Stability of Dilatonic Black hole

Navid Abbasi
Sharif University of Technology

Introduction & Motivation

1. Black hole in a thermal bath:

Equilibrium condition: $T = T_H$

Schwarschild black hole can not be stable unless by putting it in a box. Maximizing the generalized entropy for a fixed value of total energy ,we can find the volume of box.

$$S = S_H + S_m = 4\pi M^2 + \frac{4}{3}\sigma V \theta_{rad}^3$$

$$E = M + \sigma V \theta_{rad}^4$$

Reissner-Nordstorm Black hole can be stable in special cases.

Lousto (1996)

2. Dilatonic Black hole:

Low energy effective action of string theory in 4-dim:

$$S = \frac{1}{16\pi}\int d^4 x\sqrt{-g}(R - 2\partial_\mu\phi\partial^\mu\phi - e^{-2\alpha\phi}F^2)$$

Static black hole solution:

Gibbons, Maeda (1988)

Garfinkle, Horowitz, Strominger (1991)

$$ds^2 = -\frac{(r-r_+)(r-r_-)}{R^2}dt^2 + \frac{R^2}{(r-r_+)(r-r_-)}dr^2 + R^2 d\Omega_2{}^2$$

$$r_\pm = \frac{1+\alpha^2}{1\pm\alpha^2}(M \pm \sqrt{M^2-(1-\alpha^2)Q^2})$$

$$T = \frac{1}{4\pi r_+}(1-\frac{r_-}{r_+})^{\frac{1-\alpha^2}{1+\alpha^2}}$$

3. Similarity between R-N and Dilatonic black hole:

Wilczeck, Hulzhay (1992)

Although these two type of black holes are very difference in geometry structure and thermo dynamical properties, both of them reach to **extremality** after **infinite time**.

$r_+ - r_-$

Dilatonic black hole

$0 < \alpha < 1$: $T \to 0$

R-N black hole

$T \to 0$

Is there any similarity between their stability?

Stability of dilatonic black hole in thermal bath

The stability criterion:

"The existence of minimum in Helmholtz free energy"

$$F = U - TS \quad \longrightarrow \quad F = M - TS$$

From the first law of thermodynamics:

$$dM = TdS + \Phi_H dQ + \Omega_{\mathrm{H}} dJ$$

we can derive a mass formula for charged Kerr black hole:

$$M = 2TS - Q\Phi_H - J\Omega_H$$

Smarr (1973)

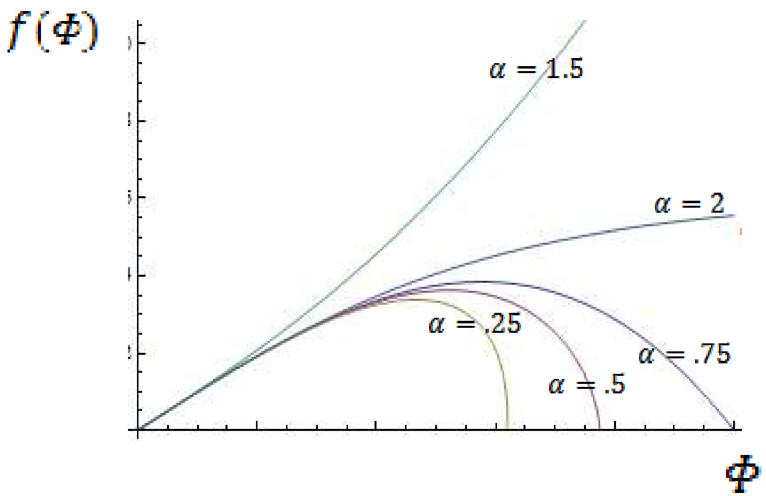
The general form of free energy function for static black hole:

$$F = TS + Q\Phi_H$$

Free energy of dilatonic black hole:

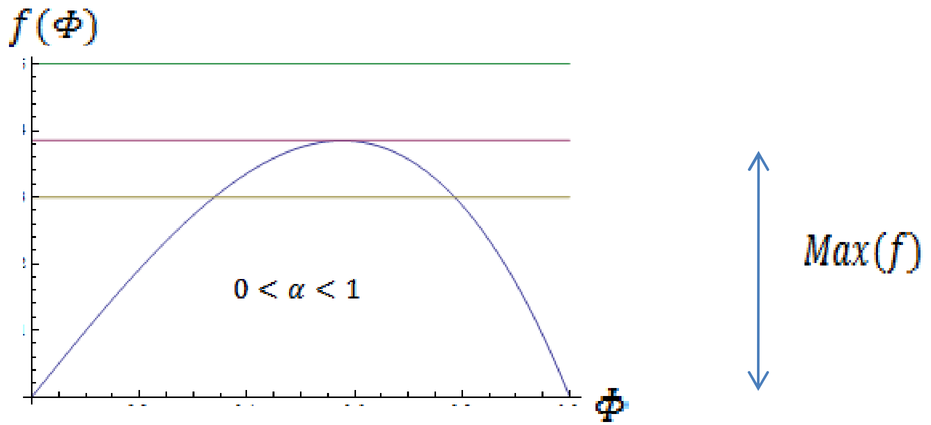
$$F = \frac{1}{16\pi T}\left(1 - \frac{1+\alpha^2}{1-\alpha^2}\Phi^2\right)^{\frac{2}{1+\alpha^2}} + Q\Phi$$

$$\left(\frac{\delta F}{\delta \Phi}\right)_Q = 0 \quad \longrightarrow \quad 4\pi T Q = \Phi\left(1 - \frac{1+\alpha^2}{1-\alpha^2}\Phi^2\right)^{\frac{1-\alpha^2}{1+\alpha^2}}$$



1 . $0 < \alpha < 1$ case:

$$4\pi T Q = f(\Phi)$$



There is a critical temperature:

$$4\pi T_c Q = Max(f)$$

$T < T_c$: Two stationary points

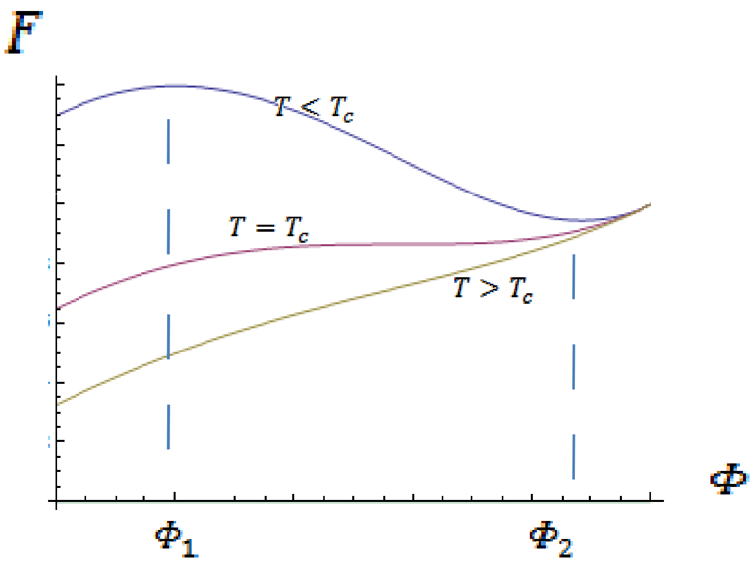
$T = T_c$: One stationary point

$T > T_c$: No stationary point

Study of stationary points:

By increasing the temperature from $T < T_c$ to $T > T_c$, a phase transition will happen.

$0 < \alpha < 1$



$$\Phi = \frac{Q}{r_+}$$

$$\Phi_1 < \Phi_2 \Rightarrow M_1 > M_2$$

What kind of phase transition?

$$S = -(\frac{\partial F}{\partial T})_Q$$ continuous

$$C = -T(\frac{\partial^2 F}{\partial T^2})_Q$$ discontinuous

⇓

second order phase transition

Critical exponents:

$$T = T_c + (\frac{\partial T}{\partial S})_Q (S - S_c) + \frac{1}{2}(\frac{\partial^2 T}{\partial S^2})_Q (S - S_c)^2 + \cdots \qquad \Rightarrow \qquad S - S_c \approx (T - T_c)^{\frac{1}{2}}$$

$$C = T_c \frac{\Delta S}{\Delta T} \approx (T_c - T)^{-\frac{1}{2}}$$

Similarly:

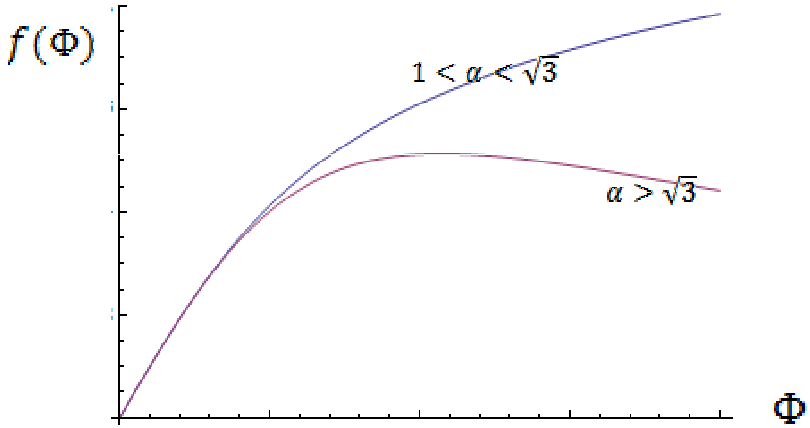
$$\Phi \approx (T_c - T)^{\frac{1}{2}}$$

2. $\alpha > 1$ case:

There is no stable state in this case.

So, this case is similar to Schwarzschild black hole.

Although, $f(\Phi)$ has two different behavior:



$1 < \alpha < \sqrt{3}$: *one stationary point, no minimum point*

$\alpha > \sqrt{3}$: *at most stationary point, no minimum point*

Summary & Conclusion

Stability of dilatonic black hole in its thermal bath depends on the value of α:

$\alpha \geq 1$: Shawrschild like

$\alpha < 1$: R-N like

The occurrence of phase transition is independent of the α.

Critical exponents are also.

Some another related questions:

1. Stability of dilatonic black hole in grand canonical ensemble.
2. The effect of quantum correction on the phase transition.

Thank you