Entanglement Wedge Cross Section: A Review

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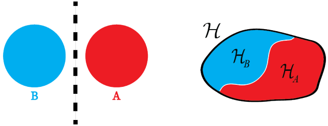
Outline

- (Holographic)Entanglement entropy
- Entanglement wedge
- holographic duals for entanglement wedge cross-section:
    - Entanglement of purification
    - Reflected entropy
    - Logarithmic negativity
    - Odd entropy

Entropy: a measure of entanglement

- Dividing a quantum system into A and B: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$.



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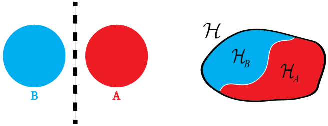

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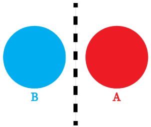

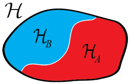

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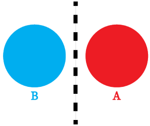

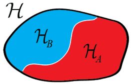

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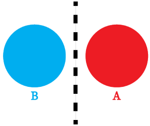

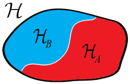

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Entanglement entropy

A measure of entanglement in a given quantum state $|\psi\rangle$.

$$S_A = -\,\mathrm{tr}(\rho_A \ln \rho_A) = \begin{cases} 0 & \Leftrightarrow \rho_A \text{ is pure} \quad \Leftrightarrow \quad \text{seprable state} \\ S > 0 & \Leftrightarrow \rho_A \text{ is mixed} \quad \Leftrightarrow \quad \text{entangled state} \end{cases}$$

Example: Thermofield double state

$$|\psi\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n_A\rangle \otimes |n_B\rangle \,, \qquad Z = \sum_n e^{-\beta E_n}$$

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The reduced density operator for subsystem A

$$\rho_A = \mathrm{tr}_B(|\psi\rangle\langle\psi|) = \frac{1}{Z} \sum_n e^{-\beta E_n} |n_A\rangle\langle n_A|$$

$$= \frac{1}{Z} e^{-\beta H_A}, \qquad H_A |n_A\rangle = E_n |n_A\rangle\,.$$

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Entanglemet Entropy: $S_A = -\mathrm{tr}_A \left[\rho_A(-\beta H_A - \ln Z)\right]$

$$= \underbrace{\beta(\langle H_A\rangle - F)}_{\text{Thermal entropy for subsystem } A} \,, \quad F = -\frac{1}{\beta} \ln Z.$$

Thermal entropy for subsystem A

Remark

This example shows that you can always purify a thermal system by doubling Hilbert space as $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$.

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Purification

You can always purify a mixed state by enlarging its Hilbert space.

Aside: Eternal black hole and Thermofield double state (I)

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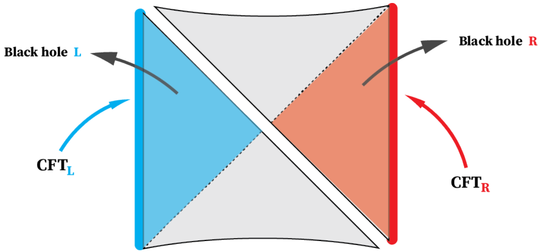

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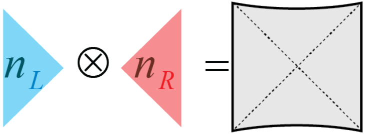
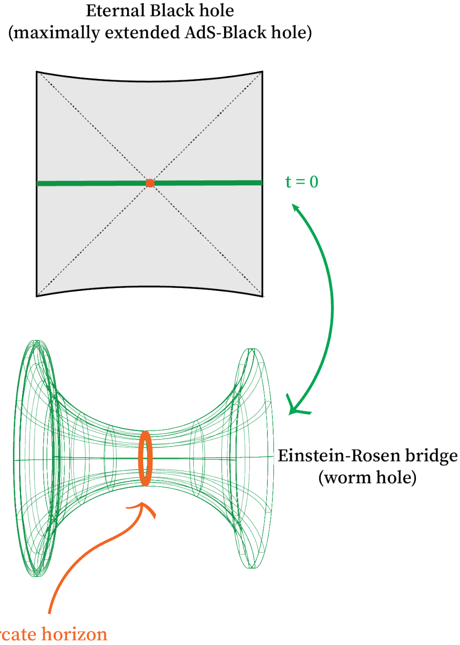
- By doubling Hilbert space one can purify a thermal state as well as thermal state of a CFT dual to black hole.
- What is the gravity description of this purification?
- The natural answer: a maximally extended black hole geometry with two disjoint asymptotic region (dual to two non-interacting CFTs)!

$$\sum_n e^{-\beta\frac{E_n}{2}} |n_L\rangle \otimes |n_R\rangle = |\psi\rangle$$

$$\sum_n e^{-\beta\frac{E_n}{2}}$$



Aside: Eternal black hole and Thermofield double state (II)



- black hole entropy is entanglement entropy in this case.
- Classical smooth connection ⇔ Quantum Entanglement

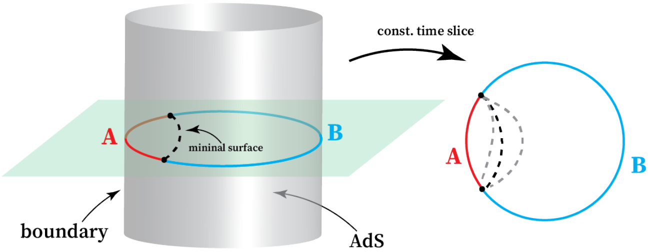
Ryu-Takayanagi (RT) prescription [Ryu-Takayangi '06]

$$S_A = -\,\mathrm{tr}(\rho_A \ln \rho_A) = \min_{\gamma_A} \frac{\text{Area}(\gamma_A)}{4G}$$

Entanglement entropy in CFT   Area of minimal surface

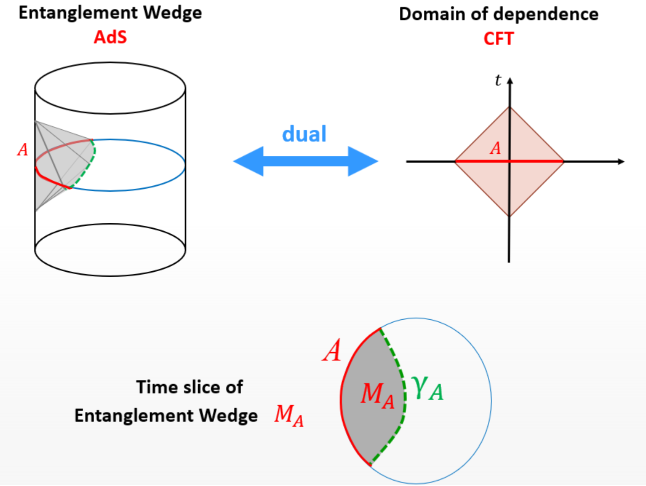
A minimal codimension-2 surface which satisfies:

i) $\partial\gamma_A = \partial A$ *ii)* γ_A is homologous to A



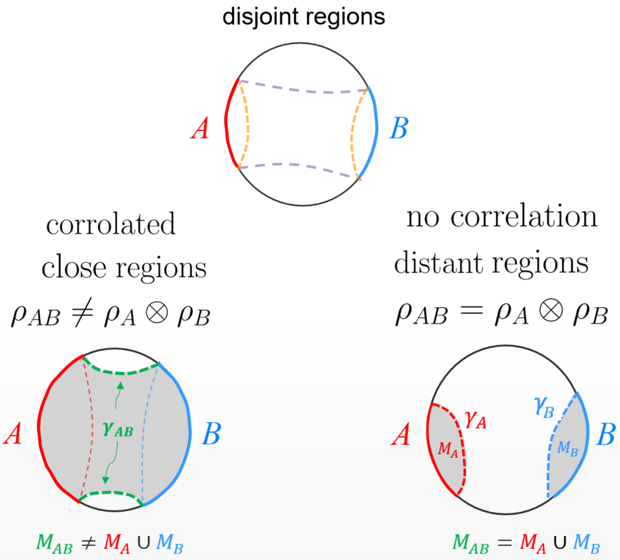
Subregion/subregion duality

Information of ρ_A is included in entanglement wedge M_A.



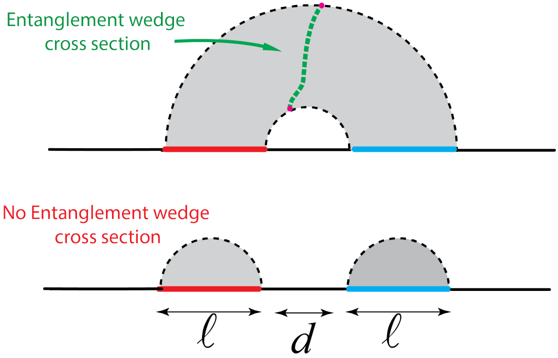
[Czech-Karczmarek-Nogueira-Raamsdonk '12][Wall '12] [Headrick-Hubeny-Lawrence-Rangamani '14]

EW of $A \cup B$



disjoint regions
A
B
corrolated
close regions
$\rho_{AB} \neq \rho_A \otimes \rho_B$
A
γ_{AB}
B
$M_{AB} \neq M_A \cup M_B$
no correlation
distant regions
$\rho_{AB} = \rho_A \otimes \rho_B$
A
γ_A
M_A
γ_B
M_B
B
$M_{AB} = M_A \cup M_B$

Entanglement wedge cross-section of $A \cup B$



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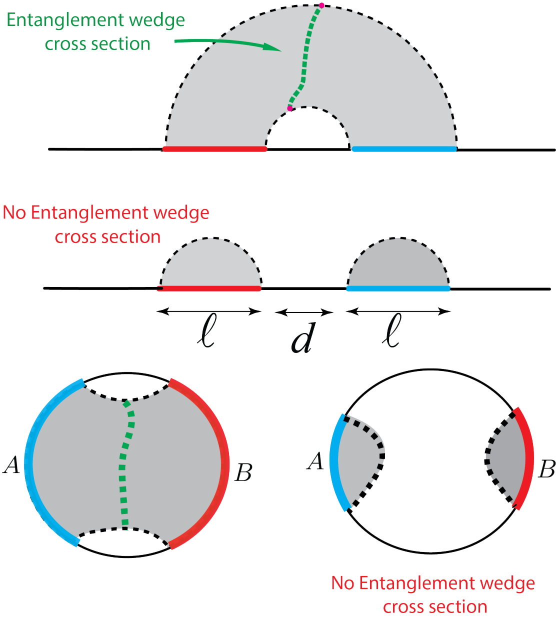

What's the dual of entanglement wedge cross-section?

Entanglement wedge and holography for mixed states?!

- Entanglement of purification? [Takayanagi-Umemoto'17] [Nguyen-Devakul-Halbasch- Zaletel-Swingle'17]
- Reflected entropy? [Dutta, Faulkner, '19]
- Logarithmic negativity? [Kudler-Flam,Ryu'18]
- Odd entropy? [Tamaoka'18]
- ...?!

Correlation measures for mixed states

$|\psi\rangle_{AB}$ pure state $\qquad \rho_{AB} = \sum_i p_i |i\rangle\langle i|$ mixed

- S_A is not a measure of correlation for mixed states!
  No correlation for $\rho_A \bigotimes \rho_B$, but $S_A > 0$ for mixed ρ_A.

- Holographic correlation measures for mixed states?

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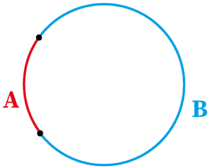

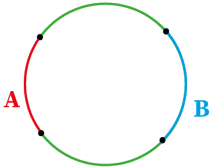

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- Holographic correlation measures for mixed states? e.g. mutual information, entanglement of purification

Purification

You can always purify a mixed state by enlarging its Hilbert space.

$$\rho_{AB} = \mathrm{tr}_{A'B'} \left(|\psi\rangle \langle\psi| \right)$$

Purification is not unique !

Entanglement of purification [Terhal- Horodecki- Leung- DiVincenzo, '02]

$$\rho_{AB} = \mathrm{tr}_{A'B'} \left(|\psi\rangle \langle\psi| \right)$$

Entanglement between AA' and BB':

$$S_{AA'} = -\,\mathrm{tr} \left(\rho_{AA'} \ln \rho_{AA'} \right)$$

Entanglement of purification:

$$E_p \equiv \min_{\text{all purification}} S_{AA'}$$

It measures both quantum and classical correlations between A and B.

It has an operational interpretation in terms of local operations and a small amount of communication.

Properties of entanglement of purification

- E_p is bounded above by S_E

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$$E_p(A:B) \geq \frac{I(A:B)}{2}$$

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- A bound for tripartite system

$$E_p(A:BC) \geq \frac{I(A:B)}{2} + \frac{I(A:C)}{2}$$

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- Polygamous for tripartite system

$$E_p(A:B) + E_p(A:C) \geq E_p(A:BC)$$

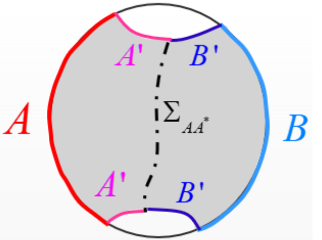
Entanglement of purification and EWC

The entanglement wedge as a new holographic geometry?
with the boundary $\partial M_{AB} = A \cup B \cup A' \cup B'$

minimal cross-section of the entanglement wedge $=$ Entanglement of Purification

$$E_W(\rho_{AB}) \equiv \min_{\text{all } A'} \underbrace{\left(\frac{\mathrm{Area}(\Sigma^{\min}_{AA'})}{4G}\right)}_{\text{HEE of } \rho_{AA'}}$$

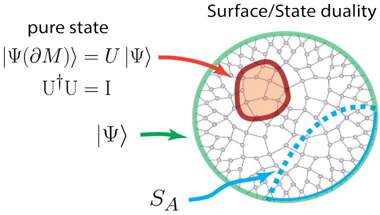
all purification over states with holographic dual?



[Takayanagi-Umemoto'17] [Nguyen-Devakul-Halbasch- Zaletel-Swingle'17]

Evidences for $E_p = E_W$

- It satisfies all mentioned inequality.
- A heuristic derivation via MERA



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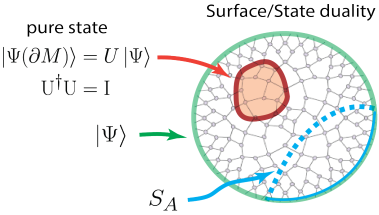

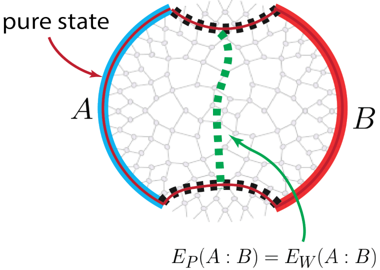

Reflected entropy [Dutta, Faulkner, '19]

For any mixed state there is a canonical purification by doubling the Hilbert space:

$$\rho_{AB} = \sum_i p_i \, |\psi_i\rangle\langle\psi_i| \quad \xrightarrow{\text{purification}} \quad \sqrt{\rho_{AB}} = \sum_i \sqrt{p_i} \, |\psi_i\rangle_{AB} \, |\psi_i\rangle_{A^* B^*}$$

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Example: thermofield double state

$$\rho_{th} = \frac{1}{Z} \sum_n e^{-\beta E_n} \, |n_A\rangle\langle n_A| \quad \xrightarrow{\text{purification}} \quad |\psi\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} \, |n_A\rangle \otimes |n_B\rangle$$

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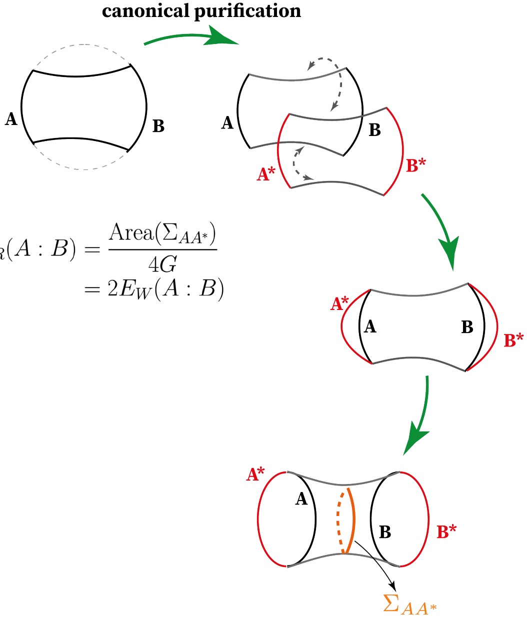
reflected entropy:

$$S_R(A:B) \equiv S_{AA^*} = -\operatorname{tr}\left(\rho_{AA^*} \ln \rho_{AA^*}\right)$$

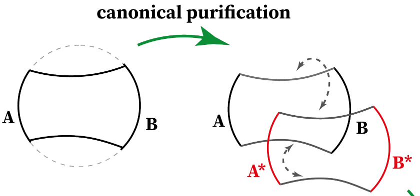
$$\rho_{AA^*} = \operatorname{tr}_{BB^*}\left(|\sqrt{\rho_{AB}}\rangle \, \langle \rho_{AB}|\right)$$

Reflected entropy and EWC [Dutta-Faulkner '19]



$$S_R(A:B) = \frac{\text{Area}(\Sigma_{AA^*})}{4G} = 2E_W(A:B)$$

Reflected entropy and EWC



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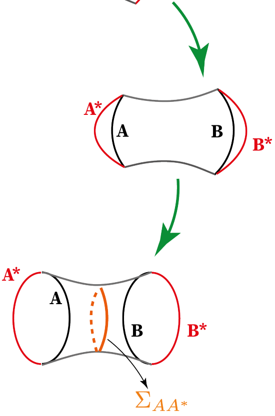

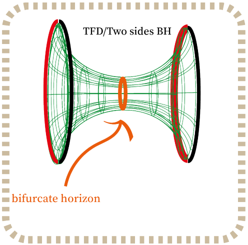

Replica trick for reflected entropy

Twice replication by taking $n \times m$ copies:

- 1st replication:

$$|\psi_m\rangle = \frac{1}{\sqrt{\operatorname{tr}\rho_{AB}^m}} \left|\rho_{AB}^{m/2}\right\rangle, \quad 2m \in \mathbb{Z}^+$$

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- 2nd replication for Renyi entropy

$$S^{(n)}(AA^*)_{\psi_m} = \frac{1}{n-1} \ln \mathrm{tr} \left(\rho_{AA^*}^{(m)}\right)^n$$

$$\rho_{AA^*}^{(m)} = \mathrm{tr}_{BB^*} |\psi_m\rangle\langle\psi_m| = \frac{1}{\mathrm{tr}\,\rho_{AB}^m} \mathrm{tr}_{BB^*} \left|\rho_{AB}^{m/2}\right\rangle\left\langle\rho_{AB}^{m/2}\right|$$

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Now one can obtain

$$Z_{n,m} \equiv \mathrm{tr}_{AA^\star} \left(\mathrm{tr}_{BB^\star} \left|\rho_{AB}^{m/2}\right\rangle \left\langle \rho_{AB}^{m/2}\right|\right)^n, \quad \text{(calculable via twist op's)}$$

where $Z_{1,m} = \mathrm{tr}\rho_{AB}^m$ and

$$S^{(n)}(AA^\star)_{\psi_m} = \frac{1}{n-1} \ln \frac{Z_{n,m}}{(Z_{1,m})^n}, \quad \longrightarrow \quad S_R = \lim_{m,n\to 1} S^{(nm)}$$

Logarithmic negativity and EWC [Kudler-Flam,Ryu'18] [Kusuki, Kudler-Flam, Ryu'19]

Renyi entropy is related to area of cosmic brane in the bulk gravity theory [Dong '16]

$$\tilde{S}^{(n)} \equiv n^2 \partial_n \left(\frac{n-1}{n} S^{(n)} \right) = \frac{\text{area(cosmic brane)}}{4G}, \quad \text{tension} = \frac{n-1}{4nG}$$

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Renyi reflected entropy $\propto$ area of back reacted brane on EW

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Partial transposition: $\langle i_A, j_B | \rho_{AB}^{T_B} | k_A, l_B \rangle \equiv \langle i_A, l_B | \rho_{AB} | k_A, j_B \rangle$

Negative Eigenvalue of $\rho_{AB}^{T_B}$ $\Longrightarrow$ Entanglement

Logarithmic Negativity $= \log\left(\mathrm{tr} \sqrt{\rho_{AB}^{T_B} \, \rho_{AB}^{T_B \dagger}} \right) = S_R^{(1/2)}$

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Odd entropy and EWC (without purification) [Tamaoka'18]

$$S_o(A:B) \equiv \lim_{n_o \to 1} \frac{1}{1-n_o}\left[\mathrm{tr}\left(\rho_{AB}^{T_B}\right)^{n_o} - 1\right],$$

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$$S_O(A:B) = \sum_{\lambda_i<0} |\lambda_i| \ln |\lambda_i| - \sum_{\lambda_i>0} |\lambda_i| \ln |\lambda_i|, \quad \lambda_i : \text{eigenvalues of} \rho_{AB}^{T_B}$$

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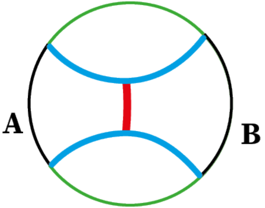
- It reduces to the EE for pure states.
- It reduces to the von Neumann entropy for product states.
- By using replica for vacuum and thermal states a holographic CFT$_2$:

$$S_o(A:B) = S(A:B) + E_W(A:B)$$



Odd entropy and EWC

A new constraint for Holographic CFTs?!

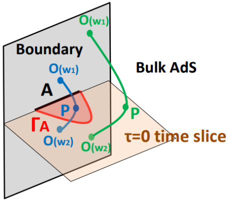
$$EW(A:B) \geq I(A:B)/2 \quad \Longrightarrow \quad S_o(A:B) - S(A:B) \geq I(A:B)/2$$

Summary: EWC is dual to ...

- Entanglement of purification?[Takayanagi-Umemoto'17] [Nguyen-Devakul-Halbasch- Zaletel-Swingle'17]
- Reflected entropy?[Dutta-Faulkner '19]
- Logarithmic negativity? [Kudler-Flam,Ryu'18]
- Odd entropy? [Tamaoka'18]
- It seems that all of them are same in holographic CFTs.

Further investigations

- bit threads description [Bao-Chatwin-Davies-Pollack-Remmen]
- entanglement wedge cross section can not be any axiomatic measures of the quantum entanglement. [Umemoto '19]
- dynamics of local quench via reflected entropy [Yuya-Tamaoka '19]
- checking the $S_R = 2E_w$ duality at higher order terms in $m-1$ for ρ_{AB}^m [Jeong- Kim-Nishida'19]
- generalization for multi-partite regions [Bao-Cheng'18] [Bao-Halpern '18] [Akers-Rath'19]
- reconstructing EW via information metrics [Kusukia-Suzukib-Takayanagia-Umemotoa'19]



[borrowed from 1908.09939]

- EoP in free scalar QFTs [Bhattacharyyaa-Takayanagi-Umemoto '18]

Some projects

- Thermal corrections to EWC (with Lifshitz and hyperscaling violating exponents) and EWCs for entangling region with singular boundaries [Babaei-Mohammadi-Vahidinia '19]

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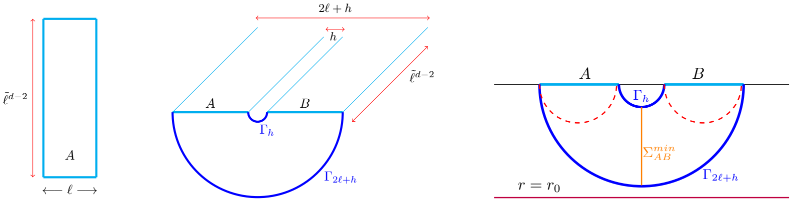
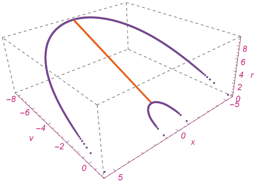
$$ds^2 = \frac{1}{r^2}\left[-f(r,v)dv^2 - 2dvdr + dx_{d-1}^2\right], \quad f(r,v) = 1 - m(v)r^d + Q^2(v)r^{2(d-1)}$$

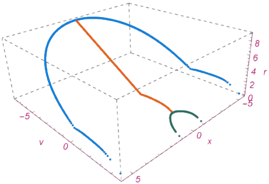
$$dv = dt - \frac{dr}{f}, \quad m(v) = m\frac{1+\tanh(v/v_0)}{2}$$



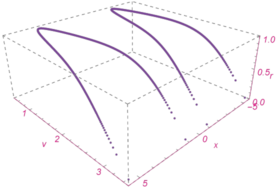
Time evolution of EWC



Initial config.

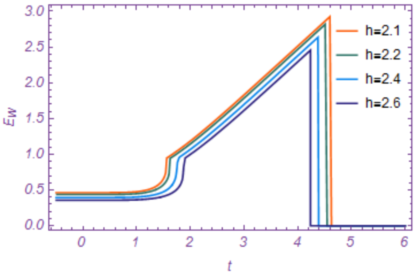


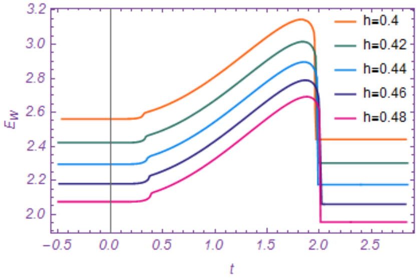
mid time

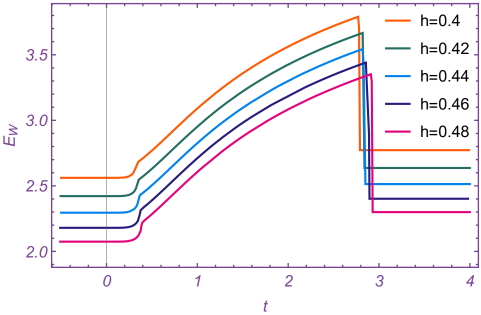


late time

[Babaei-Mohammadi-Vahidinia (to appear)]







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Thank you for your attention!