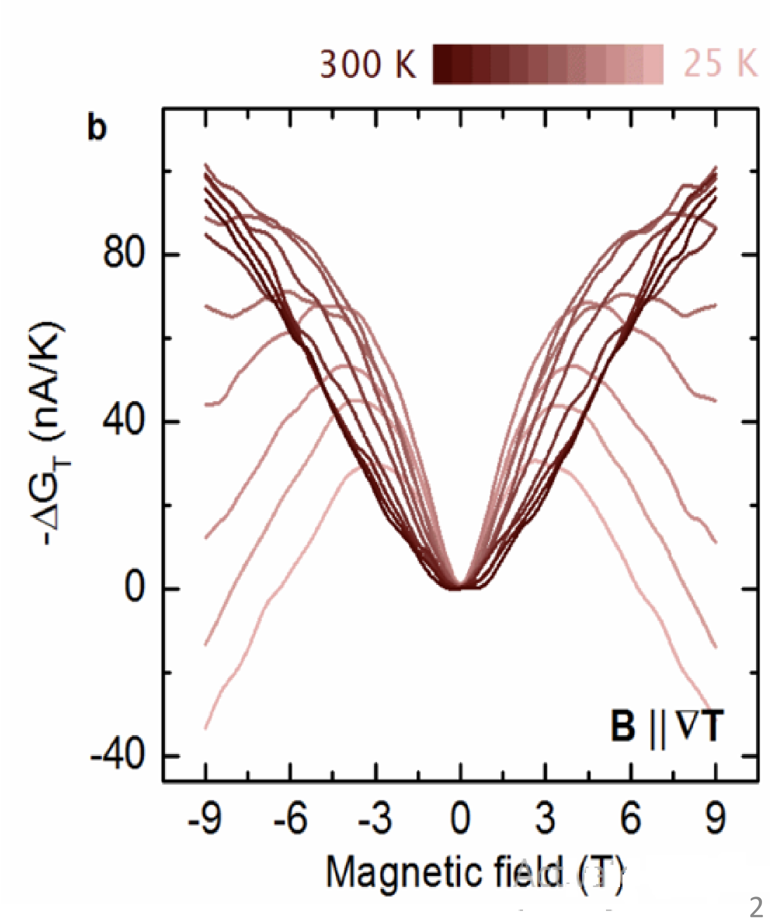
Experimental signatures of the mixed axial–gravitational anomaly in the Weyl semimetal NbP

Johannes Gooth, Anna C. Niemann, Tobias Meng, Adolfo G. Grushin, Karl Landsteiner, Bernd Gotsmann, Fabian Menges, Marcus Schmidt, Chandra Shekhar, Vicky Süß, Ruben Hühne, Bernd Rellinghaus, Claudia Felser, Binghai Yan & Kornelius Nielsch

$$J_i = G_T^{ij} \nabla_j T + G^{ij} E_j$$

$$G_T^{ij} = \tau \frac{2\ c\, c_g\, ,T}{\det(\Xi)} \frac{\partial \rho}{\partial T} B_i B_j$$



Outline:

Part 1:

- Drude model
- Weyl semimetals
- Relativistic constrains
- Effect of quantum corrections on Thermodynamic quantities
- CKT and linear response
- Ward identities for anomalous systems
- Check points

Part 2:

- Magneto hydrodynamics for an anomalous fluid
- Dynamic of linear perturbations in a weakly broken symmetry model
- transport coefficients
- Time-reversal and consequences
- Strong limit

Drude model

$$\dot{\mathbf{p}} \cdot \frac{\partial n_{\mathbf{p}}^{(e)}}{\partial \mathbf{p}} = -\frac{n_{\mathbf{p}}^{(e)} - \tilde{n}_{\mathbf{p}}^{(e)}}{\tau}$$

$$\dot{\mathbf{x}} = \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}}$$

$$\dot{\mathbf{p}} = e\,\mathbf{E}$$

$$\epsilon_p = \frac{p^2}{2m}$$

$$\delta n_p = \tau e \vec{E}.\frac{\partial n_0}{\partial \vec{p}}$$

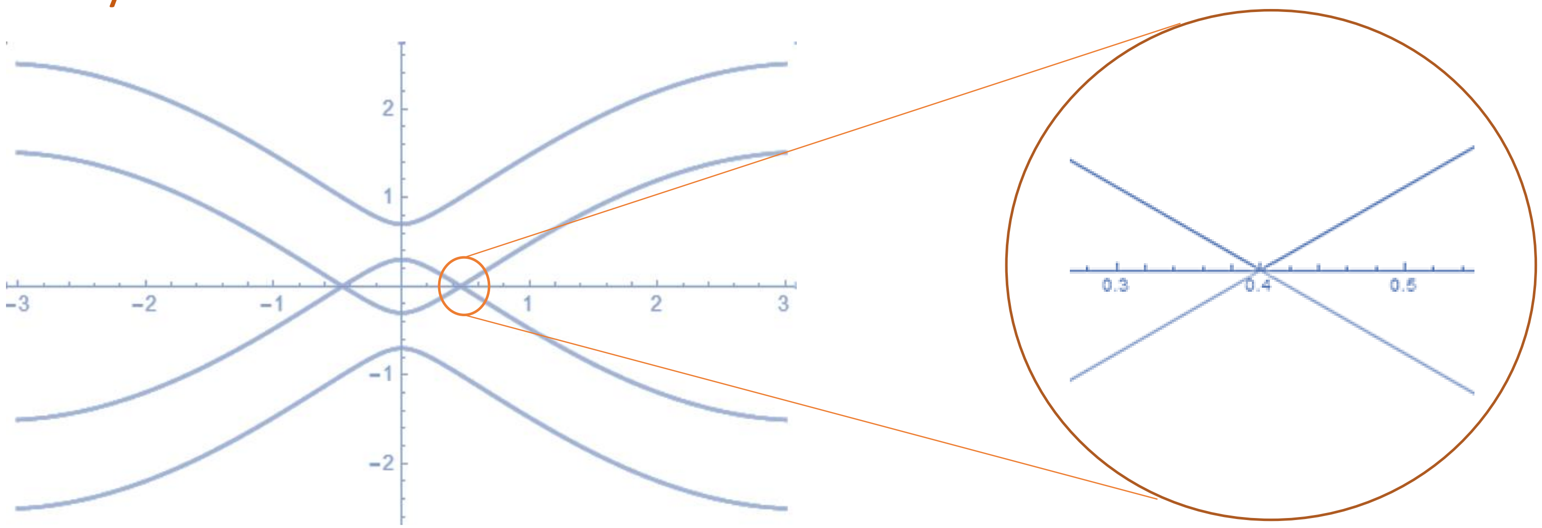
$$n_0 = \frac{1}{exp(\beta(\epsilon_p - \mu)) + 1}$$

$$J_i = e^2 \tau \int \dot{x}_i \frac{\partial n_0}{\partial p^j} \frac{d^3 p}{(2\pi)^3} E_j$$

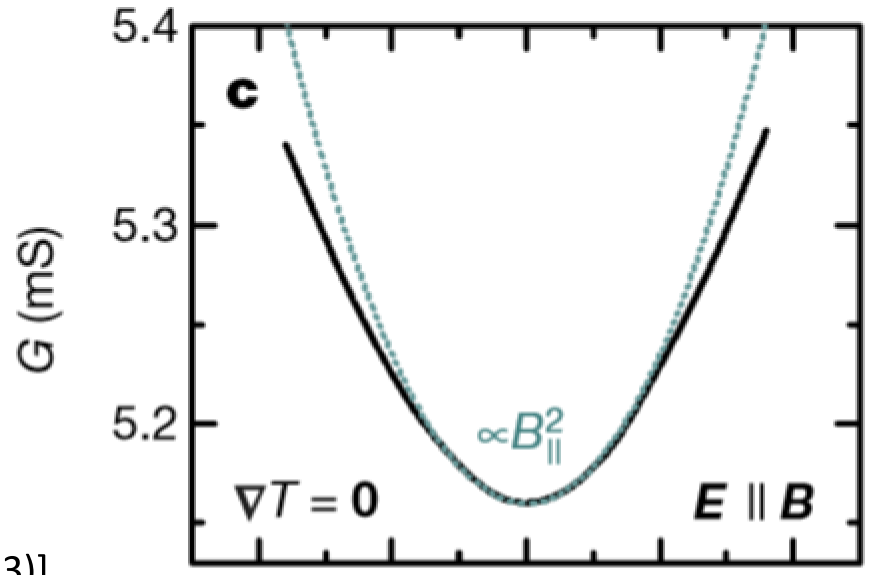
$$J_i = \sigma_{ij} E_j$$

$$\sigma_{ij} = \frac{n e^2 \tau}{m}\ \delta_{ij}$$

Weyl semimetal in condensed matter



- Metals with band-touching in Brillouin zone in the vicinity of band touching point $\omega = v\,(p - p^*)$
- P or T breaking, otherwise Dirac metal touching points with opposite chiralities



Negative – Magneto resistivity:

$$\boldsymbol{J} \sim \dot{\mathbf{x}}\,\delta n_{\mathbf{p}} \sim \left(\mathbf{B}\,(\hat{\mathbf{p}} \cdot \boldsymbol{\Omega}_{\mathbf{p}})\right)\left(\tau \boldsymbol{\Omega}_{\mathbf{p}}(\mathbf{E} \cdot \mathbf{B})\right) \sim \tau \mathbf{B}^2\,\mathbf{E}$$

[Son, Spivak, Phys.Rev.B 88 104412 (2013)]

Relativistic correction :

Lorentz Invariance: $T^{0i} = T^{i0}$ ⇨ $(\delta^{ij} + e\mathrm{B}^i\Omega^j_{\boldsymbol{p}})\epsilon(\mathbf{p})\frac{\partial\epsilon(\mathbf{p})}{\partial \mathrm{p}^j} = (1 + e\mathbf{B}\cdot\boldsymbol{\Omega}_{\mathbf{p}})\mathrm{p}^i$

$$\epsilon_p = p - \hbar\frac{B.\hat{p}}{2p}$$

[Son, Yammamoto, Phys.Rev.D87:085016, 2012]

Distribution function $$n_{\mathbf{p}} = \frac{1}{e^{\beta(\epsilon_{\mathbf{p}}-\mu)}+1}$$

WSM $$\sigma_{zz} \sim \tau\mathbf{B}^2$$

First correction to dispersion $$\epsilon_p = p - \hbar\frac{B.\hat{p}}{2p} + O(\mathrm{B}^2)$$

Second correction to dispersion $$\epsilon_p = p - \hbar\frac{B.\hat{p}}{2p} - \hbar^2\frac{(B.\hat{p})^2}{8p^3}$$

[$Abbasi, Taghi$navaz, T 2018]

$$\tilde{n}^{(e)}_{\mathbf{p}} = \tilde{n}^{(e)}_{\mathbf{p}}|_{\epsilon(\mathbf{p})=\mathrm{p}} - \left(e\frac{\mathbf{B}\cdot\mathbf{p}}{2\mathrm{p}^2} + e^2\frac{(\mathbf{B}\cdot\mathbf{p})^2}{8\mathrm{p}^5}\right)\frac{\partial\tilde{n}^{(e)}_{\mathbf{p}}}{\partial\epsilon}|_{\epsilon(\mathbf{p})=\mathrm{p}} + e^2\frac{(\mathbf{B}\cdot\mathbf{p})^2}{4\mathrm{p}^4}\frac{\partial^2\tilde{n}^{(e)}_{\mathbf{p}}}{\partial\epsilon^2}|_{\epsilon(\mathbf{p})=\mathrm{p}}$$

Effect of second order correction on Thermodynamics

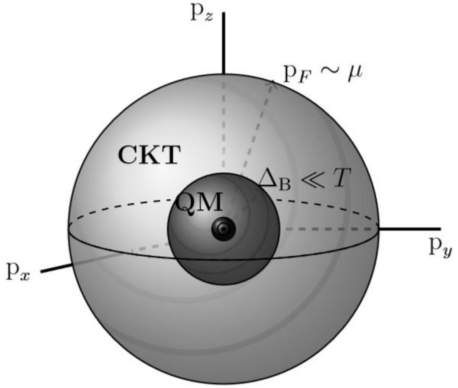
[Abbasi, Taghinavaz, T 2018]

$$\tilde{n}^{(e)}_{\mathbf{p}} = \tilde{n}^{(e)}_{\mathbf{p}}\big|_{\epsilon(\mathbf{p})=\mathrm{p}} - \left(e\frac{\mathbf{B}\cdot\mathbf{p}}{2\mathrm{p}^2} + e^2\frac{(\mathbf{B}\cdot\mathbf{p})^2}{8\mathrm{p}^5} \right) \frac{\partial \tilde{n}^{(e)}_{\mathbf{p}}}{\partial \epsilon}\Big|_{\epsilon(\mathbf{p})=\mathrm{p}} + e^2\frac{(\mathbf{B}\cdot\mathbf{p})^2}{4\mathrm{p}^4}\,\frac{\partial^2 \tilde{n}^{(e)}_{\mathbf{p}}}{\partial \epsilon^2}\Big|_{\epsilon(\mathbf{p})=\mathrm{p}}$$

non – magnetics parts:

$$\int_0^\infty d\mathrm{p}$$

magnetic parts:



$$\int_{\Delta_\mathrm{B}}^\infty d\mathrm{p}$$

$$\sqrt{e\mathrm{B}} \ll \Delta_\mathrm{B} \ll T \ll \mu$$

$$\epsilon = \sum_e \int_{\mathbf{p}} \sqrt{G}\,\epsilon(\mathbf{p})\,\tilde{n}^{(e)}_{\mathbf{p}} = T^4\left(\frac{\mu^4}{8\pi^2 T^4} + \frac{\mu^2}{4T^2} + \frac{7\pi^2}{120} \right) + \frac{e^2\mathrm{B}^2}{24\pi^2} - \left(log\frac{\mu}{\Delta_\mathrm{B}} - \frac{\pi^2}{6}\frac{T^2}{\mu^2} \right)\frac{e^2\mathrm{B}^2}{16\pi^2}$$

$$n = \sum_e \int_{\mathbf{p}} e\,\tilde{n}^{(e)}_{\mathbf{p}} = T^3\left(\frac{\mu^3}{6\pi^2 T^3} + \frac{\mu}{6T} \right) + \frac{e^2\mathrm{B}^2}{16\pi^2\mu}\left(1 + \frac{\pi^2 T^2}{3\mu^2} \right)$$

Our result satisfy:

$$\epsilon + p = T(\partial p/\partial T)_\mu + \mu(\partial p/\partial \mu)_T$$

$$w = T^4\left(\frac{\mu^4}{6\pi^2 T^4} + \frac{\mu^2}{3T^2} + \frac{7\pi^2}{90}\right) + \frac{e^2 B^2}{16\pi^2}\left(1 + O(\frac{T^5}{\mu^5})\right)$$

A physical prediction :

let′s consider the system at finite μ

B induces the CME current $J^{\parallel} = e\mu B/4\pi^2$ $\Rightarrow$ Potential difference $V^{\parallel} = \rho J^{\parallel}$ $\Rightarrow$ E

E. B anomaly decreses chirality $\Rightarrow$ $E = \mu/\tau$

power of Joule heating $J \cdot E = \sigma_L E^2 = \sigma_L \mu^2/\tau^2$

in a constant pressure $w = J \cdot E\ \tau \Rightarrow \sigma_L \frac{\mu^2}{e^2 \tau} = \frac{e^2 B^2}{16\pi^2}$ $\quad \sigma_L = \frac{\tau e^4 B^2}{16\pi^2 \mu^2}$

CKT and linear response

$$\boldsymbol{J}_e = \sigma\,\mathbf{E} + T\alpha_1\left(-\frac{\boldsymbol{\nabla}\mathrm{T}}{\mathrm{T}}\right)$$

$$\boldsymbol{J}_{th} = T\alpha_2\,\mathbf{E} + T\kappa\left(-\frac{\boldsymbol{\nabla}\mathrm{T}}{\mathrm{T}}\right)$$

$$\frac{\partial n^{(e)}_{\mathbf{p}}}{\partial t} + \dot{\mathbf{x}}\cdot\frac{\partial n^{(e)}_{\mathbf{p}}}{\partial \mathbf{x}} + \dot{\mathbf{p}}\cdot\frac{\partial n^{(e)}_{\mathbf{p}}}{\partial \mathbf{p}} = -\frac{n^{(e)}_{\mathbf{p}} - \tilde{n}^{(e)}_{\mathbf{p}}}{\tau}$$

$$\dot{\mathbf{r}} = \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} + \dot{\mathbf{p}}\times\boldsymbol{\Omega}_{\mathbf{p}}$$

$$\dot{\mathbf{p}} = e\mathbf{E} + \frac{e}{c}\dot{\mathbf{r}}\times\mathbf{B}$$

$$\epsilon(\mathbf{p}) = \mathrm{p} - e\frac{\mathbf{B}\cdot\mathbf{p}}{2\mathrm{p}^2} - e^2\frac{(\mathbf{B}\cdot\mathbf{p})^2}{8\mathrm{p}^5}$$

$$\delta n^{(e)}_{\mathbf{p}} = -\frac{\tau}{\sqrt{G}}\left[\Big(e\mathbf{E}\cdot\mathbf{v}_{\mathbf{p}} + e^2(\boldsymbol{\Omega}_{\boldsymbol{p}}\cdot\mathbf{v}_{\mathbf{p}})\,\mathbf{E}\cdot\mathbf{B}\Big) + (\epsilon(\mathbf{p}) - \hat{e}\mu)\Big(\boldsymbol{\zeta}\cdot\mathbf{v}_{\mathbf{p}} + e(\boldsymbol{\Omega}_{\boldsymbol{p}}\cdot\mathbf{v}_{\mathbf{p}})\,\boldsymbol{\zeta}\cdot\mathbf{B}\Big)\right]\frac{\partial \tilde{n}^{(e)}_{\mathbf{p}}}{\partial \mathrm{p}}$$

$$\boldsymbol{\zeta} \equiv -\boldsymbol{\nabla}T/T$$

$$\boldsymbol{J}_e = \sum_e \int_{\mathbf{p}} \sqrt{G}\,\dot{\mathbf{x}}\,\delta n^{(e)}_{\mathbf{p}}$$

$$\boldsymbol{J}_{th} = \sum_e \int_{\mathbf{p}} \sqrt{G}\,\dot{\mathbf{x}}\,(\epsilon(\mathbf{p}) - \hat{e}\mu)\,\delta n^{(e)}_{\mathbf{p}}$$

$$\boxed{\sigma_L = \frac{J^{\parallel}_e}{\mathrm{E}} = \frac{e^2\tau}{3}\left(\frac{\mu^2}{2\pi^2} + \frac{T^2}{6}\right) + e^2\tau\frac{e^2\mathrm{B}^2}{16\pi^2\mu^2}\left(1 + \frac{\pi^2T^2}{\mu^2} + O(\frac{T^4}{\mu^4})\right)}$$

$$\boxed{T\,\alpha_L = \frac{J^{\parallel}_e}{\zeta} = \frac{e\tau}{9}\,\mu T^2 - e\tau\frac{e^2\mathrm{B}^2T^2}{24\mu^3}\left(1 + O(\frac{T^2}{\mu^2})\right).}$$

$$\boxed{T\,\kappa_L = \frac{J^{\parallel}_{th}}{\zeta} = \frac{\tau(\pi T)^2}{9}\left(\frac{\mu^2}{2\pi^2} + \frac{7T^2}{10}\right) + \tau T^2\frac{e^2\mathrm{B}^2}{48\mu^2}\left(1 + O(\frac{T^2}{\mu^2})\right).}$$

Comparison with WSM

6.25% decrease in the value of conductivities

Longitudinal Conductivity	relativistic without quantum correction	WSM	relativistic with quantum corrections to second order
Electrical: σ_L	$\sigma = \frac{e^2\tau}{3}\frac{e^2\mathrm{B}^2}{5\pi^2\mu^2}$	$v^3\sigma$	$\frac{15}{16}\sigma$
Thermoelectric: α_{1L}	$\alpha = -\frac{e\tau}{9}\frac{2e^2\mathrm{B}^2T^2}{5\mu^3}$	$v^3\alpha$	$\frac{15}{16}\alpha$
Thermal: κ_L	$\kappa = \frac{\pi^2\tau}{9}\frac{e^2\mathrm{B}^2T}{5\mu^2}$	$v^2\kappa$	$\frac{15}{16}\kappa$

Some linear relation between conductivities?

Ward identities

[Herzog2009][Hartnoll 2009]

In a 2+1 dim system in the absence of anomalies:

Under the transformations:

$$\delta_\lambda A_\mu = \partial_\mu \Lambda + A_\nu\, \partial_\mu \xi^\nu + (\partial_\nu A_\mu)\xi^\nu$$
$$\delta_\lambda g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu .$$

The equilibrium generating functional changes as:

$$\delta_\lambda W = \int d^4x \sqrt{-g}\left\{ J^\mu\, \delta_\lambda A_\mu + \frac{1}{2} T^{\mu\nu}\, \delta_\lambda g_{\mu\nu} \right\}$$

In covariant theory:

Ward identities for 1-points:

$$\partial_\mu \langle J^\mu(x)\rangle = 0$$
$$\partial_\mu \langle T^{\mu\nu}(x)\rangle = F^\nu{}_\mu \langle J^\mu(x)\rangle$$

Ward identities for 2-points:

$$0 = \frac{\partial}{\partial x^\mu}\langle \mathcal{T}_*(J^\alpha(y)T^{\mu\nu}(x))\rangle + F_\mu{}^\nu \langle \mathcal{T}_*(J^\alpha(y)J^\mu(x))\rangle - \frac{\partial}{\partial x^\beta}\delta(x-y)\delta^{\beta\nu}\langle J^\alpha(y)\rangle$$
$$+\frac{\partial}{\partial x^\mu}\delta(x-y)\delta^{\alpha\nu}\langle J^\mu(y)\rangle ,$$

$$0 = D_\mu \Big(\langle \mathcal{T}_*(T^{\alpha\beta}(y)T^{\mu\nu}(x))\rangle + \delta(x-y)\langle g^{\alpha\nu}T^{\beta\mu}(y) + g^{\beta\nu}T^{\alpha\mu}(y) - g^{\mu\nu}T^{\alpha\beta}(y)\rangle \Big)$$
$$+\delta(x-y)g^{\beta\nu}D_\mu\langle T^{\mu\alpha}(x)\rangle + \delta(x-y)g^{\alpha\nu}D_\mu\langle T^{\mu\beta}(x)\rangle + F_\mu{}^\nu\langle \mathcal{T}_*(T^{\alpha\beta}(y)J^\mu(x))\rangle$$

Constraints from ward identities

$$\begin{pmatrix} J_i \\ Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & T\alpha_{ij} \\ T\alpha_{ij} & T\kappa_{ij} \end{pmatrix} \begin{pmatrix} E_j \\ -\nabla_j T/T \end{pmatrix}$$

$$E_i = -i\omega(\delta A_i + \mu \delta g_{ti})$$

$$-\nabla_i T/T = i\omega \delta g_{ti}$$

$$\delta_\lambda W = \int d^4x \sqrt{-g} \left\{ J^\mu \, \delta_\lambda A_\mu + \frac{1}{2} T^{\mu\nu} \, \delta_\lambda g_{\mu\nu} \right\}$$

$$\delta W = -i \int d^3x dt \sqrt{-g} \left\{ (T^{ti} - \mu J^i) \frac{-\nabla_i T}{i\omega T} + J^i \frac{E_i}{i\omega} \right\}$$

Heat current

$$G_{\mathcal{R}}^{J_i J_j} = -\frac{\delta \mathcal{J}_i}{\delta A_j}, \qquad G_{\mathcal{R}}^{Q_i J_j} = -\frac{\delta \mathcal{Q}_i}{\delta A_j}, \qquad G_{\mathcal{R}}^{Q_i Q_j} = -\frac{\delta \mathcal{Q}_i}{\delta g_{tj}}$$

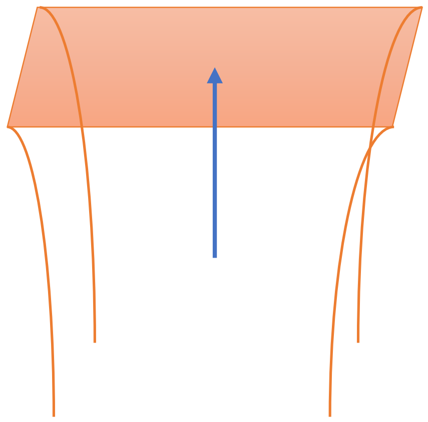
We obtain:

$$\sigma_{ij}(\omega) = \frac{e^2 \, G_{\mathcal{R}}^{J_i J_j}(\omega)}{i\omega}, \qquad T\alpha_{ij}(\omega) = \frac{e \, G_{\mathcal{R}}^{Q_i J_j}(\omega)}{i\omega}, \qquad T\kappa_{ij}(\omega) = \frac{G_{\mathcal{R}}^{Q_i Q_j}(\omega)}{i\omega}$$

From Ward:

$$T\alpha(\omega) = \frac{i\rho}{\omega} - \mu\sigma(\omega)\,; \quad T\bar{\kappa}(\omega) = \frac{i(\epsilon + P - 2\mu\rho)}{\omega} + \mu^2 \sigma(\omega) .$$

Anomaly inflow in an anomalous system



$$W_{cov} = W[\partial \mathcal{M}_5] + \int_{\mathcal{M}_5} I_5^{CS}.$$

Consistent currents: $$J^\mu_{cons} = \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta A_\mu}, \qquad T^{\mu\nu}_{cons} = \frac{2}{\sqrt{-g}} \frac{\delta W}{\delta g_{\mu\nu}}$$

Covariant currents: $$J^\mu_{cov} = \frac{1}{\sqrt{-g}} \frac{\delta W_{cov}}{\delta A_\mu} = J^\mu_{cons} + P^\mu_{BZ}$$

$$T^{\mu\nu}_{cov} = \frac{2}{\sqrt{-g}} \frac{\delta W_{cov}}{\delta g_{\mu\nu}} = T^{\mu\nu}_{cons} + P^{\mu\nu}_{BZ}.$$

[Harvey, 2001]

Constraint equations in an anomalous system

We have found that

$$\delta W_{cov} = -i \int d^3x dt \sqrt{-g} \left\{ (T^{ti}_{cov} - \mu J^i_{cov}) \frac{-\nabla_i T}{i\omega T} + J^i_{cov} \frac{E_i}{i\omega} \right\}$$

We find following two constraints:

$$T\,\alpha_L = \frac{e}{i\omega} \left\{ G_{\mathcal{R}}^{T_{03}J_z} - e\mu\, G_{\mathcal{R}}^{J_3 J_3} \right\} = -\frac{e\,n}{i\omega} - \frac{\mu}{e}\,\sigma_L$$

$$T\,\kappa_L = \frac{1}{i\omega} \left\{ G_{\mathcal{R}}^{T_{03}T_{03}} - 2\mu G_{\mathcal{R}}^{T_{03}J_3} + \mu^2\, G_{\mathcal{R}}^{J_3 J_3} \right\} = -\frac{\epsilon + p - 2\mu n}{i\omega} + \frac{\mu^2}{e^2}\,\sigma_L$$

1) Our kinetic theory conductivities satisfy them.
2) While neither do the WSM ones.

Hydrodynamic theory for transports in WSM

[Lucas , Davison , Sachdev arXiv:1604.08598, 2016]

$$\nabla_\mu J^\mu_a = -\frac{C_a}{8}\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} - \frac{G_a}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\sigma} - \sum_b\left[\mathcal{R}_{ab}\nu_b + \mathcal{S}_{ab}\beta_b\right],$$

$$\nabla_\mu T^{\mu\nu}_a = F^{\nu\mu}J_{\mu a} - \frac{G_a}{16\pi^2}\nabla_\mu\left[\varepsilon^{\rho\sigma\alpha\beta}F_{\rho\sigma}R^{\nu\mu}{}_{\alpha\beta}\right] + u^\nu_a\sum_b\left[\mathcal{U}_{ab}\nu_b + \mathcal{V}_{ab}\beta_b\right],$$

Lucas: results in weakly inter valley scattering regime

$$\alpha_{zz} = \frac{\pi^2 T}{3}\frac{\partial\sigma_{zz}(\mu, T=0)}{\partial\mu}$$

$$\bar{\kappa}_{zz} = \frac{\pi^2 T}{3}\sigma_{zz}.$$

+

us: ward identity of anomalous systems

$$T\,\alpha_L = \frac{e}{i\omega}\left\{G_{\mathcal{R}}^{T_{03}J_z} - e\mu\, G_{\mathcal{R}}^{J_3J_3}\right\} = -\frac{e\,n}{i\omega} - \frac{\mu}{e}\,\sigma_L$$

$$T\,\kappa_L = \frac{1}{i\omega}\left\{G_{\mathcal{R}}^{T_{03}T_{03}} - 2\mu G_{\mathcal{R}}^{T_{03}J_3} + \mu^2\, G_{\mathcal{R}}^{J_3J_3}\right\} = -\frac{\epsilon + p - 2\mu n}{i\omega} + \frac{\mu^2}{e^2}\sigma_L$$

$$\sigma_L^{sol} = e^2\frac{\mu^2\tau}{6\pi^2}\left(1 - \frac{2\pi^2T^2}{3\mu^2}\right) + e^{-\frac{3\mu^2}{2\pi^2T^2}}\left(C + e^2\tau\frac{3e^2\mathrm{B}^2}{32\pi^4T^2}\,\mathrm{Ei}\left(\frac{3\mu^2}{2\pi^2T^2}\right)\right)$$

$$\sigma_L(\mu, T=0) = e^2\tau\frac{\mu^2}{6\pi^2} + e^2\tau\frac{e^2\mathrm{B}^2}{16\pi^2\mu^2}$$

Part 2

Second hydrodynamic in an anomalous system

[Dmitri E. Kharzeev and Ho-Ung Yee
Phys. Rev. D **84**, 045025, 2011]

$U(1)$ Anomalous charge + non-conservation equation of the charge due to anomaly

$$\partial_\mu T^{\mu\nu} = F^{\nu\alpha} J_\alpha \qquad T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p\,\eta^{\mu\nu} + \tau^{\mu\nu},$$

$$\partial_\mu J^\mu = c\, E^\mu B_\mu \qquad J^\mu = n\, u^\mu + \nu^\mu.$$

$$\tau^{\mu\nu} = \sigma^\epsilon_{\mathcal{B}}(u^\mu B^\nu + u^\nu B^\mu) + C_1(B^\mu B^\nu - \frac{1}{3} P^{\mu\nu} B^2) + \Pi^{\mu\nu}_{\alpha\beta} B^\alpha \left(C_2 E^\beta - C_3 \mu \frac{\nabla^\beta T}{T} \right)$$

$$\nu^\mu = \sigma_E\, E^\mu + \sigma_{\mathcal{B}}\, B^\mu$$

$$\Pi^{\mu\nu}_{\alpha\beta} = \frac{1}{2}(P^\mu{}_\alpha P^\nu{}_\beta + P^\mu{}_\beta P^\nu{}_\alpha - \frac{2}{3} P^{\mu\nu} P_{\alpha\beta})$$

$k \to 0$ limit

$$\tilde{T}^{tt} = \epsilon, \qquad \tilde{T}^{zz} = p, \qquad \tilde{T}^{ii} = p - \frac{C_1}{3} B^2 \ (i = x, y), \qquad \tilde{J}^t = n$$

$$\tilde{T}^{tz} = \frac{1}{2}(c_g T^2 + c\mu^2) B, \qquad \tilde{J}^z = c\mu\, B$$

Response of the system

external sources $\delta \boldsymbol{E} \quad \boldsymbol{\nabla} T$

system resposnse

$$
\begin{aligned}
u^\mu &\to \big(1, \delta u_x(\boldsymbol{x},t), \delta u_y(\boldsymbol{x},t), \delta u_z(\boldsymbol{x},t)\big) \\
T &\to T + \delta T(\boldsymbol{x},t) \\
\mu &\to \mu + \delta\mu(\boldsymbol{x},t)
\end{aligned}
\quad \to
$$

$$
\begin{aligned}
\delta T^{ti} &= w\delta u_i, \quad (i = x, y) \\
\delta T^{tz} &= w\delta u_z + \big(c_g T\,\delta T + c\mu\,\delta\mu\big)B \\
\delta T^{tt} &= \big(c_g T^2 + c\mu^2\big)\delta u_z\,B + e_1\delta\mu + e_2\delta T \\
\delta T^{ii} &= s\,\delta T + n\,\delta\mu - \frac{C_2}{3}B\,\delta E_z + \frac{C_3}{3}\,B\,\frac{\mu}{T}\frac{\nabla_z T}{T}, \quad (i = x, y) \\
\delta T^{zz} &= \big(c_g T^2 + c\mu^2\big)B\,\delta u_z + n\delta\mu + s\delta T + \frac{2C_2}{3}B\,\delta E_z - \frac{C_3}{3}\,B\,\frac{\mu}{T}\frac{\nabla_z T}{T} \\
\delta T^{zi} &= \frac{1}{2}\big(c_g T^2 + c\mu^2\big)B\,\delta u_i, \quad (i = x, y).
\end{aligned}
$$

$$
\begin{aligned}
\delta J^t &= c\mu\,\delta u_z B + f_1\,\delta\mu + f_2\,\delta T \\
\delta J^x &= n\,\delta u_x + \sigma_E\delta E_x + 2\sigma_E\delta u_y B \\
\delta J^y &= n\,\delta u_y + \sigma_E\delta E_y - 2\sigma_E\delta u_x B \\
\delta J^z &= n\delta u_z + \sigma_E\delta E_z + \sigma_E\,\mu\frac{\nabla_z T}{T} + cB\,\delta\mu.
\end{aligned}
$$

Dynamic of linear perturbation in hydrodynamic model

weakly broken symmetry model

$$\partial_\mu \delta T^{\mu 0} = \delta(F^{0\nu} J_\nu) - \frac{\delta T^{0\mu}}{\tau_e}\tilde{u}_\mu$$

$$\partial_\mu \delta T^{\mu i} = \delta(F^{i\nu} J_\nu) - \frac{\delta T^{i\mu}}{\tau_m}\tilde{u}_\mu$$

$$\partial_\mu \delta J^{\mu} = c\,\delta E_\mu B^\mu - \frac{\delta J^{\mu}}{\tau_c}\tilde{u}_\mu$$

[Landsteiner, Liu and Sun. JHEP 1503, 127 (2015)]

Dissipative process in the fluid via three different scattering mechanisms even in spatially uniform state

Laplace transformation:

$$\left(-i\omega + \frac{1}{\tau_e}\right)\left(e_1\delta\hat{\mu} + e_2\delta\hat{T} + (c_g T^2 + c\mu^2) B\,\delta\hat{u}_z\right) = c\mu\, B\,\delta E_z^{(0)} - c_g T\, B\,\nabla_z T^{(0)} + \cdots$$

$$\left(-i\omega + \frac{1}{\tau_m}\right)\left(c_g T\,\delta\hat{T} + \cdots\right) + \cdots$$

$\begin{pmatrix} a_{11} & \cdots \\ \vdots & \vdots \\ a_{51} & \cdots \end{pmatrix}$ $\begin{pmatrix} \delta u_x \\ \vdots \\ \delta\mu \end{pmatrix}$

Conductivities $\sim B^2\left(c^2\mathcal{A} + c\,c_g\,\mathcal{B} + c_g^2\,\mathcal{C}\right)$ $\begin{pmatrix} \delta E \\ \delta T \end{pmatrix}$

$$\left(-i\omega + \frac{1}{\tau_c}\right)\left(f_1\delta\hat{\mu} + f_2\delta\hat{T} + c\mu B\,\delta\hat{u}_z\right) = c\,B\,\delta E_z^{(0)} + \cdots\,.$$

Weak regime: hydrodynamic magneto transports in chiral kinetic theory

$$\epsilon = T^4\left(\frac{\mu^4}{8\pi^2T^4}+\frac{\mu^2}{4T^2}+\frac{7\pi^2}{120}\right)+\frac{e^2\mathrm{B}^2}{24\pi^2}-\left(log\frac{\mu}{\Delta_\mathrm{B}}-\frac{\pi^2}{6}\frac{T^2}{\mu^2}\right)\frac{e^2\mathrm{B}^2}{16\pi^2}$$

$$p = T^4\left(\frac{\mu^4}{24\pi^2T^4}+\frac{\mu^2}{12T^2}+\frac{7\pi^2}{360}\right)+\frac{e^2\mathrm{B}^2}{48\pi^2}+\left(log\frac{\mu}{\Delta_\mathrm{B}}-\frac{\pi^2}{6}\frac{T^2}{\mu^2}\right)\frac{e^2\mathrm{B}^2}{16\pi^2}$$

$$n = T^3\left(\frac{\mu^3}{6\pi^2T^3}+\frac{\mu}{6T}\right)+\frac{e^2\mathrm{B}^2}{16\pi^2\mu}\left(1+\frac{\pi^2T^2}{3\mu^2}\right).$$

$$\sigma = \sigma_E - \frac{\mu^2\tau_m}{6\pi^2}\left(1+\frac{24\pi^4}{45}\left(\frac{T}{\mu}\right)^4-\frac{48\pi^6}{45}\left(\frac{T}{\mu}\right)^6\right) + \frac{B^2}{16\pi^2\mu^2}(2\tau_c-2\tau_e+\tau_m)+\frac{B^2T^2}{\mu^4}\left(\frac{13}{40}\tau_c-\frac{7}{60}\tau_e-\frac{1}{8}\tau_m\right)$$

$$\alpha_1 = -\mu\,\sigma_E+\frac{\mu T^2\tau_m}{6}\left(1-\frac{24\pi^2}{45}\left(\frac{T}{\mu}\right)^2+\frac{48\pi^4}{45}\left(\frac{T}{\mu}\right)^4\right)-\frac{B^2T_0^2}{48\mu^3}(2\tau_e+\tau_m)$$

$$\alpha_2 = -\mu\,\sigma_E+\frac{\mu T^2\tau_m}{6}\left(1-\frac{24\pi^2}{45}\left(\frac{T}{\mu}\right)^2+\frac{48\pi^4}{45}\left(\frac{T}{\mu}\right)^4\right) + \frac{B^2}{8\pi^2\mu}(\tau_e-\tau_c)+\frac{B^2T^2}{\mu^3}\left(\frac{7}{48}\tau_m+\frac{7}{60}\tau_e-\frac{13}{40}\tau_c\right)$$

$$\kappa = \mu^2\sigma_E+\frac{\pi^2T^4\tau_m}{6}\left(1-\frac{48\pi^2}{45}\left(\frac{T}{\mu}\right)^2\right)+\frac{B^2T^2\tau_e}{24\mu^2}$$

Constraints from time-reversal

$\alpha_1 \overset{?}{=} \alpha_2$

$$O(\frac{1}{\mu}) : \qquad \tau_e - \tau_c = 0$$

$$O(\frac{1}{\mu^3}) : \qquad \frac{7}{48}\tau_m + \frac{7}{60}\tau_e - \frac{13}{40}\tau_c = -\frac{1}{48}(2\tau_e + \tau_m)$$

$$\boxed{\tau_e = \tau_m = \tau_c}$$

→ *Comparable with kinetic theory in specific limit* $B \ll T^2 \ll \mu^2$

interestingly by this constraint in general we find:

$$T\,\alpha = -\frac{i}{\tilde{\omega}}\,n - \mu\,\sigma$$

$$T\,\kappa = -\frac{i}{\tilde{\omega}}(\epsilon + p - 2\mu n) + \mu^2\,\sigma$$

Conclusion

- It is necessary to consider the second quantum correction

1. Consistency of thermo
2. Obeying the Ward identities
3. Agreement with Lucas-Divison-Sachdev

- From quantum kinetic theory

$$p^2 - \epsilon_p^2 + \hbar e B.\hat{p} = O(\hbar^2 e^2 B^2)$$

$$p^2 - \epsilon_p^2 + \hbar e B.\hat{p} = O(\hbar^3 e^3 B^3)$$

one loop exact ABJ anomaly?

$$p^2 - \epsilon_p^2 + \hbar e B.\hat{p} = 0$$

- For general fluid in the present of chiral anomaly $\sim B^2 \left(c^2 \mathcal{A} + c\, c_g\, \mathcal{B} + c_g^2\, \mathcal{C}\right)$
- Onsager ⟶ $\tau_e = \tau_m = \tau_c \equiv \tau$
- Obeying new Ward identities consistent by model
- Strong regime gauge/Gravity duality

Thank you

Transport Coeffitions:

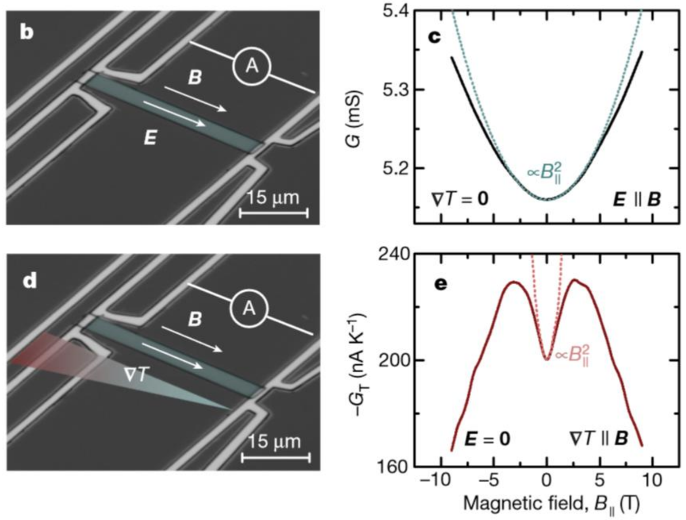
$$\begin{pmatrix} J_i \\ Q_i \end{pmatrix} = \begin{pmatrix} \sigma_{ij} & T\alpha_{ij} \\ T\alpha_{ij} & T\kappa_{ij} \end{pmatrix} \begin{pmatrix} E_j \\ -\nabla_j T/T \end{pmatrix}$$

$$\begin{aligned} \sigma =& \sigma_E + \frac{i}{\omega_m}\frac{n^2}{w} \\ &+ \frac{iB^2 T}{w^2(e_2 f_1 - f_2 e_1)}\Bigg\{ c_g^2 \frac{f_1 T^2 n}{\omega_m} + c^2 \mu s \left(e_2(\frac{w}{\omega_C} - \frac{\mu n}{\omega_m}) - f_2(\frac{w}{\omega_E} - \frac{\mu n}{\omega_m}) \right) \\ &+ c c_g n \left(e_1(\frac{w}{\omega_C} - \frac{\mu n}{\omega_m}) - f_1(\frac{w}{\omega_E} - \frac{\mu n}{\omega_m})\mu + \frac{f_2 T^2 s}{\omega_m} \right) \Bigg\} \end{aligned}$$

$$\begin{aligned} \alpha_1 =& -\mu\sigma_E + \frac{i}{\omega_m}\frac{Tns}{w} \\ &+ \frac{iB^2}{(e_2 f_1 - e_1 f_2)w^2}\Bigg\{ c^2 \frac{\mu s^2 T^2}{\omega_m}(\mu f_2 - e_2) - c_g^2\, T^3 n\Big(w(\frac{1}{\omega_e} - \frac{1}{\omega_m}) + \frac{\mu n}{\omega_m}\Big) f_1 \\ &+ c\, c_g\, T^2 s \left(\frac{\mu n}{\omega_m}(f_1\mu - e_1) - wT(\frac{1}{\omega_m} - \frac{1}{\omega_e}) f_2 - \frac{\mu n T f_2}{\omega_m} \right) \Bigg\} \end{aligned}$$

$$\begin{aligned} \alpha_2 =& -\mu\sigma_E + \frac{i}{\omega_m}\frac{Tns}{w} \\ &+ \frac{iB^2}{(e_2 f_1 - e_1 f_2)w^2}\Bigg\{ c^2 \mu s \left(w(\frac{\mu f_2}{\omega_e} - \frac{e_2}{\omega_c}) - \frac{\mu n}{\omega_m}(\mu f_2 - e_2) \right) - c_g^2 \frac{T^3 \mu n^2 f_1}{\omega_m} \\ &+ c\, c_g\, T \mu n \left(-\frac{sT^2 f_2}{\omega_m} + w(\frac{\mu f_1}{\omega_e} - \frac{e_1}{\omega_c}) - \frac{\mu n}{\omega_m}(\mu f_1 - e_1) \right) \Bigg\} \end{aligned}$$

$$\begin{aligned} \kappa =& \mu^2 \sigma_E + \frac{i}{\omega_m}\frac{s^2}{w} \\ &+ \frac{iB^2}{(e_2 f_1 - e_1 f_2)w^2}\Bigg\{ c^2 \frac{T^2 \mu^2 s^2}{\omega_m}(e_2 - f_2\mu) + c_g^2\, T^3 \mu n \left(w(\frac{1}{\omega_e} - \frac{1}{\omega_m}) + \frac{\mu n}{\omega_m} \right) f_1 \\ &- c\, c_g\, T^2 \mu s \left(T w f_2(\frac{1}{\omega_m} - \frac{1}{\omega_e}) - \frac{\mu}{\omega_m}(e_1 + T f_2 - \mu f_1) \right) \Bigg\} \end{aligned}$$



$$\sim B^2 \left(c^2 \mathcal{A} + c\, c_g\, \mathcal{B} + c_g^2\, \mathcal{C} \right)$$

$\alpha_{hydro} = \alpha_{kinetic}$

$\sigma_{hydro} = \sigma_{kinetic}$

$k_{hydro} = k_{kinetic}$

σ_E in transports

All equations should give same result

quantum conductivity which we could not find in CKT

$$\sigma_E = \frac{T^2 \tau}{18}\left(1 - \frac{8\pi^2}{5}(\frac{T}{\mu})^2 + \frac{16\pi^4}{5}(\frac{T}{\mu})^4\right) - \frac{B^2 \tau}{48\mu^2}\frac{T^2}{\mu^2}$$

Ward identities in a system with weakly broken symmetry

$$\tau_e = \tau_m = \tau_c \equiv \tau$$

$$\sigma = \sigma_E + \frac{i}{\tilde{\omega}}\frac{n^2}{w} + \frac{i}{\tilde{\omega}}\frac{B^2}{w^2(e_2 f_1 - e_1 f_2)}\left\{c^2(e_2 - f_2\mu)T^2 s^2 + c c_g T(e_1 + f_2 T - f_1\mu) n T s + c_g^2 f_1 T^3 n^3\right\} \qquad \tilde{\omega} = \omega + i/\tau$$

Ward identity with generating function by modified gauge and diffeomorphism symmetry

$$\begin{aligned}
\partial_\mu \delta T^{\mu 0} &= \delta(F^{0\nu} J_\nu) - \frac{\delta T^{0\mu}}{\tau}\tilde{u}_\mu \\
\partial_\mu \delta T^{\mu i} &= \delta(F^{i\nu} J_\nu) - \frac{\delta T^{i\mu}}{\tau}\tilde{u}_\mu \\
\partial_\mu \delta J^{\mu} &= c\,\delta E_\mu B^\mu - \frac{\delta J^{\mu}}{\tau}\tilde{u}_\mu
\end{aligned}$$