Introduction to Relativistic Heavy-Ion Collisions
Lecture 4: Glauber model (part 3)

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figure: https://www.bnl.gov/newsroom

Input of the Glauber modelling

$$\sigma_{\mathrm{AB}} = \int d^2b \left\{1 - \left[1 - \sigma_{\mathrm{inel}}^{\mathrm{NN}} \hat{T}_{\mathrm{AB}}(b)\right]^{AB}\right\}$$

- nuclear charge densities (measured in low-energy electron-ion scattering experiments)
- energy dependence of the inelastic nucleon-nucleon cross section (measured in high-energy p+p collisions)

Nuclear charge density

- Woods-Saxon/2-parameter Fermi profile

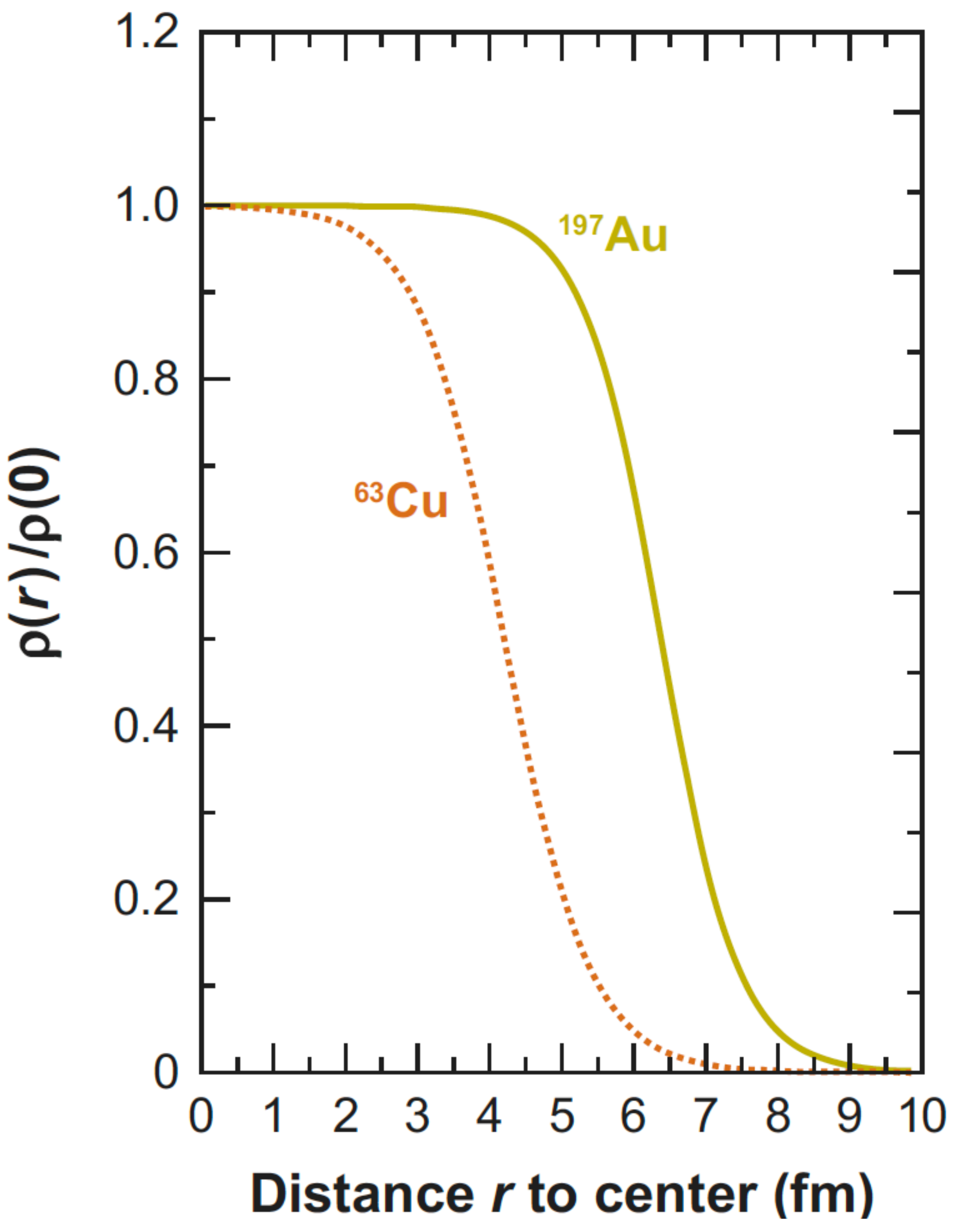
$$\rho(r) = \rho_0 \frac{1 + w(r/R)^2}{1 + \exp\left(\frac{r-R}{a}\right)}$$

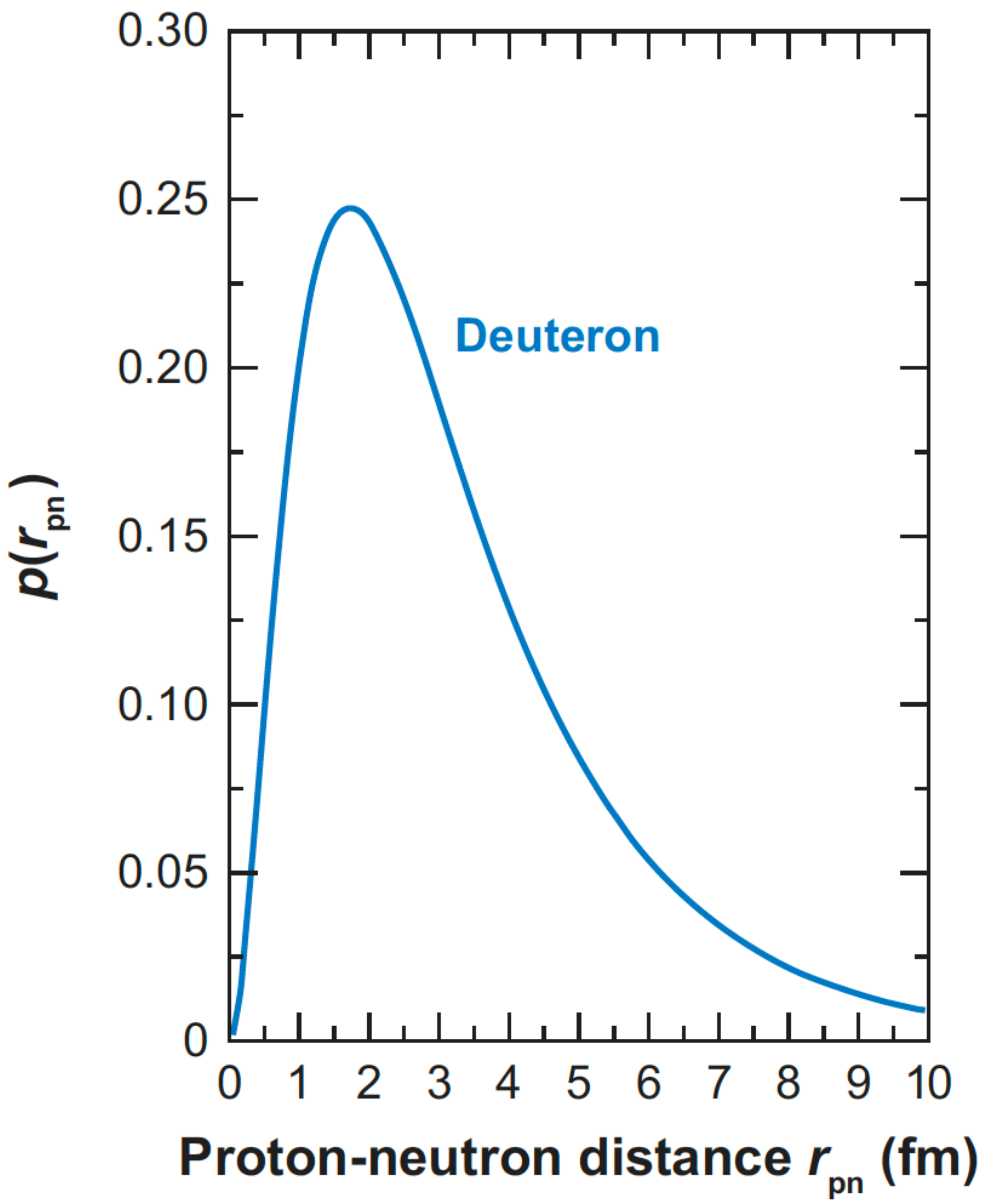
$$^{197}\text{Au } (R = 6.38 \text{ fm}, a = 0.535 \text{ fm}, w = 0)$$
$$^{63}\text{Cu } (R = 4.20641 \text{ fm}, a = 0.5977 \text{ fm}, w = 0)$$
$$4\pi r^2 \rho(r)$$

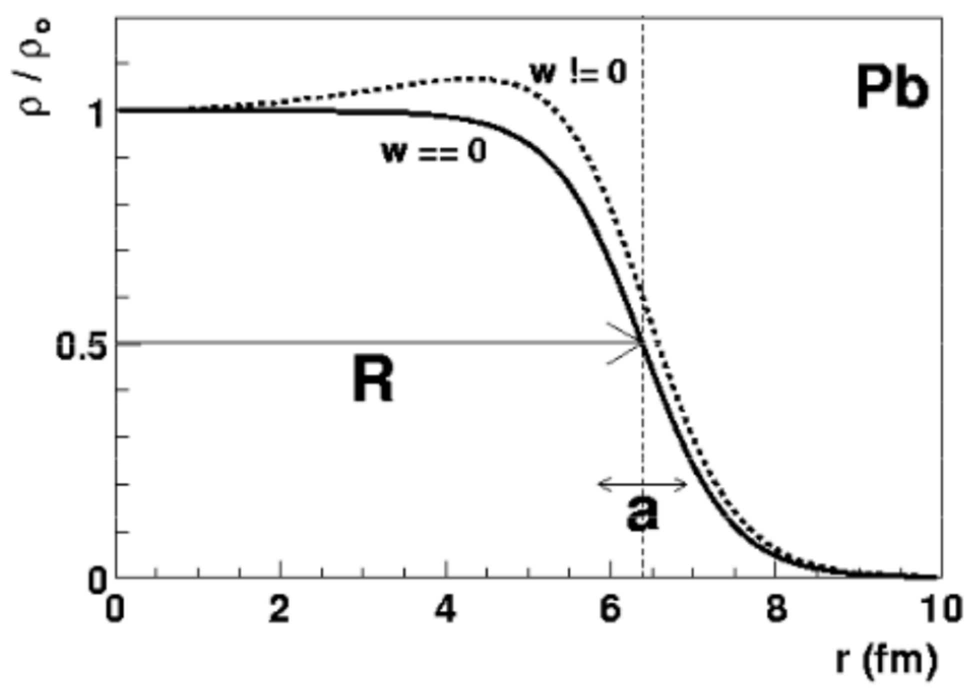
- Hulthen wave function

$$\phi(r_{pn}) = \frac{1}{\sqrt{2\pi}} \frac{\sqrt{ab(a+b)}}{b-a} \left(\frac{e^{-ar_{pn}} - e^{-br_{pn}}}{r_{pn}}\right)$$

$$p(r_{pn}) = 4\pi r_{pn}^2 \phi^2(r_{pn})$$

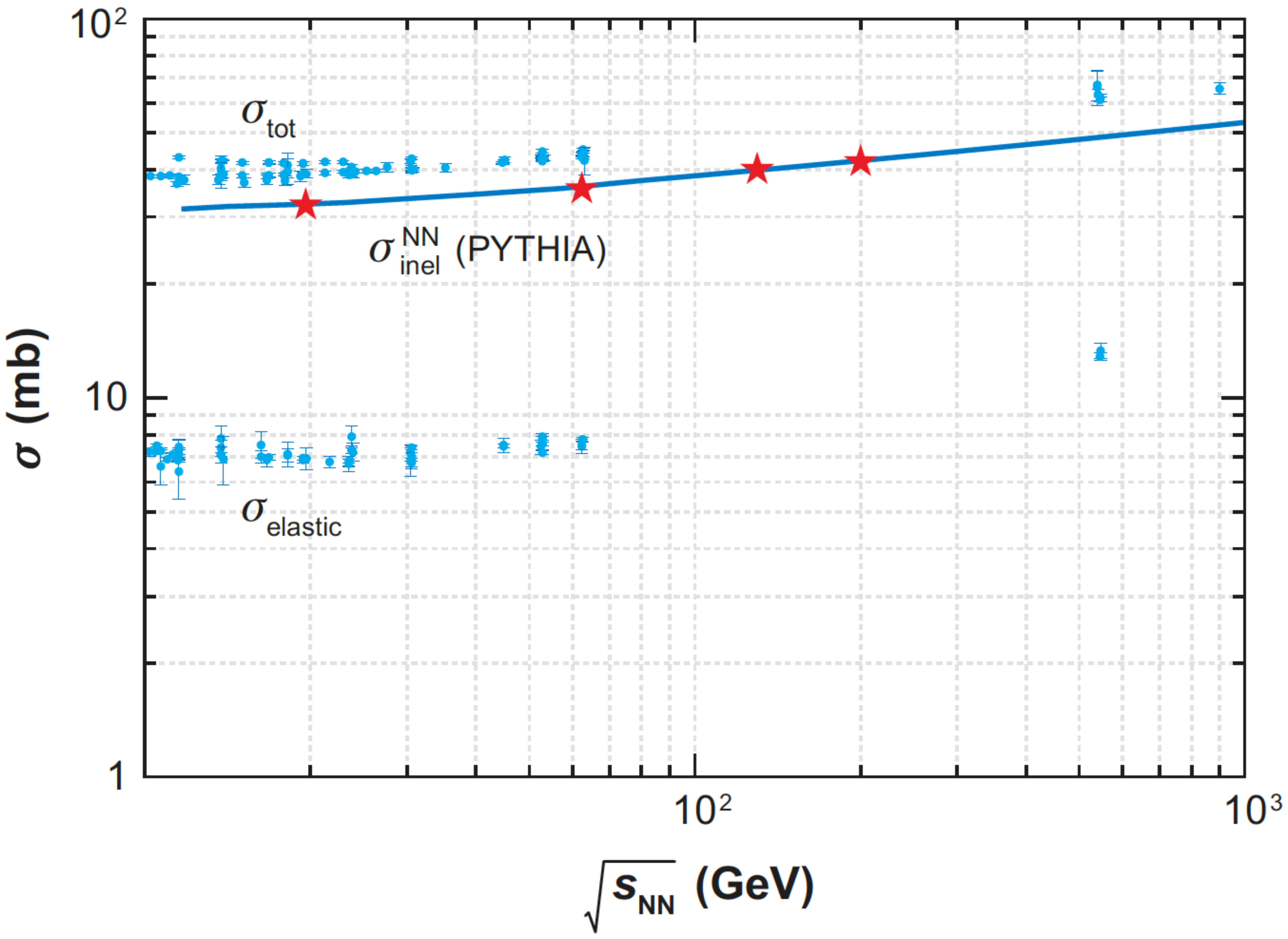




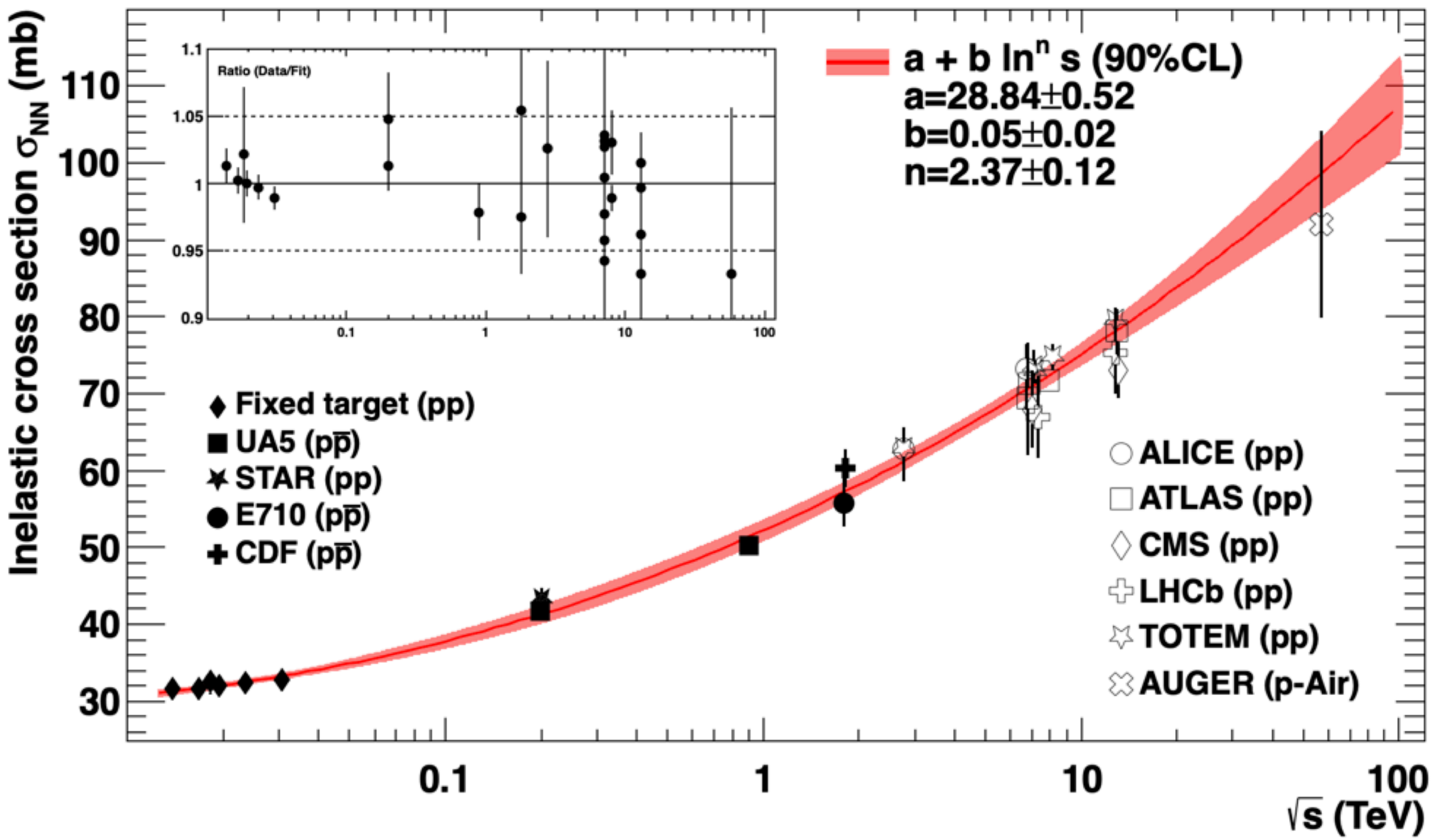


Inelastic nucleon-nucleon cross section

at RHIC ($\sigma_{inel}^{NN} = 32.3$, 35.6, 40, and 42 mb at $\sqrt{s_{NN}} = 19.6$, 62.4, 130, and 200 GeV,



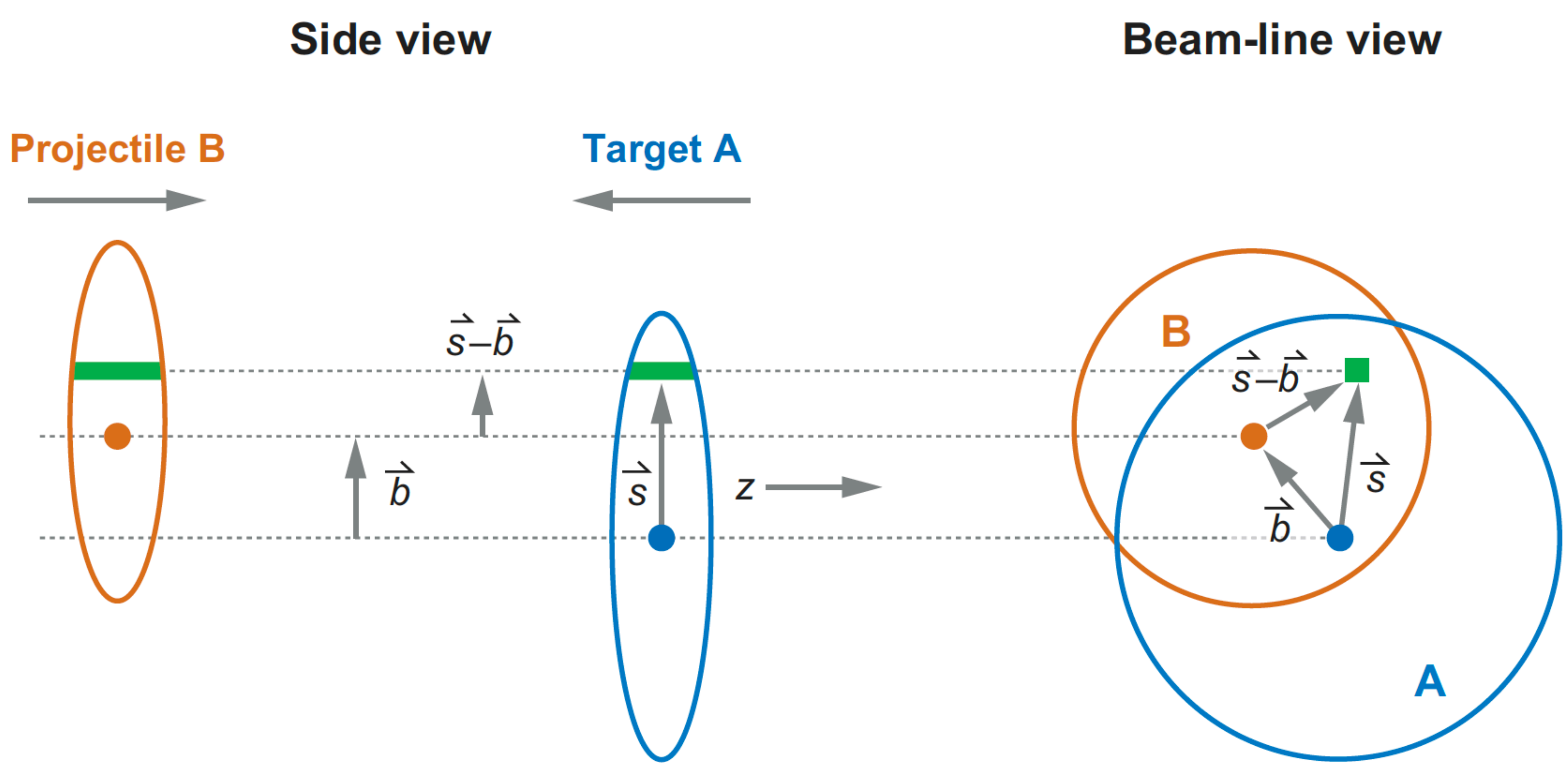
Inelastic nucleon-nucleon cross section



$$\sigma_{NN}(s) = a + b\ln^n(s)\,,$$

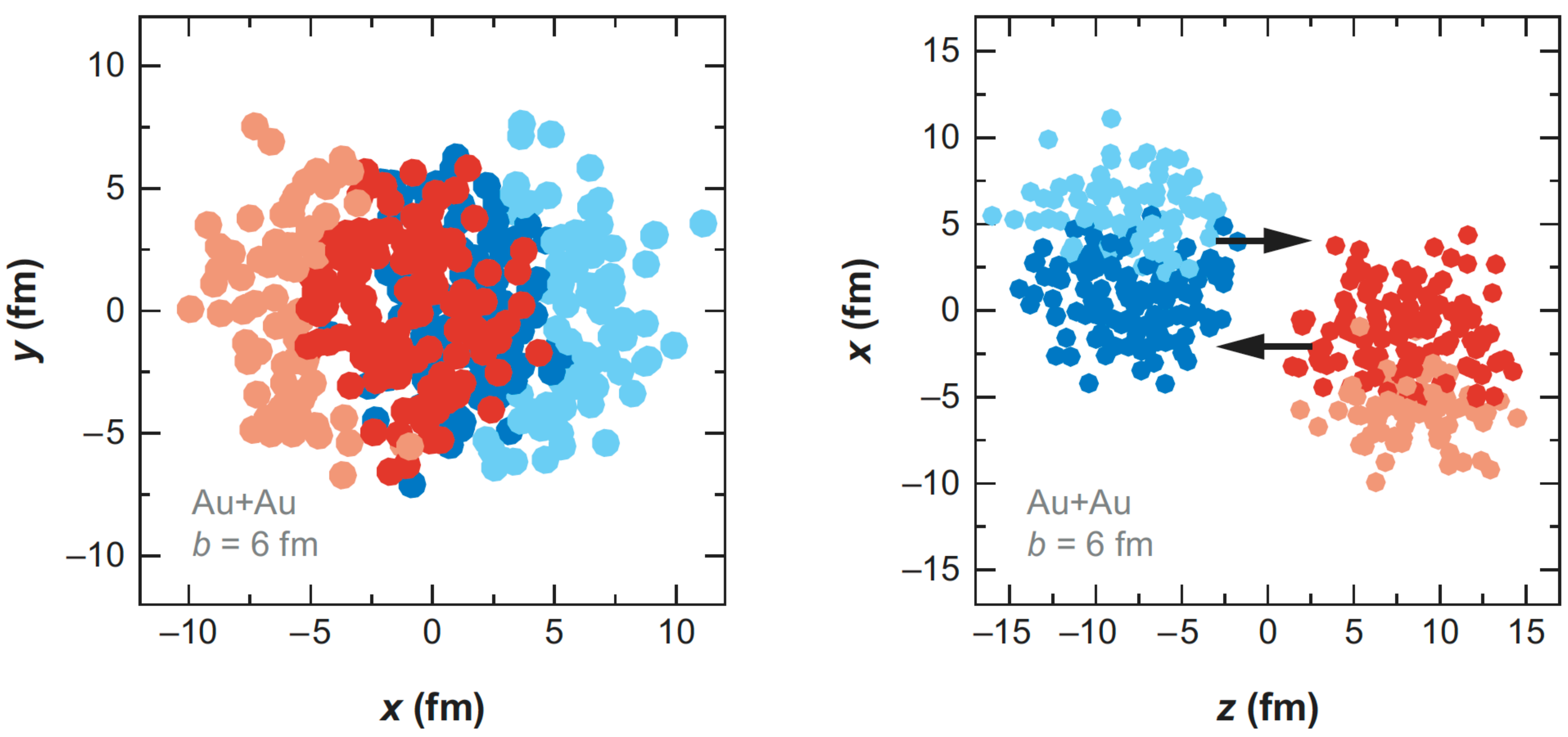
$$a = 28.84 \pm 0.52,\ b = 0.0458 \pm 0.0167 \text{ and } n = 2.374 \pm 0.123$$

Optical limit of the Glauber model



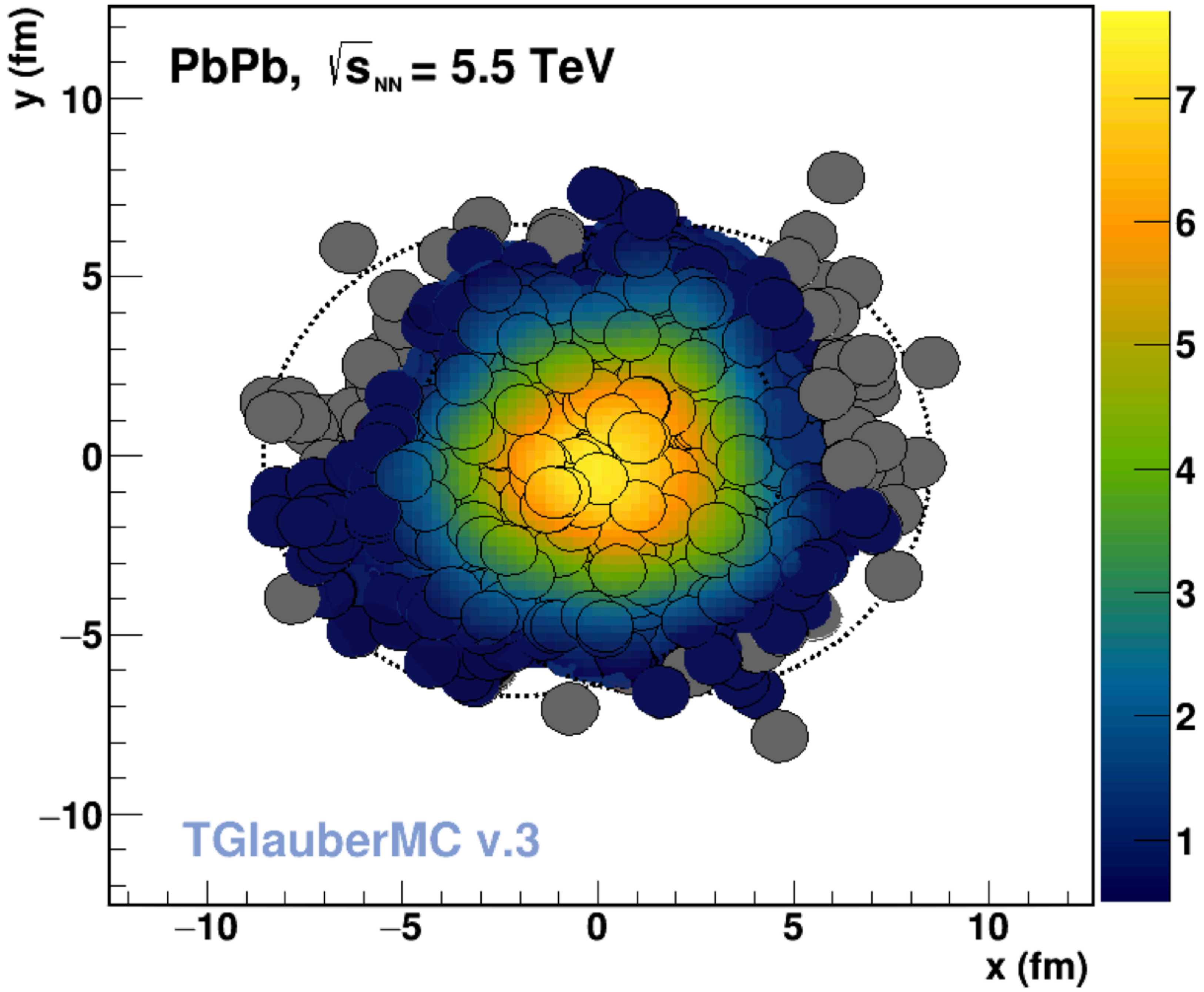
incoming nucleons see the target as a smooth density;
does not locate nucleons at specific spatial coordinates

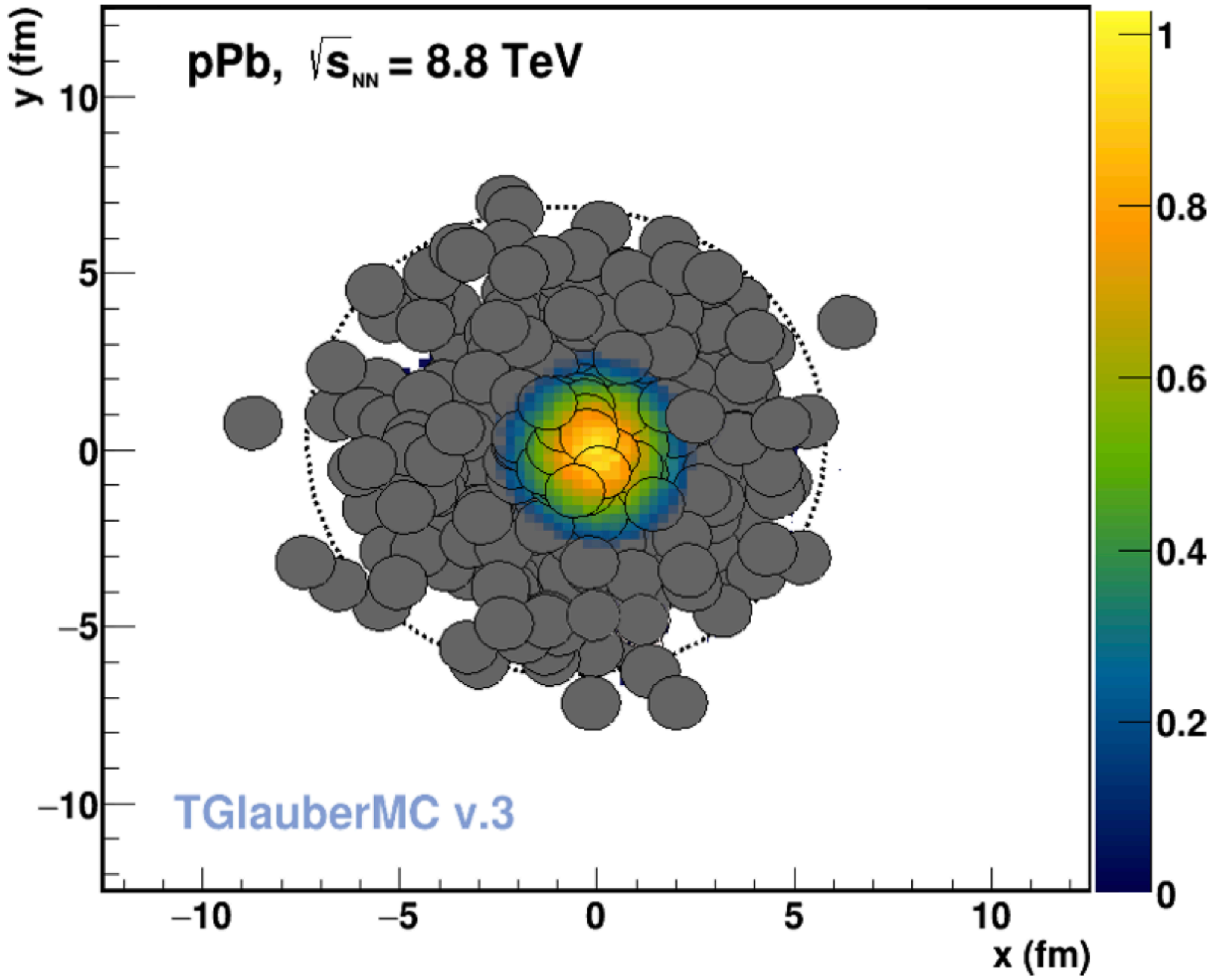
Glauber Monte Carlo modelling



- colliding nuclei are assembled by distributing the A nucleons of nucleus A and the B nucleons of nucleus B in a three-dimensional coordinate system according to the respective nuclear density distribution
- random impact parameter is then drawn from the distribution $\mathrm{d}\sigma/\mathrm{d}b = 2\pi b$
- a sequence of independent binary nucleon-nucleon collisions
- NN collision takes place if the nucleons' distance in the plane orthogonal to the beam axis satisfies $d \leq \sqrt{\sigma_{\mathrm{inel}}^{\mathrm{NN}}/\pi}$

Glauber Monte Carlo modelling





one of the most typical applications of the Glauber model is to provide
the initial conditions for the number density (or entropy or energy densities) of the medium
formed in nuclear collisions as input for hydrodynamic calculations

Glauber Monte Carlo modelling

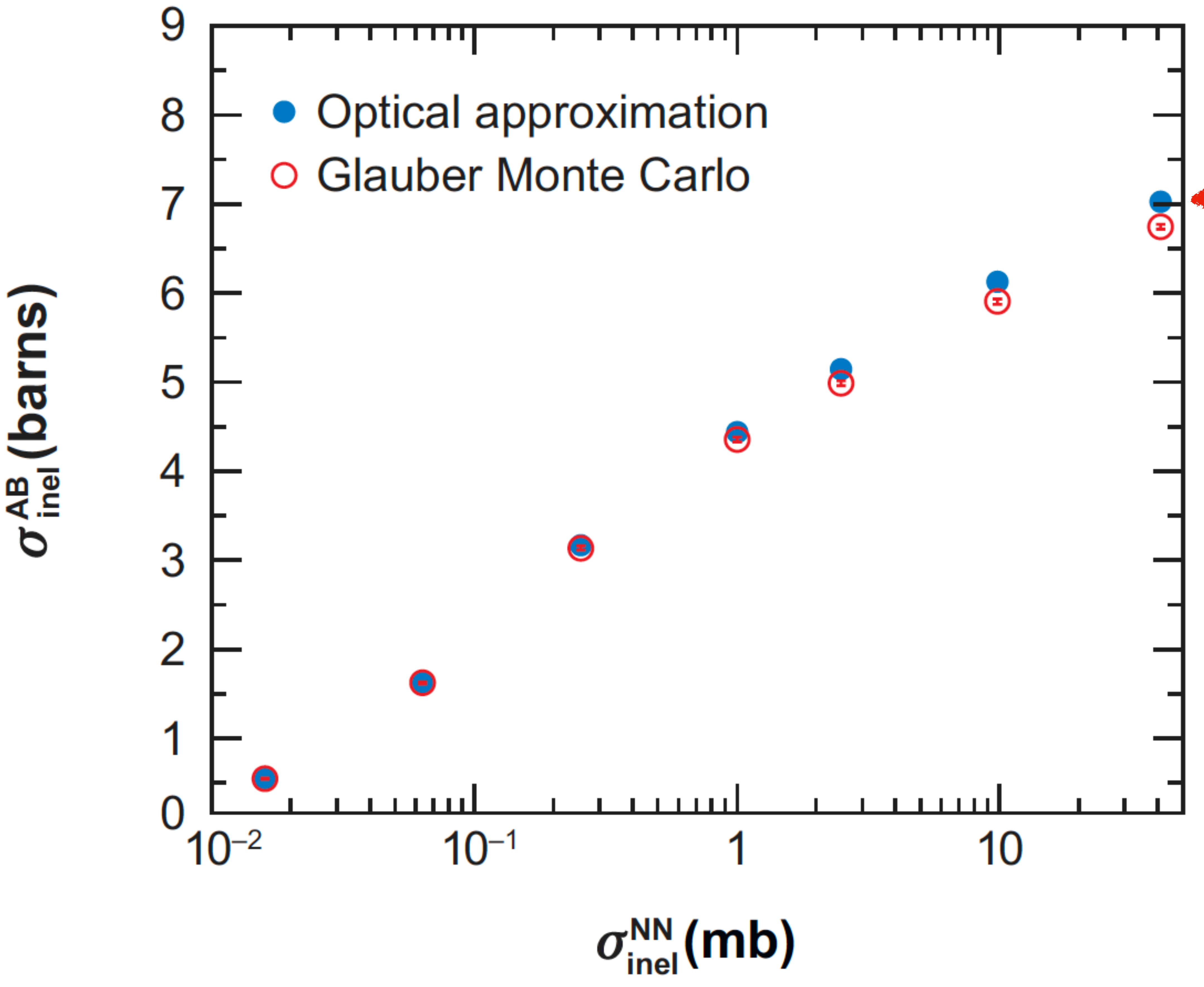
$$
\begin{aligned}
\sigma_{\text{inel}}^{\text{AB}} &= \int d^2b \int d^2s_1^A \cdots d^2s_A^A d^2s_1^B \cdots d^2s_B^B \times \hat{T}_{\text{A}}\left(\mathbf{s_1^A}\right) \cdots \hat{T}_{\text{A}}\left(\mathbf{s_A^A}\right) \hat{T}_{\text{B}}\left(\mathbf{s_1^B}\right) \cdots \hat{T}_{\text{B}}\left(\mathbf{s_B^B}\right) \\
&\times \left\{1 - \prod_{j=1}^{B} \prod_{i=1}^{A} \left[1 - \hat{\sigma}\left(\mathbf{b} - \mathbf{s_i^A} + \mathbf{s_j^B}\right)\right]\right\},
\end{aligned}
$$

$$
\sigma_{\text{AB}} = \int d^2b \left\{1 - \left[1 - \sigma_{\text{inel}}^{\text{NN}} \hat{T}_{\text{AB}}(b)\right]^{AB}\right\}
$$

These expressions are generally the same for:

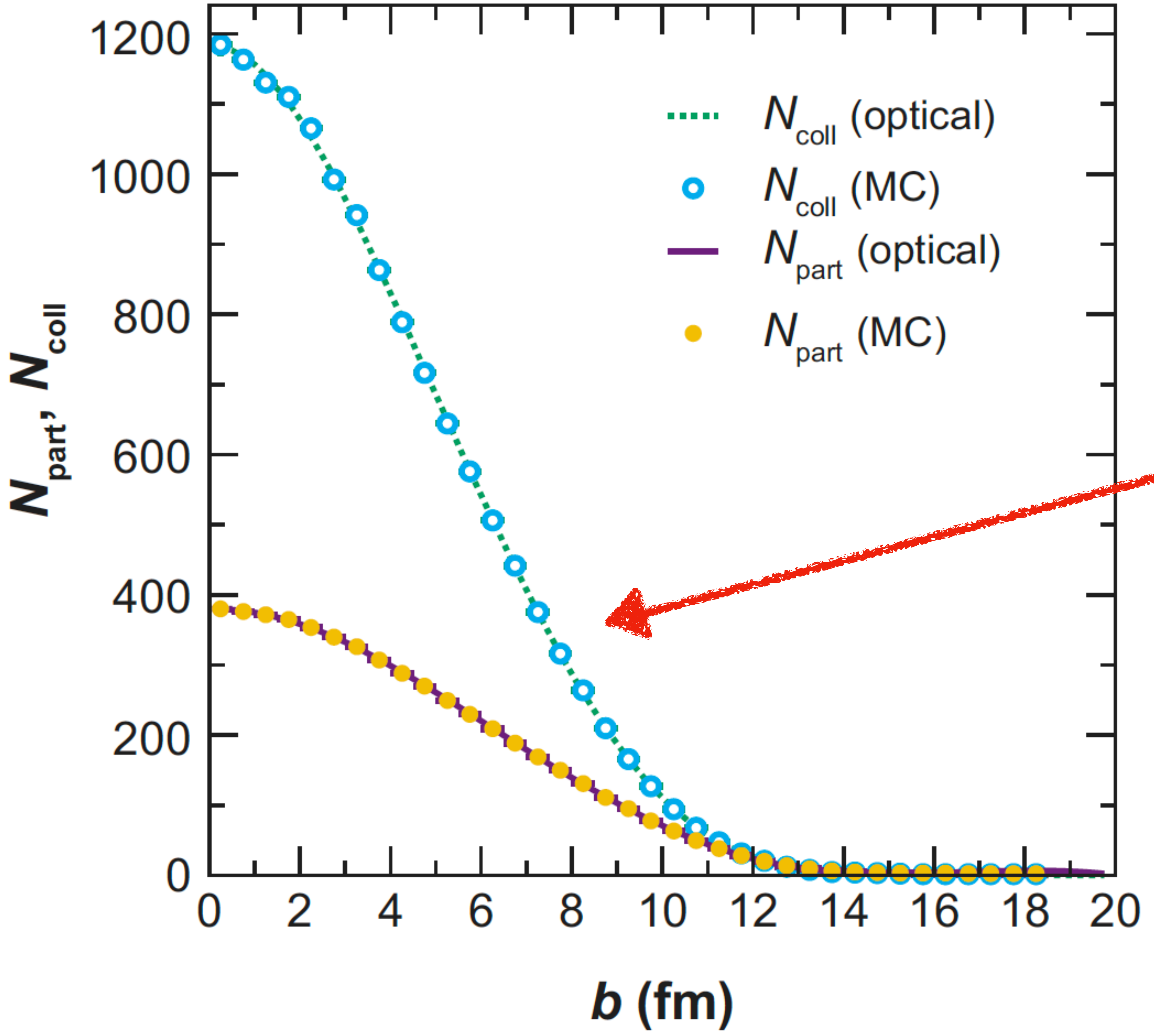
- large mass numbers A (and B)
- sufficiently small inelastic nucleon-nucleon cross section

GMC versus optical limit



extra terms in GMC introduce shadowing corrections that reduce the cross section

GMC versus optical limit



GMC results agree with optical limit up to large impact parameters

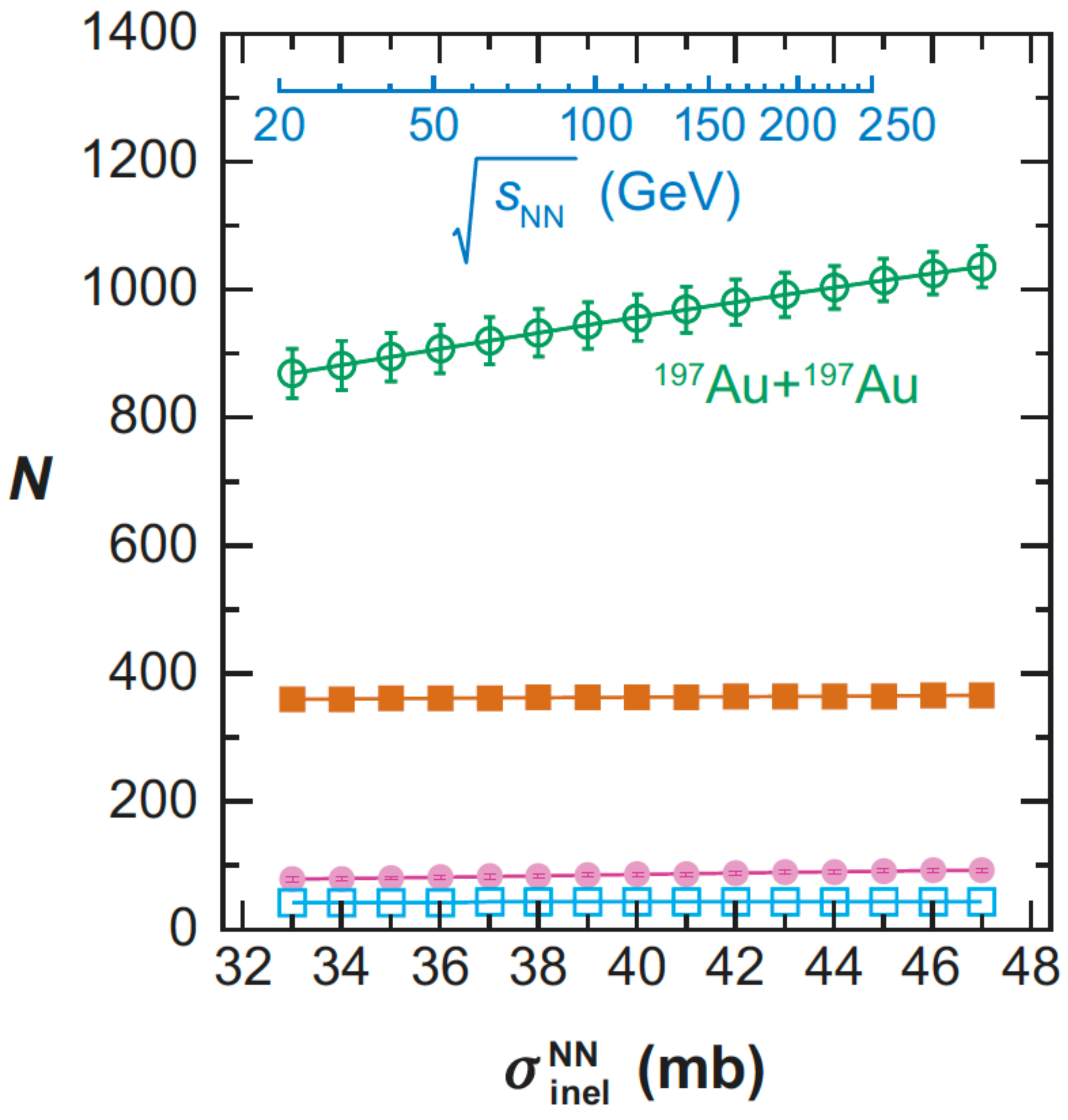
Systematics

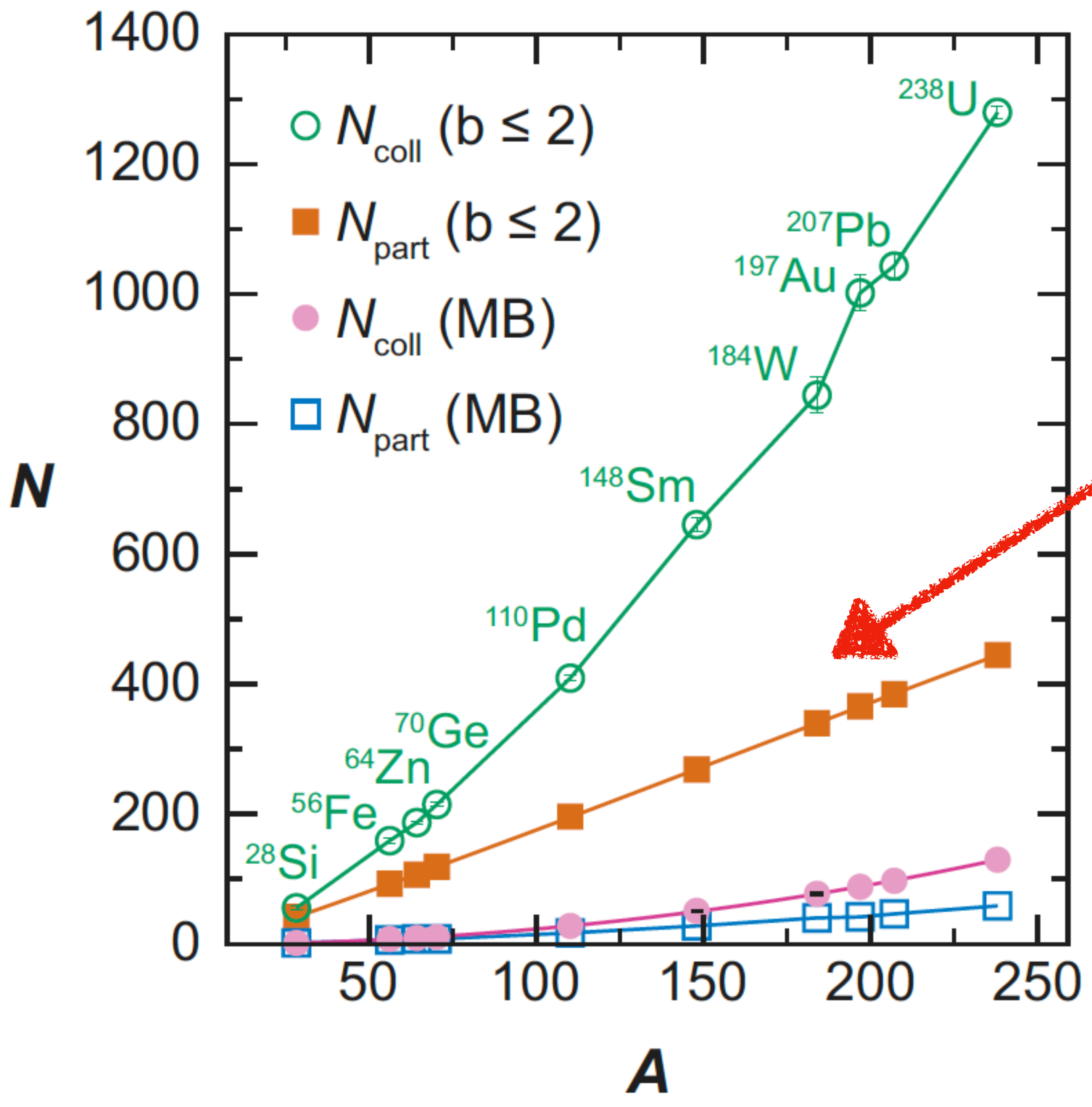
show only a weak energy dependence within RHIC energy range

largely dependent on the geometry of the colliding nuclei

Npart should scale with the volume of the interaction region

common volume of the largely overlapping nuclei in central collisions is proportional to A for a saturated nuclear density



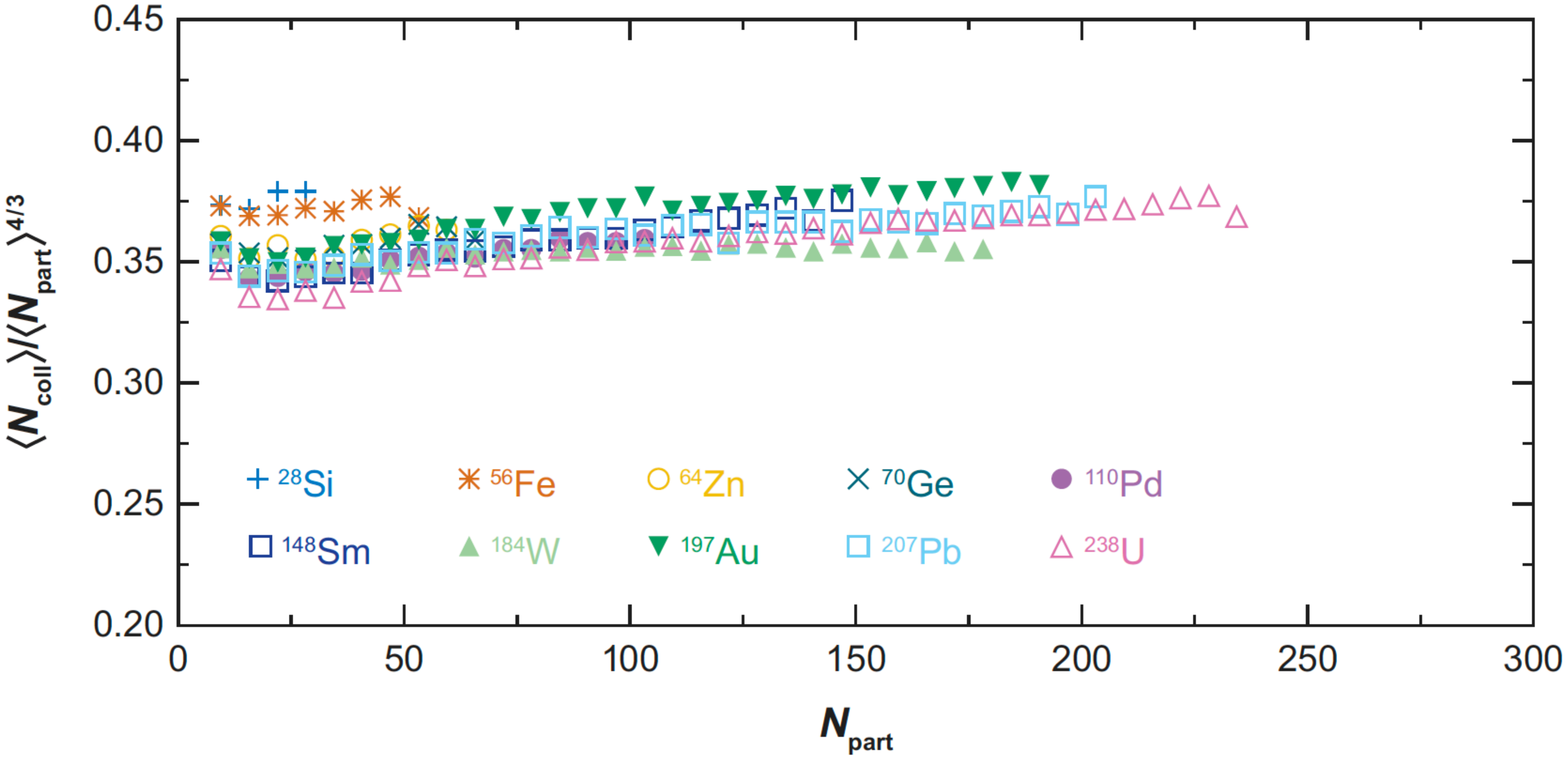


fixed colliding system

fixed cross section

$\sqrt{s_{NN}} = 200$ GeV

Systematics



the average number of collisions per participant
scales as the length of the interaction volume so one observes

$$N_{\rm coll} \propto N_{\rm part}^{4/3}$$

Relating GM to experimental data

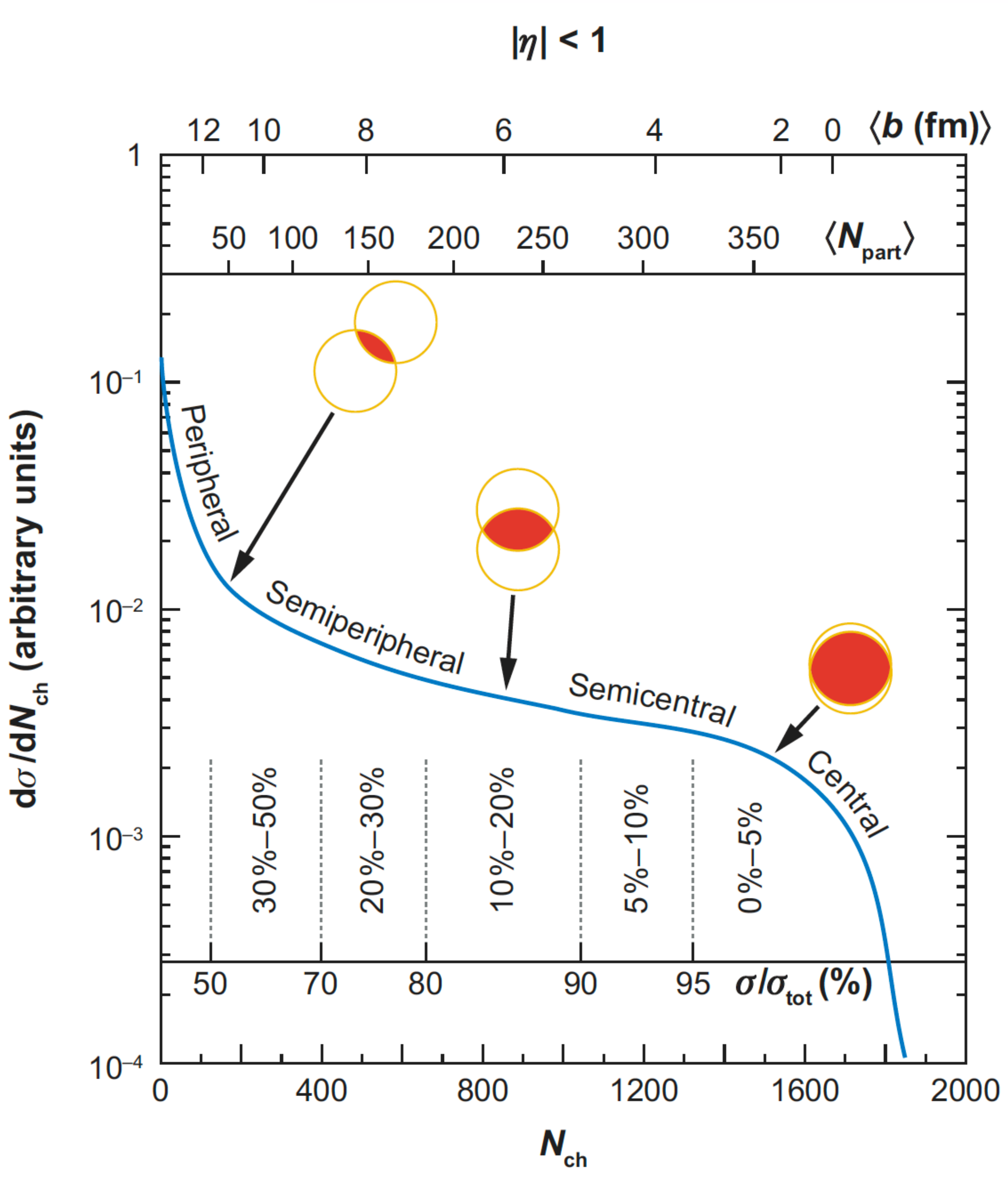
- Neither Npart nor Ncoll can be measured directly in experiment

- Mean values of such quantities can be extracted for **classes of measured events** via a mapping procedure

- Measured distribution is mapped to the corresponding distribution obtained from Glauber calculations.

- This is done by defining **centrality classes** in both the measured and calculated distributions and then **connecting the mean values from the same centrality class in the two distributions**.

Relating GM to experimental data



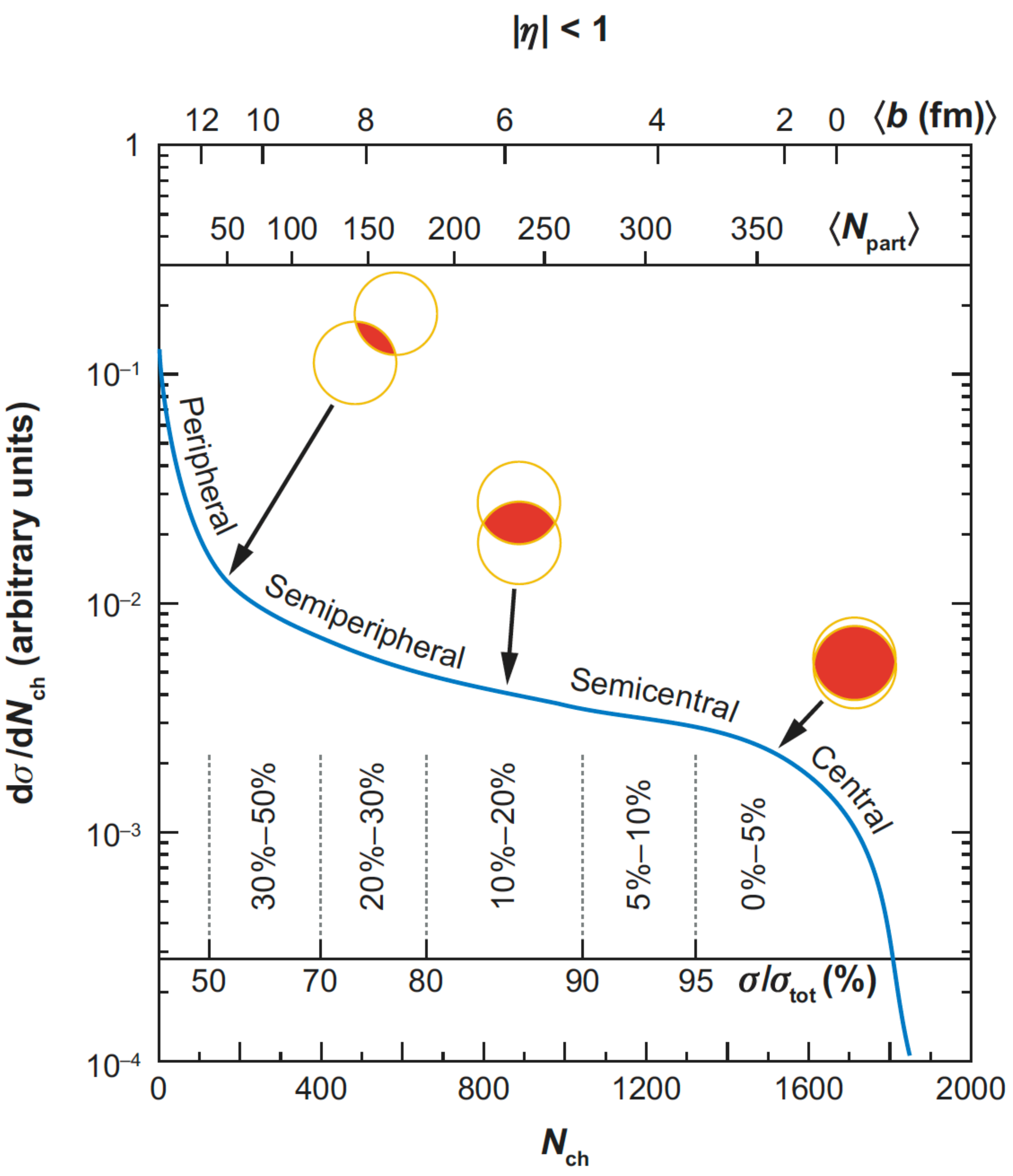
The basic assumption underlying centrality classes is that the impact parameter b is monotonically related to particle multiplicity

one measures the per-event charged-particle multiplicity for an ensemble of events

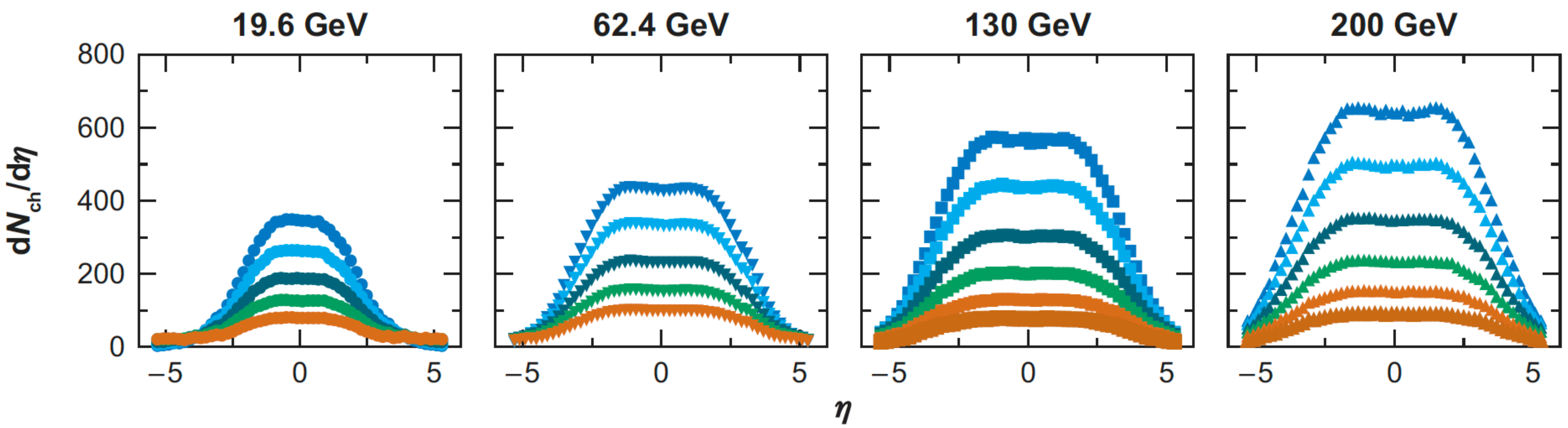
when the total integral of the distribution is known, centrality classes are defined by binning the distribution on the basis of the fraction of total

same procedure is then applied to a calculated distribution, often derived from a large number of Monte Carlo trials

Relating GM to experimental data



Experiment



Theory:

Nch can be simulated via various methods
but all require the coupling of a Glauber calculation to a model of charged-particle production,
either dynamic or static

For an optical Glauber calculation, the simulated multiplicity can be calculated semi-analytically, assuming that each participant contributes a given value of Nch, which is typically drawn from one of the static probability distributions

Relating GM to experimental data

Białas, Bleszyński and Czyż argued (in 1976) that the average multiplicity in a collision of two nuclei with the mass numbers A and B is

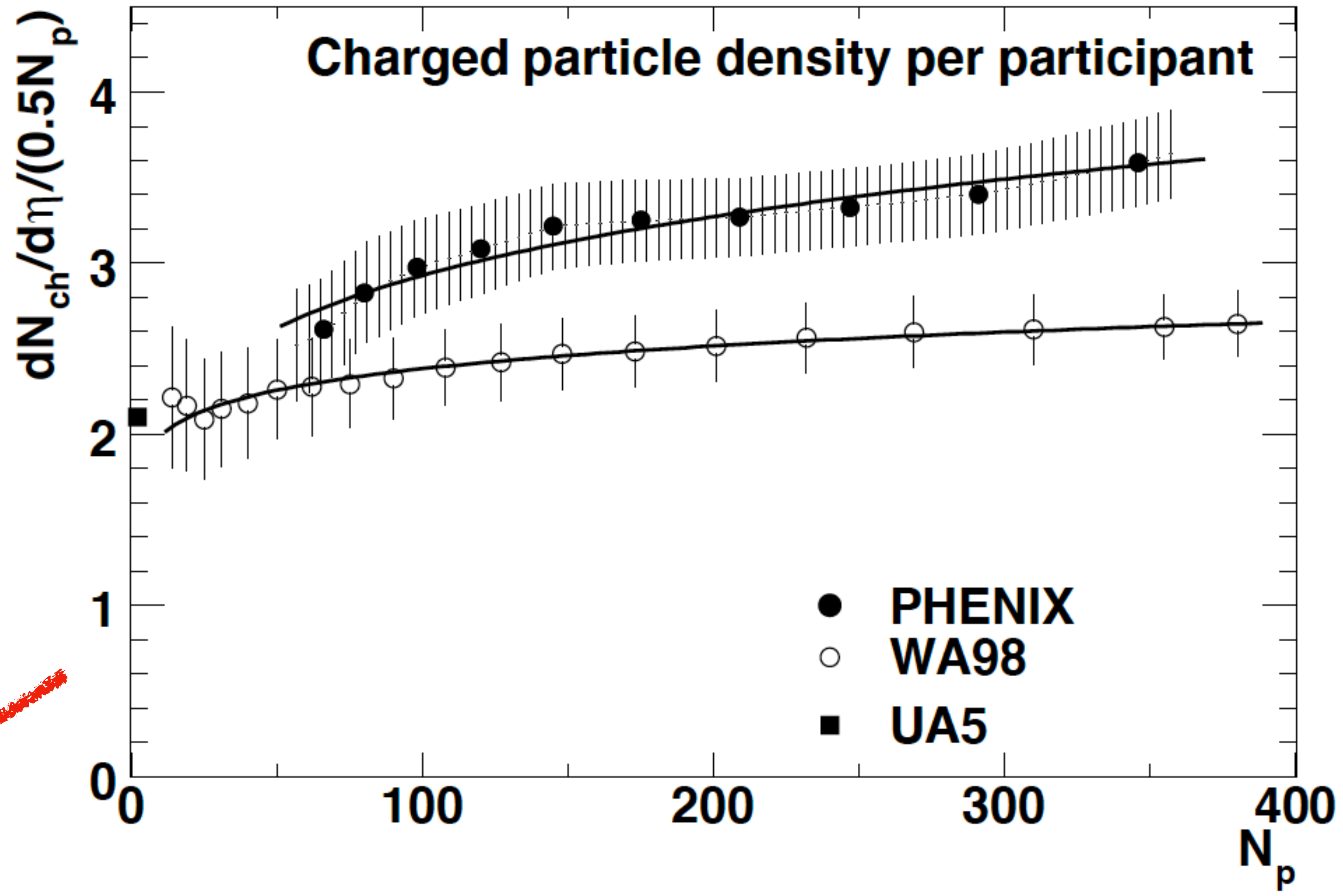
$$\overline{N}_{AB} = \frac{1}{2}\,\overline{w}_{AB}\,\overline{N}_{\mathrm{NN}}\,,$$

Estimates of the charged particle multiplicities obtained from the wounded nucleon model, $\frac{1}{2}\,\overline{w}_{AA}\,\overline{N}_{\mathrm{NN}}$, compared with the measured multiplicities, $\overline{N}_{AA}$, for different reactions studied by the NA49 and PHOBOS Collaborations. The last column shows the ratio of the measured multiplicity and the model prediction.

Expt.	E_{lab}/A [GeV]	$\sqrt{s_{\mathrm{NN}}}$ [GeV]	$\overline{N}_{AA}$	$\overline{w}_{AA}$	$\frac{1}{2}\,\overline{w}_{AA}\,\overline{N}_{\mathrm{NN}}$	r
NA49	40	8.8	693	349	875	0.79
NA49	80	12.3	1029	349	1059	0.97
NA49	158	17.3	1413	362	1307	1.08
PHOBOS	(9000)	130.0	4200	355	2902	1.45

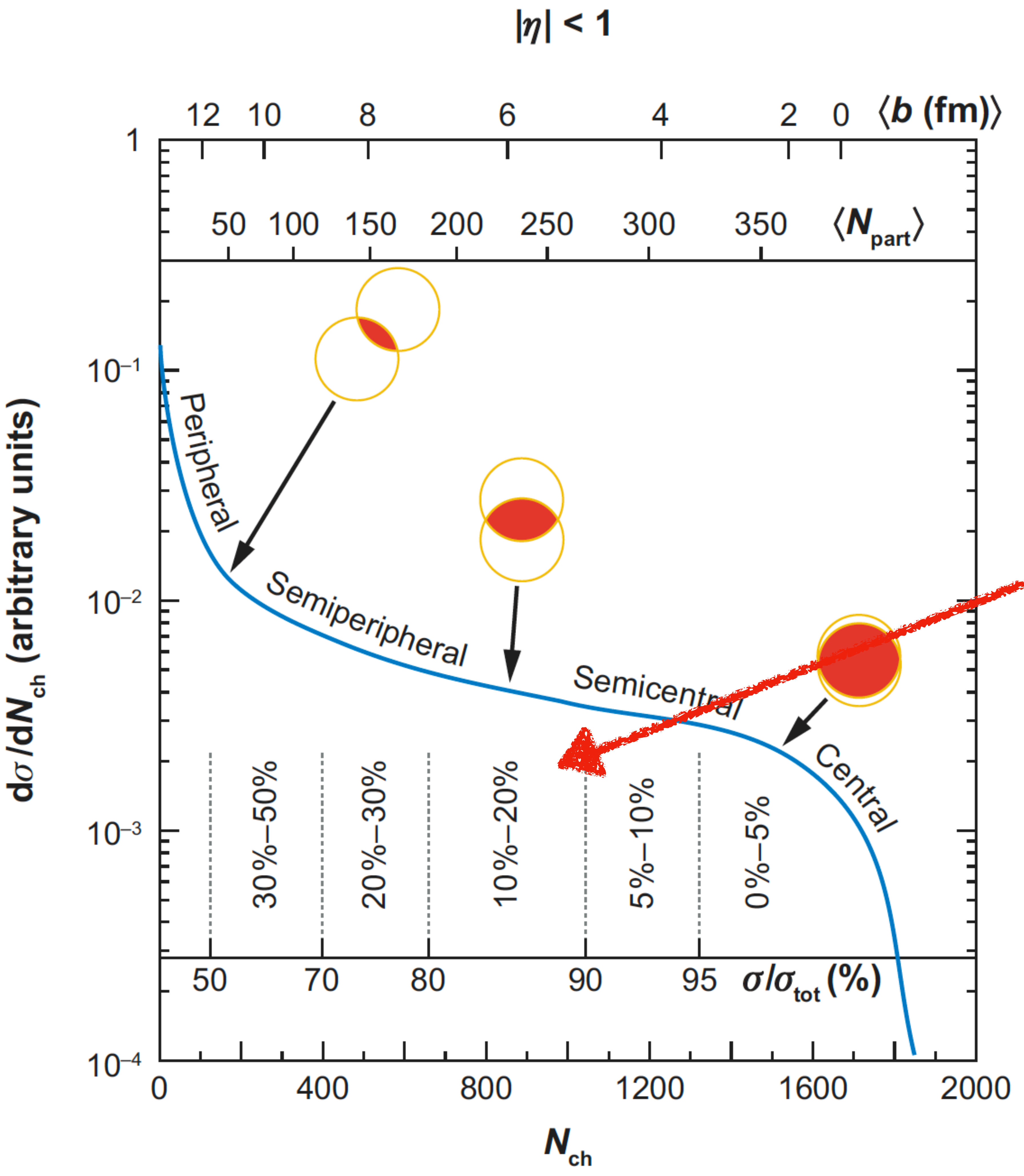
Relating GM to experimental data

The charged particle pseudorapidity density as a function of the number of the participants. The measurement of the PHENIX group at RHIC, $\sqrt{s_{\mathrm{NN}}}$ = 130 GeV, is compared to the measurement done by the WA98 group at the SPS, $\sqrt{s_{\mathrm{NN}}}$ = 17.3 GeV.



$$\boxed{\left.\frac{d\overline{N}_{AB}}{d\eta}(\mathbf{b})\right|_{\eta=0} = \frac{\alpha}{2}\overline{w}_{AB}(\mathbf{b}) + \beta\overline{n}_{AB}(\mathbf{b})}$$

Relating GM to experimental data

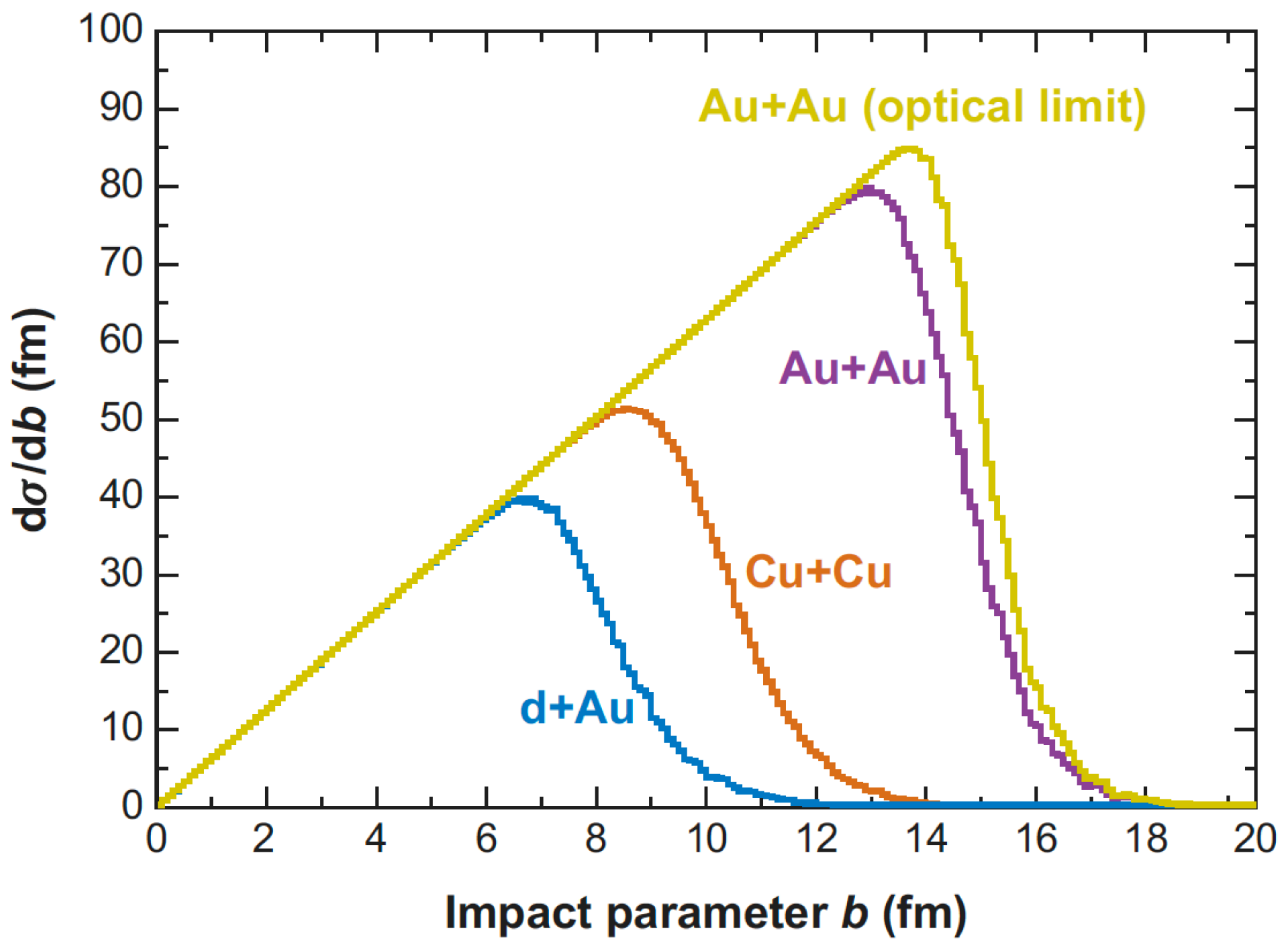


the mapping procedure is robust to an overall scaling of the simulated Nch distribution compared to the measured distribution

$$\frac{\int_{\infty}^{n10} \frac{dN_{evt}}{dN_{ch}}\, dN_{ch}}{\int_{\infty}^{0} \frac{dN_{evt}}{dN_{ch}}\, dN_{ch}} = 0.1 \quad \text{and} \quad \frac{\int_{\infty}^{n20} \frac{dN_{evt}}{dN_{ch}}\, dN_{ch}}{\int_{\infty}^{0} \frac{dN_{evt}}{dN_{ch}}\, dN_{ch}} = 0.2.$$

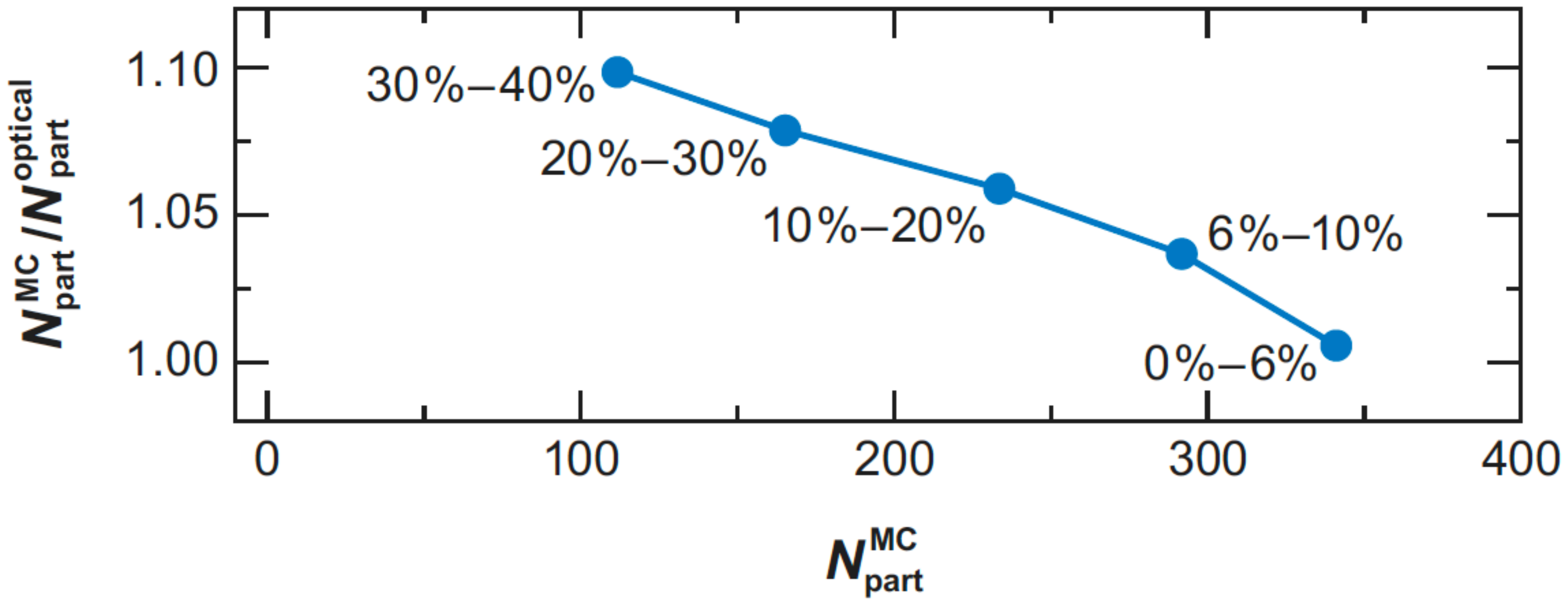
once a centrality class is defined in simulation,
the mean values of quantities such as Npart
can be calculated for events that fall in that centrality bin

Estimating Geometric Quantities

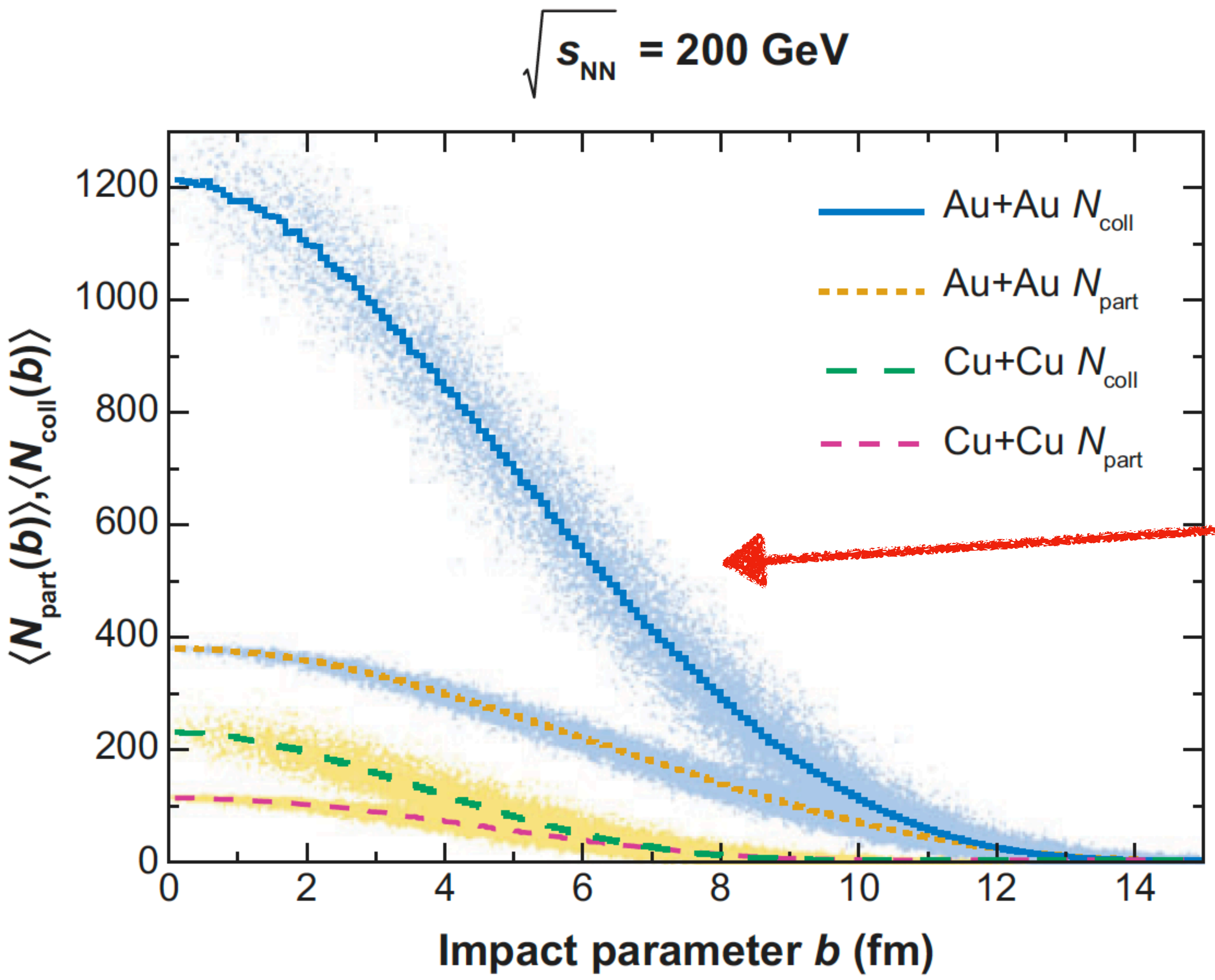


Optical-limit calculations do not naturally contain the terms in the multiple-scattering integral, where nucleons hide behind each other.
This leads to a slightly larger cross section

$$\sigma_{\mathrm{geo}}^{\mathrm{AuAu}} \approx 6840\ \mathrm{mb} \qquad \sigma_{\mathrm{geo,\ optical}}^{\mathrm{AuAu}} \approx 7280\ \mathrm{mb}$$

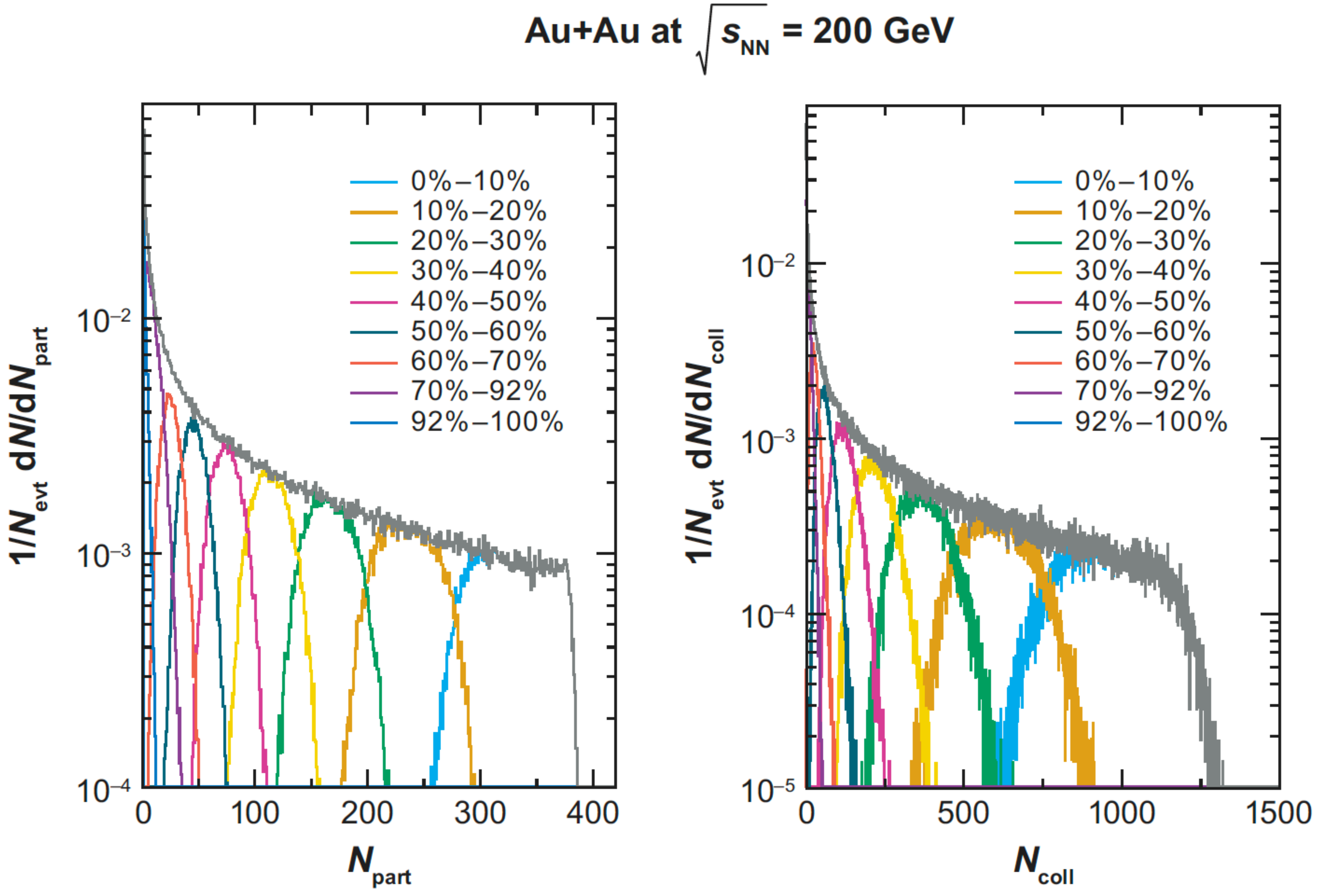


Estimating Geometric Quantities



The event-by-event fluctuations for a fixed impact parameter

Estimating Geometric Quantities



reflect the fact that peripheral A+B collisions are more likely than central collisions

Thank you!

All kinds of feedback regarding this course is welcome: radoslaw.ryblewski@ifj.edu.pl

Self-study for this lecture:

On Glauber model: W. Florkowski, *Phenomenology of Ultra-Relativistic Heavy-Ion Collisions* (World Scientific, Singapore, 2010), Sec.3
Progress in the Glauber Model at Collider Energies, https://arxiv.org/pdf/2011.14909.pdf
Glauber Modeling in High-Energy Nuclear Collisions, Annu. Rev. Nucl. Part. Sci. 2007.57:205-243.

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