RF acceleration

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Charged particle acceleration

$$F = qE + q[v \times B]$$

$$\Delta E_k = \int F ds = \int (qE + q[v \times B])\, ds$$

$$ds = vdt$$

$$\Delta E_k = \int qE.vdt + \underbrace{\int q[v \times B].vdt}_{=0}$$

The work done by the magnetic field is zero!

Charged particle acceleration

- We can only use electric field for charged particle acceleration.

- To change a charged particle trajectory, both electric and magnetic fields can be used. Although…

$$F = qE + q[v \times B]$$

$$E \approx v\,B\,sin\phi$$

- A magnetic field of 1 T is equal to an electric field of 300’000’000 V/m!

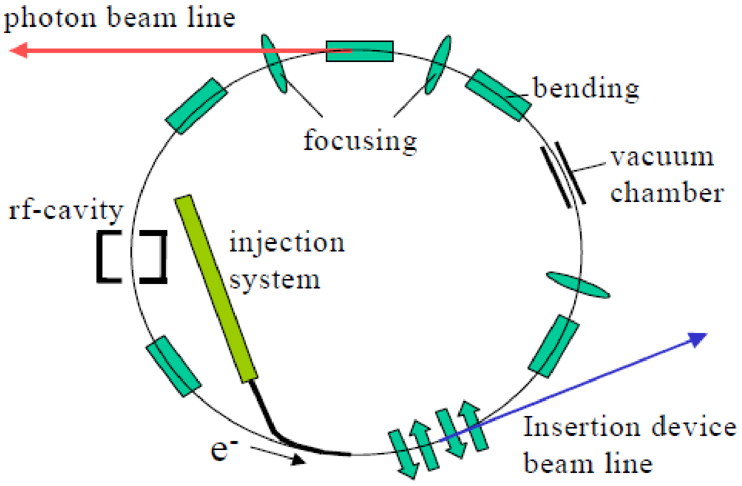
Radio frequency fields: History

- When the electrostatic accelerators reached their limit in restraining high voltages, the first RF linear accelerator was proposed and experimented by *Rolf Wideröe* in 1928.

- It was not under much study until after the developments of microwave technology during World War II, stimulated by radar programs

Necessity of Radio Frequency system

- Lorentz Force and Energy gain:



$$F = qE$$

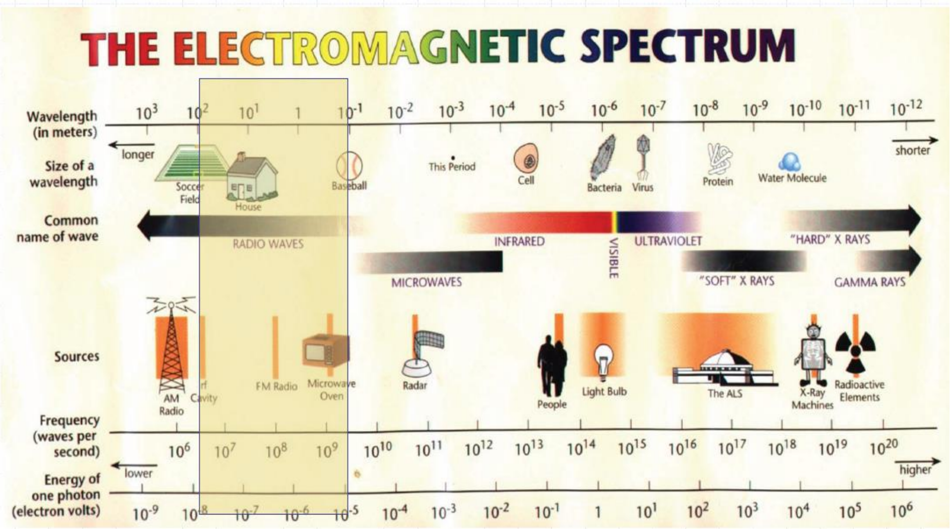
$$\Delta E_k = \int F ds = \int qE\, ds$$

  - In circular machines:

    => DC acceleration is impossible, since $\oint \vec{E} \cdot d\vec{s} = 0$ ➡ Time-varying Field

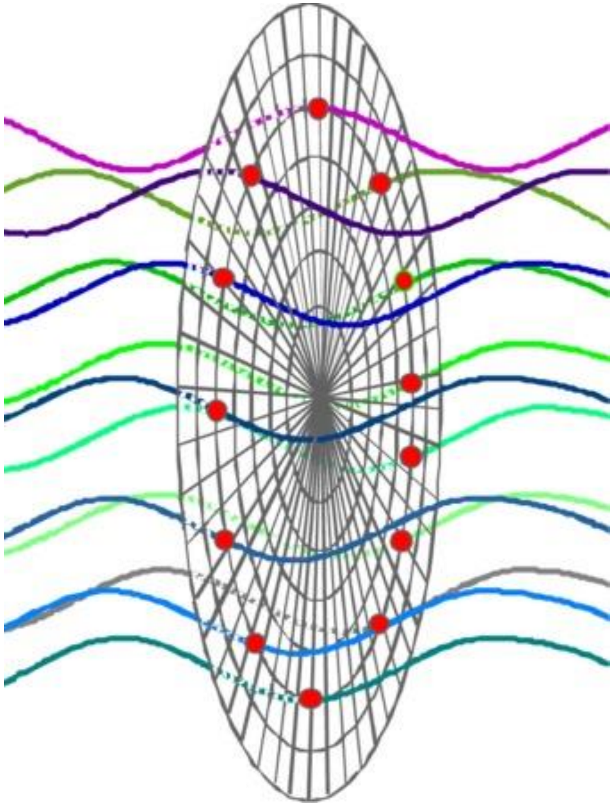
  - In non-circular machines: DC acceleration is possible but limited by break down

Which frequency?



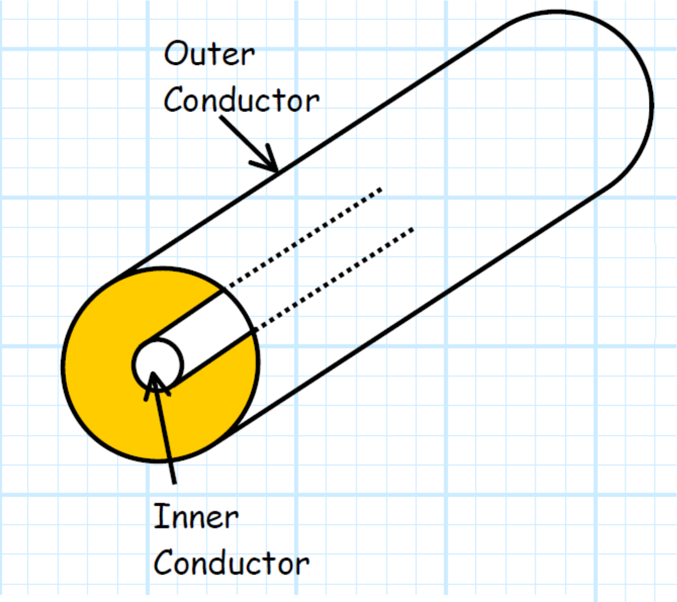
  - Structure size, power source availability, etc. ➡ Radio Frequency Field
- How to apply the field to the electrons?
  - Passing the electrons through a field resonator which is called cavity

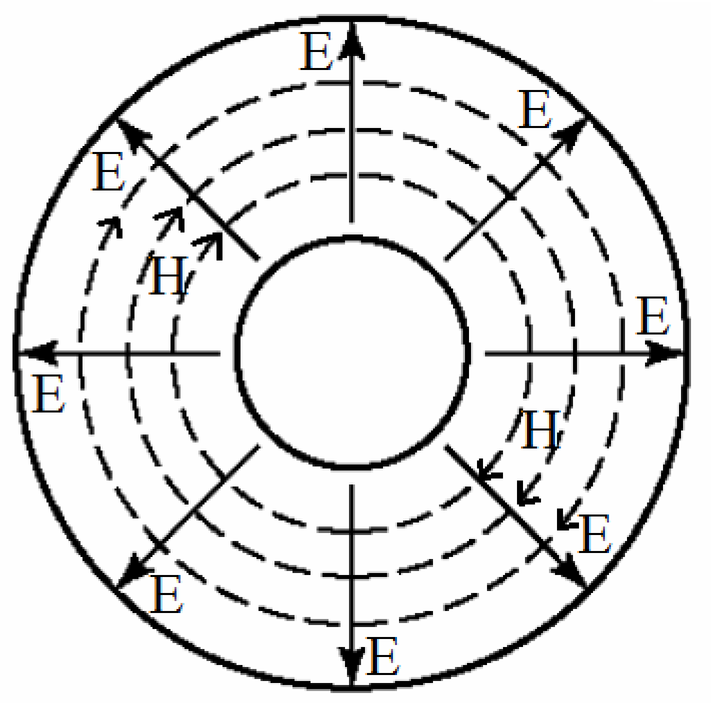
Transverse plane



TEM : Transverse Electric Magnetic

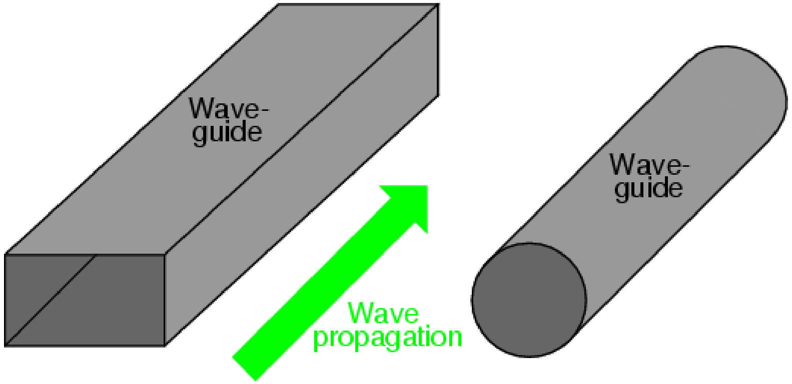
- A good example : Coaxial transmission line

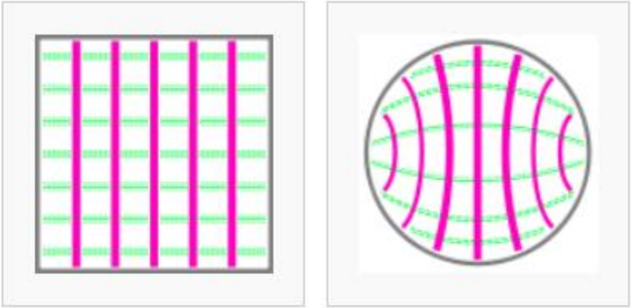


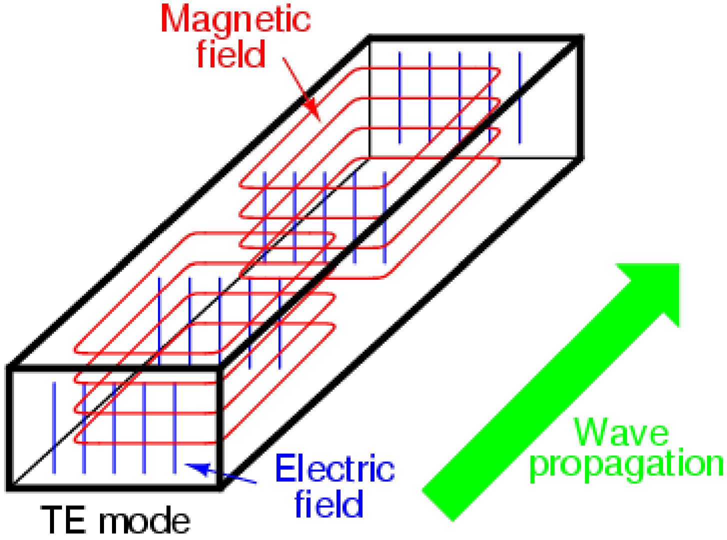


TE : Transverse Electric

- A good example : Waveguide

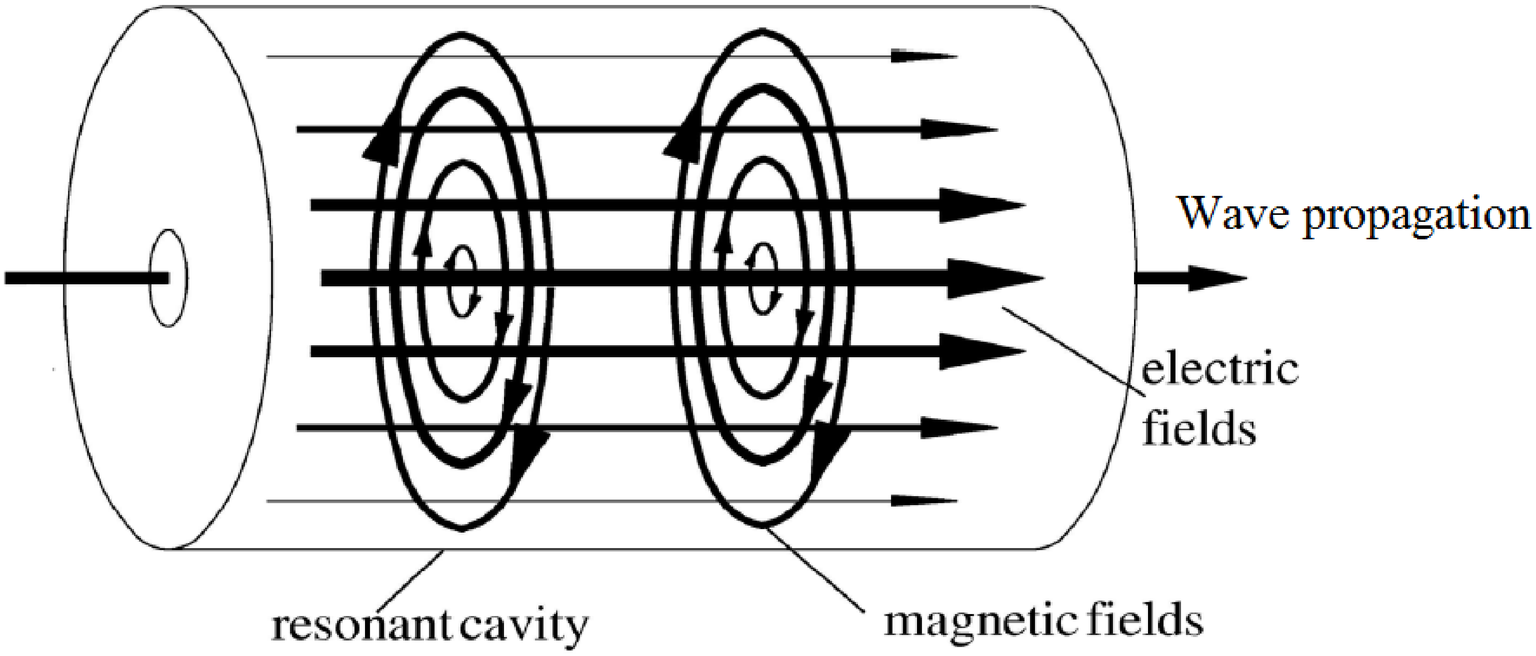






TM : Transverse Magnetic

- A good example : Resonant cavity



Order of modes

TM_{01}

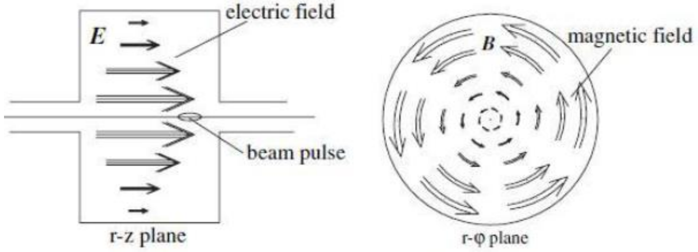


Fig. 19.2. Longitudinal parasitic mode in a pill box cavity

Most suitable
for
particle acceleration

TM_{11}

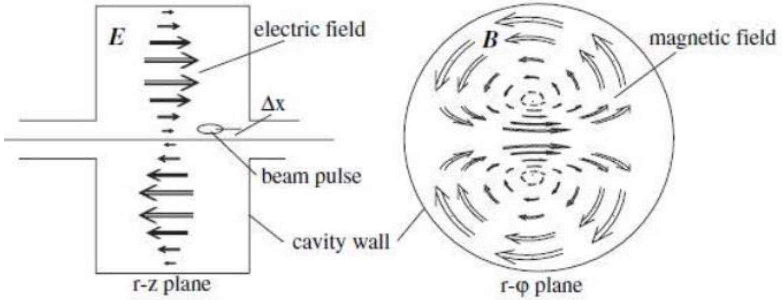


Fig. 19.3. Transverse parasitic mode in a pill box cavity

Band frequencies, band names

- Older band names:

Frequency range	Code name	description
300 Hz~3 kHz	ELF	Extra Low Frequency
3 kHz~30 kHz	VLF	Very Low Frequency
30 kHz~300 kHz	LF	Low Frequency
300 kHz~3 MHz	MF	Medium Frequency
3 MHz~30 MHz	HF	High Frequency
30 MHz~300 MHz	VHF	Very High Frequency
300 MHz~3 GHz	UHF	Ultra High Frequency
3 GHz~30 GHz	SHF	Super High Frequency
30 GHz~300 GHz	EHF	Extra High Frequency

Band frequencies, band names

- New IEEE standards:

Frequency range [GHz]	Band name
1~2	L
2~4	S
4~8	C
8~12.5	X
12.5~18	Ku
18~26.5	K
26.5~40	Ka

Phase velocity, group velocity

- The **phase velocity** of a wave is the rate at which the phase of the wave propagates in space. This is the speed at which the phase of any one frequency component of the wave travels. For such a component, any given phase of the wave (for example, the crest) will appear to travel at the phase velocity. In terms of the wavelength λ and period T, or equivalently, in terms of the wave's angular frequency ω, and angular wave number k (usually expressed in radians per meter)

$$v_\mathrm{p} = \frac{\lambda}{T}.$$

$$v_\mathrm{p} = \frac{\omega}{k}.$$

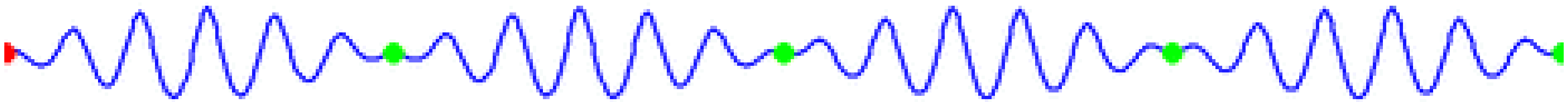
Phase velocity, group velocity

- The group velocity of a wave is the velocity with which the overall shape of the wave's amplitudes (known as the modulation or envelope of the wave) propagates through space.
- The group velocity vg is defined by the equation:

$$v_g \equiv \frac{\partial \omega}{\partial k}$$

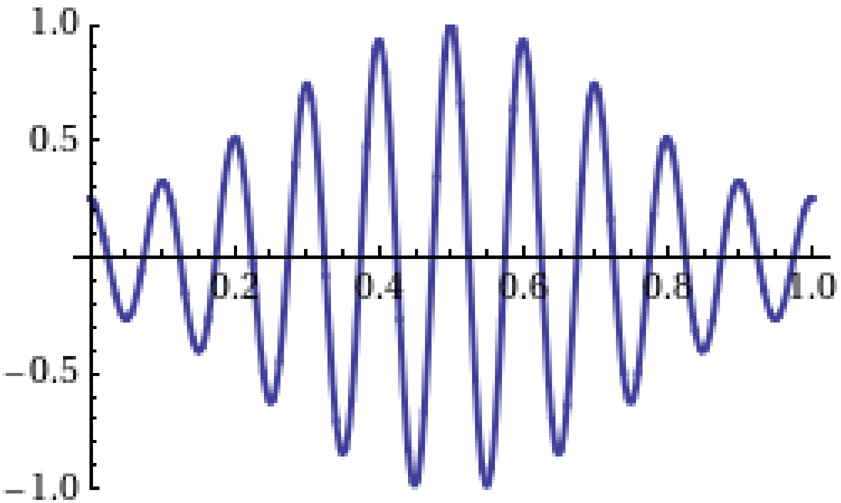
- where:
- ω is the wave's angular frequency and k is the angular wave number (usually expressed in radians per meter).

Phase velocity, group velocity



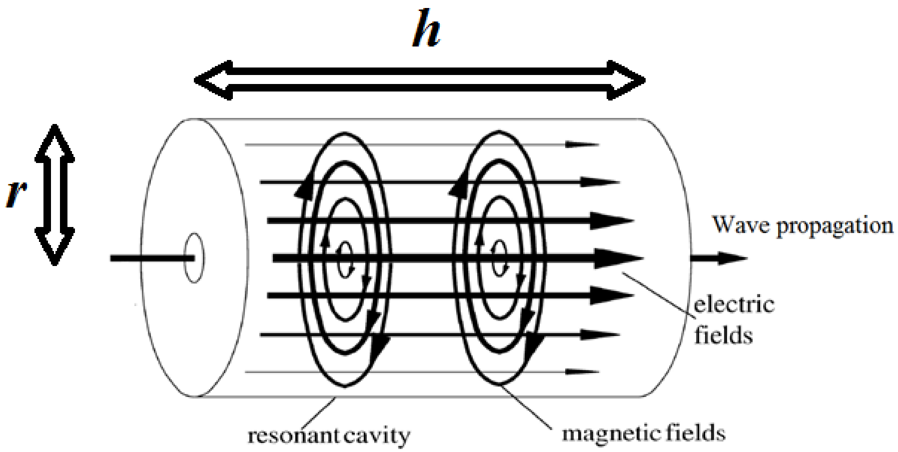
- The red dot moves with the phase velocity, and the green dots propagate with the group velocity.

Phase velocity, group velocity



- A wave with the group velocity and phase velocity going in different directions. (The group velocity is positive and the phase velocity is negative.)

RF cavity as an accelerating structure



- General cavity parameters for a given frequency can be derived by solving the wave equation (numerical calculations, finite element analysis).
- For design purposes, we follow a simpler way to derive the fundamental field for a pillbox cavity.

Pill box cavity

- Electrical field is zero at radius:

$$r = \frac{2.405}{k} = \frac{2.405}{2\pi}\lambda_{RF}$$

- And this is the location for the wall to make a pill box cavity (TM01 mode only, for higher order modes the order of Bessel's function will increase)

- Cavity length (h) is determined by RF phase velocity (i.e. particle velocity)

RF voltage

- The definition of voltage:

$$V = \int E dl$$

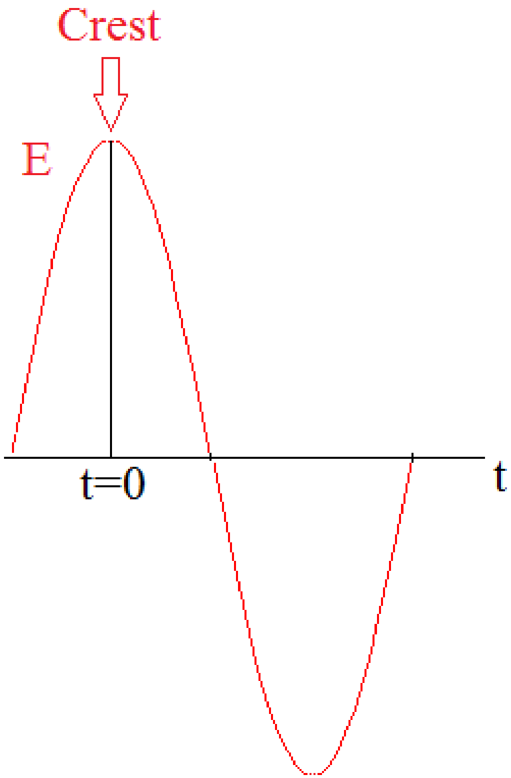
- RF voltage :

$$V_{RF} = \int_{-d/2}^{d/2} E(z,0) dz \quad (E(z,t)\ at\ t = 0)$$

$$V_{RF} = E_0 d\ ,$$

$$E_0 = \frac{1}{d} \int_{-d/2}^{d/2} E(z,0) dz : Average\ accelerating\ gradient$$

Field crest, time and phase origin



Energy gain in cavity

$$\Delta E_{kin} = qE_0 \int_{-d/2}^{d/2} \cos\left(\omega \frac{z}{v} + \delta\right) dz$$

- For simplicity we assume $\delta=0$

$$\Delta E_{kin} = qV_{RF} \frac{\sin \frac{\omega d}{2v}}{\frac{\omega d}{2v}} = qV_{RF} T$$

Transit Time Factor

- We introduce transit time factor as:

$$T = \frac{\sin \frac{\omega d}{2v}}{\frac{\omega d}{2v}}$$

Power and efficiency

- Magnetic fields generate current on cavity walls
- These currents result in power losses due to limited conductivity of walls
- The well-known quality factor of a resonator is defined in terms of the average power loss P and electromagnetic stored energy U as:

$$Q = \frac{\omega U}{P}$$

- We also define an impedance for the cavity walls, called shunt impedance:

$$r_s = \frac{V_{RF}^{\;2}}{P}$$

Power and efficiency

- It is more convenient to define the shunt impedance as impedance per unit length of cavity:

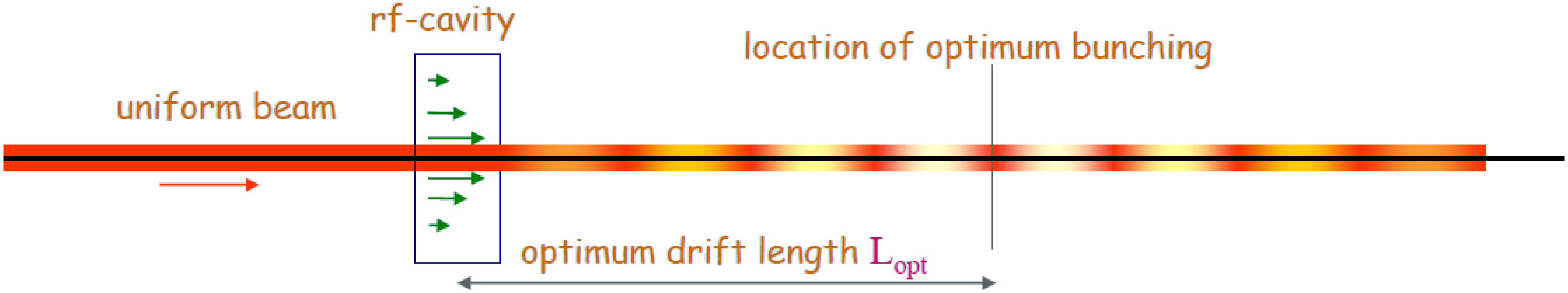
$$r_s = \frac{{V_{RF}}^2}{Pd}$$

- It is also common to include the transit time factor in shunt impedance and define effective shunt impedance:

$$r_{s,eff} = \frac{\left(V_{RF}T\right)^2}{Pd} = r_s T^2$$

Bunching

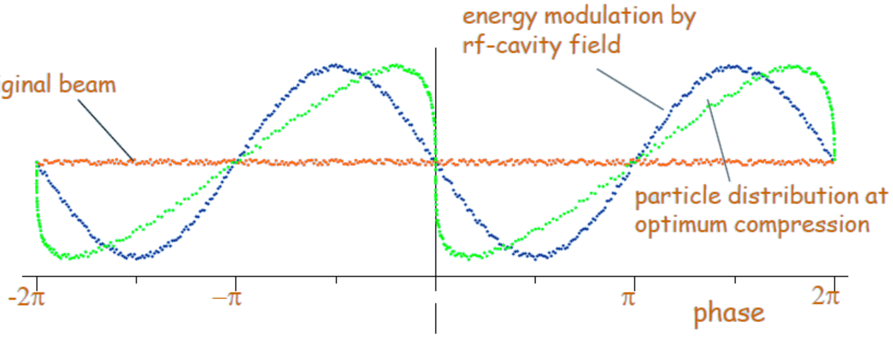
- Based on velocity modulation and works only for non-relativistic beam:



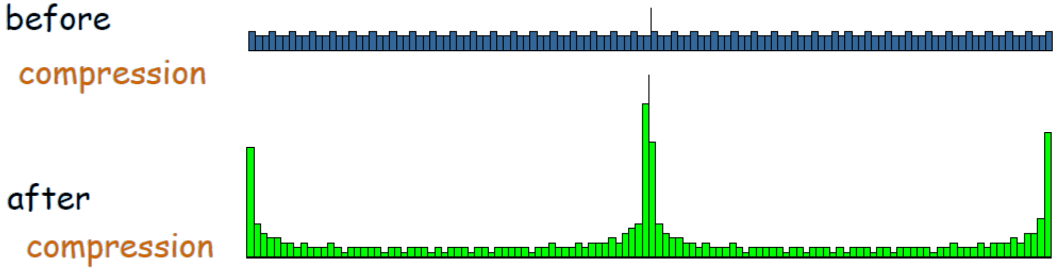
- Energy modulation:

$$\Delta E_{kin} = qTV_{RF} \sin\varphi = mc^2 \beta\gamma^3 \Delta\beta$$

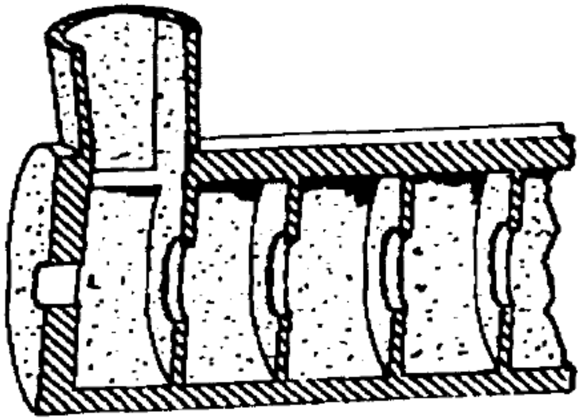
Bunching

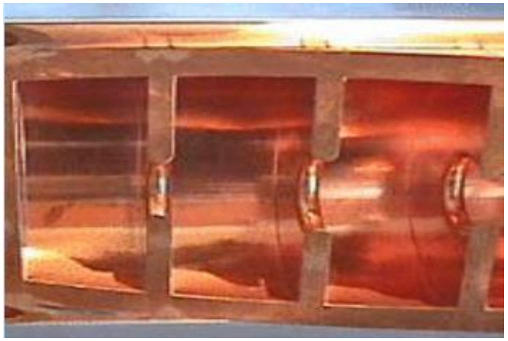


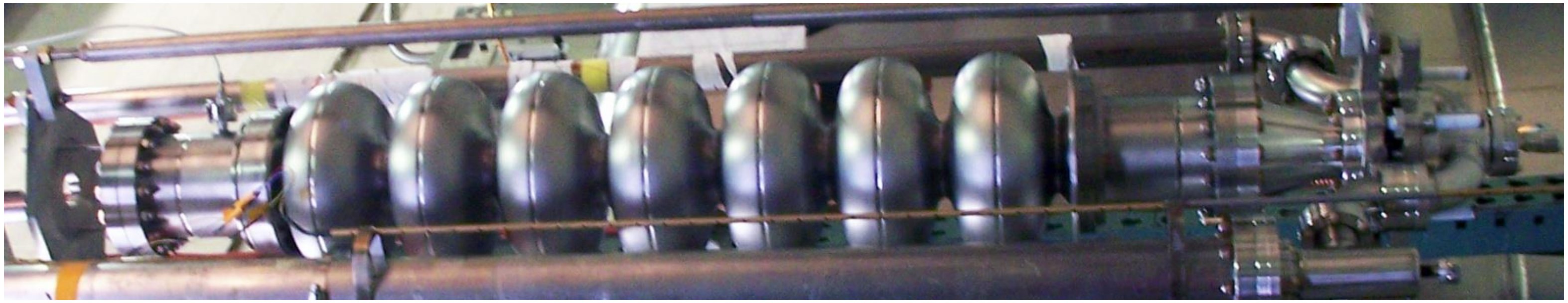
histograms:



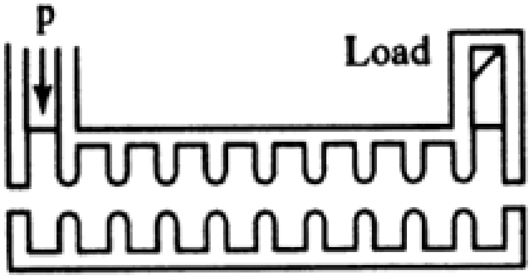
Multi-cell cavities, Linacs



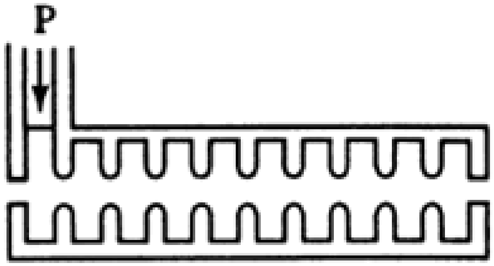




Multi-cell cavity types



Travelling wave



Standing wave

Thank you for your attention