Holography in 2D Flat Spacetime

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Hamid R. Afshar

Ferdowsi University of Mashhad

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Based on

- Warped-Schwarzian theory `1908.08089` *H.A.*
- Flat space holography and complex SYK `1911.05739` *H.A., D. Grumiller, H. González, D. Vassilevich*
- Flat space holography in spin-2 extended dilaton-gravity `2012.15807` *H.A., E. Esmaeili, H. Safari*

Our goal is ...

- How/if holography works beyond AdS.
- Construct models of quantum gravity in flat spacetime.

Our approach is ...

- Lowest possible dimensions; 1+1 spacetime dim.
- Gauge theory (BF) formulation of 1+1 gravity.
- Coadjoint orbit method. (Geometric quantization).

Some sharp questions ...

- Extend the JT/SYK correspondence to the case of flat.
- What is the flat-spacetime analogue of the Schwarzian action.
- Condensed matter generalizations to SYK model.
- A quantum toy model description of black holes.

Outlook

- Dilaton-grvaity in 2D
- JT gravity and SYK model
- (Warped) Schwarzian theory
- Flat space analogue(s) of JT gravity

2D dilaton-gravity

- Rich enough to accommodate holographic features e.g. JT/SYK correspondence.
- Provide models for semi-classical BH formation/evaporation e.g. page curve developments.
- An effective description of the near horizon physics.

An effective description

- 2D Dilaton-gravity can be considered as dimensional reduction from higher dimensions where the Dilaton plays the role of radial direction $R = \ell e^{-\phi}$;

$$ds_d^2 = ds_2^2 + e^{-2\phi} d\Omega_{d-2}^2$$

- In the near horizon limit

$$R \sim r_H + r\,, \qquad e^{-2\phi} \sim e^{-2\phi_H}(1 + X(r,t))$$

- Expand and keep only $\mathcal{O}(X)$

$$S_{\text{JT}} = S_0 + \frac{r_H^2}{2} \int d^2x \sqrt{-g}\, X(R + r_H^{-2}) \qquad \text{AdS}$$

$$S_{\text{CGHS}} = S_0 + \frac{r_H^2}{2} \int d^2x \sqrt{-g}\, (XR + r_H^{-2}) \qquad \text{Flat}$$

1D Hologram

- The holographic description of the near horizon demands finite temperature in the boundary and a Euclidean time

$$\tau \sim \tau + \beta$$

- All fields of the system living on the circle is in some representation of $Diff(S^1)$. They are arranged as vectors, functions, ...

$$f(\tau) \in Vec(S^1)\,, \qquad g(\tau) \in C^\infty(S^1)\,, \quad \cdots$$

- These fields form a centrally extended symmetry group $\hat{G}(S^1)$ on the circle e.g. Virasoro, Warped Virasoro, ...

1D Hologram

- Semi-classical holographic degrees of freedom usually have a description in terms of Goldstone modes of the broken symmetry to a global subgroup G_0.

$$f(\tau), g(\tau), \cdots \in \mathcal{M} = \frac{\hat{G}(S^1)}{G_0(S^1)}$$

- These quotient subspaces are called *coadjoint orbits* of the group and can be constructed and studied systematically.

Sachdev-Ye Kitaev model

- A statistical quantum mechanical system of N real fermions ψ^a;

$$\psi^a(\tau)\psi^b(\tau) + \psi^b(\tau)\psi^a(\tau) = \delta^{ab}$$

- Described by the Euclidean action;

$$S \sim \int d\tau(\psi^a \partial_\tau \psi^a - H)$$

- The Hamiltonian contains random fermion interactions e.g.;

$$H = -J^{abcd}\psi^a\psi^b\psi^c\psi^d\,, \qquad \langle J^2_{abcd}\rangle \sim J^2/N^3\,, \quad N \gg 1\,.$$

- A class of large N theories.

Symmetry aspects

- In the strict UV ($J\tau \ll 1$) and IR ($J\tau \gg 1$) limit, the theory has reparametrization symmetry.

$$\tau \to f(\tau)\,, \qquad f(\tau) \in \text{Diff}(S^1)\,.$$

- In the IR, this symmetry is spontaneously broken to SL(2,$\mathbb{R}$);

$$H \sim \partial_\tau\,, \qquad D \sim \tau\partial_\tau\,, \qquad K \sim \tau^2\partial_\tau\,.$$

- Soft directions are parametrized by Goldstone modes $f(\tau)$ living in Diff(S^1)/SL(2, $\mathbb{R}$) space.
- These Goldstone modes map solutions to solutions without any cost.

Virasoro group

$$\hat{G} = Diff(S^1) \times \mathbb{R}$$

Elements of the group $Diff(S^1)$ are smooth maps $f : \tau \to f(\tau)$ such that $f'(\tau) > 0$ and $f(\tau + \beta) = f(\tau) + \beta$. The corresponding Lie algebra consists of vector fields on the circle $L = L(\tau)\partial_\tau$ with $L(\tau) = \sum_n L_n \exp(\frac{2\pi}{\beta} in\tau)$. It admits a universal central extension, the Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0},$$

The Virasoro Lie-algebra contains the $sl(2)_m$ Lie subalgebras

$$\frac{L_m}{m}, \qquad \frac{L_{-m}}{m}, \qquad \frac{L_0}{m} + \frac{c(m^2-1)}{24m}.$$

Coadjoint orbits

The coadjoint action on the quadratic differential on the circle $b = b(\tau)d\tau^2$ looks as

$$b \to b^f = \left(b(f(\tau))f'(\tau)^2 + \frac{c}{12}S(f)\right) d\tau^2 .$$

where $S(f) = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$ is the Schwarzian derivative. Coadjoint orbits are

$$\mathcal{O}_b = \{b^f; f \in Diff(S^1)\}.$$

The geometry of the coadjoint orbit is symplectic. We are interested in coadjoint orbits passing through constant representatives b_0.

Schwarzian theory

$$I_{Schw} = \oint \left(b_0 f'^2 + \frac{c}{12} S(f) \right)$$

- In order to perform the 1-loop path integral we need two ingredients; the action and the measure of integration which both can be obtained by the fact that coadgoint orbits are simplectic manifolds. The full semi-classical description of the model is in terms of a partition function which turns out to be one-loop exact; *Stanford, Witten 2017*

$$Z = \int_{\mathcal{M}} Df \, \exp\left(\tfrac{i}{\hbar} I[b_0, c, f]\right) = e^{\frac{i}{\hbar} I_{on-shell}} Z_{one-loop}$$

- When G_0 is the maximal 3 dim global subgroup $SL(2,R)$ corresponding to symmetries of the the hyperbolic disk

$$b_0 = -\frac{\pi^2 c}{6\beta^2}, \qquad Z_{one-loop} \sim \frac{1}{\beta^{3/2}}$$

Warped Schwarzian theory

H.A. 2019

$$I_{WSchw} = I_{Schw} + \oint \left[T_0^\kappa f'^2 + \left(\frac{f''}{f'} + P_0^\kappa f'\right) g' - \kappa g''\right] + \oint (k\, g'^2 + P_0^k g' f' + T_0^k f'^2)$$

- The Warped-Virasoro group is the group of diffeomorphism of the circle acting naturally on smooth functions of the circle

$$\hat{G} = Diff(S^1) \ltimes C^\infty(S^1) \times \mathbb{R}^3$$

- If G_0 is the maximal 4 dim global subgroups $SL(2,R) \times U(1)$ or $ISO(2)_c$ we get

$$T_0^\kappa = \alpha P_0^\kappa = -\frac{2\pi i \kappa \alpha}{\beta}, \qquad T_0^k = 2\alpha P_0^k = \frac{\alpha^2 k}{4}, \qquad Z_{one-loop} \sim \frac{1}{\beta^2}$$

Warped Conformal symmetry *Detournay Hartman Hofman 2012*

From the 2D point of view they are symmetries of a non-relativistic conformal field theory. In general it admits three non-trivial cocycles. *H.A. Grumiller Detournay Oblak 2015*

$$
\begin{aligned}
[L_n, L_m] &= (n-m)L_{n+m} + \frac{c}{12}\, n^3 \delta_{n+m,0}\,, \\
[L_n, P_m] &= -mP_{n+m} - i\kappa\, n^2 \delta_{n+m,0}\,, \\
[P_n, P_m] &= k\, n \delta_{n+m,0}\,.
\end{aligned}
$$

Example: Jackiw-Teitelboim gravity

In the first order form JT-model can be written as a BF theory of SL(2,$\mathbb{R}$).

$$S_{JT} = \int [X_a(de^a + \epsilon^a{}_b\omega e^b) + X(d\omega - \tfrac{1}{2}\lambda\,\epsilon^{ab}e_a e_b)] + I_B = \int \langle B, F\rangle + I_B$$

where $A = e^a P_a + \omega J$ and $B = X^a P_a + XJ$ and

$$[P_a, J] = \epsilon_a{}^b P_b\,, \quad [P_a, P_b] = -\epsilon_{ab} J$$

with the Killing form $\langle P_a, P_b\rangle = \eta_{ab}$ and $\langle J, J\rangle = 1$.

- The bulk term is zero and the boundary term is determined by imposing AdS_2 boundary conditions and requiring having a well-defined variational principle. The boundary term is the 1D Schwarzian action.

Cangemi-Jackiw construction

In the first order form we can write a BF theory for centrally extended Poincaré algebra.

$$S_{\widehat{CGHS}} = \int \left[X_a(de^a + \epsilon^a{}_b \omega e^b) + X d\omega - Y(dC + \tfrac{1}{2}\epsilon_{ab} e^a e^b)\right] = \int \langle B, F \rangle$$

where $A = e^a P_a + \omega J + CZ$ and $B = X^a P_a + YJ + XZ$ and

$$[P_a, J] = \epsilon_a{}^b P_b\,, \quad [P_a, P_b] = \epsilon_{ab} Z$$

with the bilinear form $\langle P_a, P_b \rangle = \eta_{ab}$ and $\langle J, Z \rangle = -1$ and $\langle J, J \rangle = \gamma_0$. On-shell $\widehat{CGHS}$ and $CGHS$ are equivalent

$$S_{\widehat{CGHS}} = \frac{1}{2}\int \sqrt{-g}(XR - 2Y) + \int Y dC + \frac{\gamma_0}{2}\int \sqrt{-g} Y R$$

H.A., D. Grumiller, H. González, D. Vassilevich & H.A., E. Esmaeili, H. Safari

Asymptotic symmetries in 2D flat spacetime

$$R=0\,,\qquad \varepsilon^{\mu\nu}\partial_\mu C_\nu=1\,,\qquad \nabla_\mu\nabla_\nu X+g_{\mu\nu}Y=0\,,\qquad Y=\text{const.}$$

A general solution

$$X=x_1(u)r+x_0(u)\,,\qquad C=r du\,,\qquad ds^2=-2(r\,P(u)-T(u))du^2-2dudr$$

These solutions are *preserved* by the diff

$$\xi=\epsilon(u)\,\partial_u-\big(\epsilon'(u)r+\sigma'(u)\big)\,\partial_r$$

The (P,T) transform infinitesimally as;

$$\delta_\xi P=(\epsilon P)'-\epsilon''\,,\qquad \delta_\xi T=\epsilon T'+2\epsilon' T+\sigma''+\sigma' P$$

Warped Virasoro algebra

The Fourier mode generators on the circle

$$L_n = \oint T(\tau)\, e^{\frac{2\pi}{\beta} in\tau}\,, \qquad P_n = \oint P(\tau)\, e^{\frac{2\pi}{\beta} in\tau}$$

satisfy the twisted warped-Witt algebra;

$$\begin{aligned}
[L_n, L_m] &= (n-m)L_{n+m} + \gamma_0 n^3 \delta_{n+m,0}\,, \\
[L_n, P_m] &= -mP_{n+m} - i\kappa\, n^2 \delta_{n+m,0}\,, \\
[P_n, P_m] &= 0\,.
\end{aligned}$$

The $\widehat{CGHS}$ model with appropriate boundary conditions is a Warped-Schwarzian theory.

Boundary action

We derive the boundary action in the BF formulation.

- Translating boundary conditions in the BF-gauge theory up to a gauge transformation

$$
\begin{aligned}
A_r &= 0 \\
A_\tau &= P_- + T(\tau)P_+ + P(\tau)J \\
B &= [x_0' + Tx_1]P_+ + [x_1' + Px_1]J + x_1P_- + x_0Z
\end{aligned}
$$

- We aim to cancel the boundary term which remains after variation of a boundary action. The variation of the BF-action is (on-shell)

$$
\delta I_{BF} = -\oint \langle B, \delta A\rangle = -\oint (x_1(\tau)\delta T(\tau) - x_0(\tau)\delta P(\tau))
$$

Boundary action

$$\delta I_{BF} = -\oint \langle B, \delta A \rangle = -\oint (x_1(\tau) T(\tau) - x_0(\tau) P(\tau))$$

Using the field equations can write this boundary term in terms of linear and bilinear Casimirs

$$\oint \left(\delta \frac{C_1}{x_1} + C_1 \delta \frac{1}{x_1} - C_0 \delta \frac{x_0}{x_1} + \delta x_0' - [x_0 \delta \ln x_x]' \right)$$

where

$$C_0 = Y = x_1' + P x_1 \,, \qquad C_1 = \frac{1}{2} \langle B, B \rangle = x_0 C_0 - (x_0' + T x_1) x_1$$

Boundary action

$$\oint \Big[(\delta \frac{C_1}{x_1} + C_1 \delta \frac{1}{x_1} - C_0 \delta \frac{x_0}{x_1} + \delta x_0' - [x_0 \delta \ln x_x]') \\ + \tfrac{1}{2}\gamma_0 \big(\delta \frac{C_0^2}{x_1} + C_0^2 \delta \frac{1}{x_1} - 2C_0 [\delta \ln x_1]'\big) \Big]$$

This boundary term is integrable provided that the following zero modes are fixed

$$f = \oint \frac{1}{x_1} \qquad\qquad g = \oint \frac{x_0}{x_1}$$

These (quasi)-periodic functions should have fixed periodicity. Plugging in we have

$$I \to I + \partial I\,, \qquad \partial I = -\kappa \oint \frac{1}{x_1}\left(C_1 + \tfrac{1}{2}\gamma_0 C_0^2\right)\,.$$

It reproduces the Warped-Schwarzian action at level zero.

$$I_{WSchw} = \kappa \oint \left(\mathcal{T}_0 h'^2 + \gamma_0\, S(h) - g'\,(i\mathcal{P}_0 h' + \frac{h''}{h'}) \right)$$

We can evaluate the on-shell action

$$I_{\mathrm{E}}\big|_{\text{on-shell}} = \kappa\beta(\mathcal{T}_0 + \tfrac{1}{2}\gamma_0 \mathcal{P}_0^2) - i\kappa\mathcal{P}_0 \oint g'$$

The entropy is obtained from the free energy $F = -T\ln Z$,

$$S = -\frac{\partial F}{\partial T} = -I_{\mathrm{E}} - T\frac{\partial I_{\mathrm{E}}}{\partial T}$$

Holonomy condition and entropy

On the disk the holonomy of $\mathcal{A}_\tau$ along the thermal cycle should belong to the center of the group

$$\text{Hol}(\mathcal{A}_\tau) = \exp\Big[\oint \mathcal{A}_\tau\Big] \in Z(G)$$

which fixes one of the charges in terms of the temperature

$$\mathcal{P}_0 = \frac{2\pi}{\beta} = Y$$

The entropy is

$$S = 2\pi\kappa\Big(-\frac{\mathcal{T}_0}{\mathcal{P}_0} + x_0 - \gamma_0\mathcal{P}_0\Big) = 2\pi\kappa\big(X_{\text{H}} - \gamma_0 Y\big)$$

Thermodynamics

Specific heat is is finite and linear in T;

$$C = T\frac{\partial S}{\partial T} = -(2\pi)^2\kappa\gamma_0 T$$

Extensions

Spin-2 extension to the bulk (in flat spacetime) — vector-extension on the circle

$$\mathrm{Diff}(S^1) \ltimes C^\infty(S^1) \ltimes \mathrm{Vec}(S^1)$$

The geometric action turns out to be;

$$I^{\mathsf{E}}_{(2)} = -\kappa \oint \Big(s_0 h'^2 + \gamma_0 S(h) + g' \big(i\mathcal{P}_0 h' - \frac{h''}{h'} \big) + \frac{w'}{h'} \big(t_0 h'^2 + \gamma_1 S(h) \big) + \tfrac{1}{2} \big[\big(\frac{w'}{h'} \big)' \big]^2 \Big)$$

The orbit is represented by five constants $(s_0, t_0, \mathcal{P}_0, \gamma_0, \gamma_1)$. *H.A., E. Esmaeili, H. Safari 2012.15807*

Extended algebra

$$[L_n, L_m] = (n-m)L_{n+m} + \gamma_0 n^3 \delta_{n+m,0}$$
$$[L_n, T_m] = (n-m)T_{n+m} + \gamma_1 n^3 \delta_{n+m,0}$$
$$[L_n, P_m] = -mP_{n+m} + i\kappa\, n^2 \delta_{m+n,0}$$
$$[T_n, T_m] = (n-m)M_{n+m} + \frac{c}{12} n^3 \delta_{n+m,0}$$
$$[T_n, P_m] = 0\,, \qquad [P_m, P_n] = 0$$

with

$$M_n = \sum_q P_{n-q} P_q + 2i\kappa\, n P_n\,, \qquad \text{and} \qquad \kappa^2 = \frac{c}{24}$$

Summary and remarks

- Holographic aspects of dilaton-gravity in flat spacetime.
- Spin-1 and s[in-2 extension in the bulk corresponds to extension of the Schwarzian action.
- BF-theory formulation very useful.
- What could be the SYK-like model?
- We considered the BH solutions how about Generalized black holes?
- Wormholes and their partition function? Ensemble of theories?
- Relation to random matrix model?

متشکرم