Higher Spins

And Holography

RAJESH GOPAKUMAR

Harish-Chandra Research Institute

Allahabad, India

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Outline of the talk

- Introduction
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- A Little About Higher Spin Theories
  - The Vasiliev Framework
- Free Yang-Mills Theory
  - Limit of Tensionless Strings on AdS
- Non-SUSY AdS-CFT
  - $O(N)$ model in 3d, $\mathcal{W}_{\mathcal{N}}$ models in 2d
- Toy Models of Stringy Gravity
  - Singularities, Black hole Thermodynamics
- Questions

1 Introduction

Massless Higher Spin Theories

- Vasiliev has developed a classical theory of higher spin fields.
- An infinite tower (typically) of massless symmetric tensors of rank s.
- Exists only in AdS (or dS) space times and not in flat space.
- Highly non-linear and nonlocal theory with an infinite dimensional gauge invariance. Strongly constrains the structure of the theory.
- A vast extension of the diffeomorphism invariance of conventional gravity.
- But does not reduce to Einstein equations.
- Why are these theories interesting and attracting so much attention?
- AdS-CFT

Higher Spin Theories and AdS/CFT

- Theories of gravity on AdS are dual to CFTs on the boundary
- Classical limit $G_N \to 0 \leftrightarrow N \to \infty$.
- Thus expect Vasiliev type theories to be dual to the $N \to \infty$ limit of some CFTs.
- Conventional Einstein theories are dual to large N CFTs with $\lambda \to \infty$.
- Most bulk calculations in AdS/CFT are in this regime - ultra strong coupling in the CFT.
- What if we are interested in the CFT with $\lambda \sim \mathcal{O}(1)$?
- We need to quantize string theory on AdS with $\frac{R_{AdS}}{\ell_s} \sim \lambda^{\frac{1}{4}}$.
- Currently outside analytic control even for SUSY theories.
- We need a different expansion point rather than $\lambda \to \infty$.

Higher Spin Theories and AdS/CFT *(contd.)*

- This is where Vasiliev theories can play a role. A different powerful symmetry.
- Consider a free (massless) large N gauge theory $(\lambda = 0)$.
- This has a much larger set of global symmetries than generic interacting theory.
- An infinite number of conserved currents of arbitrary spin.
- $$J_{(\mu_1\ldots\mu_s)}(x) = \sum_{k=0}^{s} c_k^{(s)} \mathrm{Tr}[\partial_{(\mu_1} \ldots \partial_{\mu_k} \Phi^\dagger(x) \partial^{\mu_{k+1}} \ldots \partial_{\mu_s)} \Phi(x)] - (Traces)$$
- $\partial_\mu J_{(\mu\mu_2\ldots\mu_s)}(x) = 0$ by free equations of motion $\partial^2\Phi(x) = 0$.
- The bulk dual should have gauge fields corresponding to these symmetries.
- $$\phi_{(\alpha_1\ldots\alpha_s)} \sim \phi_{(\alpha_1\ldots\alpha_s)} + \nabla_{(\alpha_1}\xi_{\alpha_2\ldots\alpha_s)}.$$
- Exactly the (linearised) gauge invariances of the Vasiliev higher spin theories.

Higher Spin Theories and AdS/CFT *(contd.)*

- How much of this symmetry survives for $\lambda \neq 0$ gauge theories?
- Start by looking at free vector like models - have exactly the same global symmetries.
- Conjectures for *non-supersymmetric* 3d and 2d conformal field theories.
- 3d $O(N)$ vector models at free and interacting fixed points (Klebanov-Polyakov, Sezgin-Sundell). Also 3d Gross-Neveu model..
- 2d minimal model CFTs with $\mathcal{W}_N$ symmetry (Gaberdiel- R.G.). A fixed line $0 < \lambda < 1$ of CFTs in the 'tHooft limit.
- Potentially decodable cases of AdS/CFT. How does holography work?
- Can perhaps use the CFTs to gain a sharp understanding of conceptual questions in quantum gravity? (especially in non-supersymmetric situations).
- How to characterize black holes and their thermodynamics in stringy theories? How do they resolve singularities?

2 A Little About Higher Spin Theories

A Little About Higher Spin Theories

- Start with non-interacting theory of massless higher spin fields $\phi_{(\alpha_1 \ldots \alpha_s)}$ (Fronsdal).

$$\phi^{\beta\gamma}_{\beta\gamma\alpha_1\ldots\alpha_{s-4}} = 0 \qquad \phi_{\alpha_1\ldots\alpha_s} \sim \phi_{\alpha_1\ldots\alpha_s} + \nabla_{\alpha_1}\xi_{\alpha_2\ldots\alpha_s}$$

- Gauge parameter is traceless $\xi^{\alpha}_{\alpha\alpha_3\ldots\alpha_{s-1}} = 0$
- Linearised equation of motion given by

$$\nabla^2_{(s)}\phi_{\alpha_1\ldots\alpha_s} - \nabla_{\alpha_1}\nabla^{\lambda}\phi_{\lambda\alpha_2\ldots\alpha_s} + \nabla_{\alpha_1}\nabla_{\alpha_2}\phi^{\lambda}_{\lambda\alpha_3\ldots\alpha_s} - \frac{a_{s,D}}{R^2_{AdS}}\phi_{\alpha_1\ldots\alpha_s} = 0.$$

- Generalisation of Maxwell and linearised (about AdS) Einstein equations.
- Challenge is to generalize this to interacting theory preserving gauge invariance.
- Move to a frame like formulation: generalization of vielbein and connection

$$e^a_\alpha, \omega^{ab}_\alpha \rightarrow e^{a_1\ldots a_{s-1}}_\alpha, \omega^{a_1\ldots a_{s-1},b}_\alpha.$$

- Enlarged gauge invariance - generalized local lorentz rotations $\rightarrow$ more gauge fields.

A Little About Higher Spin Theories *(contd.)*

- Linearised gauge transformations are:

$$\delta_\xi e_\alpha^{a_1 \ldots a_{s-1}} = \partial_\alpha \xi^{a_1 \ldots a_{s-1}}$$

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$$\delta_\Lambda e_\alpha^{a_1 \ldots a_{s-1}} = \bar{e}_{\alpha,b} \Lambda^{a_1 \ldots a_{s-1},b}; \quad \delta_\Lambda \omega_\alpha^{a_1 \ldots a_{s-1},b} = \partial_\alpha \Lambda^{a_1 \ldots a_{s-1},b}$$

- Leads to a whole set of extra fields $\omega_\alpha^{a_1 \ldots a_{s-1},b_1 \ldots b_k}$. $(k \leq s-1)$ to get rid of all the extra degrees of freedom.

- All these can be conveniently packaged in terms of (grassmann even) spinor oscillators for $D = 3, 4, 5$ (somewhat like in a superfield).

- E.G. in $D = 4$, the components of a spin s field will be multiplied by $2(s-1)$ oscillators $Y_k, Y_{\dot{k}}$.

- Combine all spins s into one generating function field $W_\alpha(X|Y)$.

A Little About Higher Spin Theories *(contd.)*

✦

$$W(X|Y) = \sum_s \sum_{n,m;n+m=2(s-1)} W^{(s)}(X)_{k_1 \ldots k_n;\dot{p}_1 \ldots \dot{p}_m} Y^{k_1} \ldots Y^{k_n} Y^{\dot{p}_1} \ldots Y^{\dot{p}_m}$$

✦ The generalized gauge symmetry acts (linearly) as

$$\delta_\epsilon W = dW + \epsilon \star W - W \star \epsilon$$

✦ $\epsilon = \epsilon(X|Y)$ and the star product acts on the oscillators via a Moyal like product.

✦ But also need another field (scalar) $B(X|Y)$ whose lowest component is a scalar in AdS.

✦ Derivatives of the scalar packaged into other components (unfolded formulation).

A Little About Higher Spin Theories *(contd.)*

- This is convenient to express equations of motions as *constraints* on the infinite number of fields.

- $$\phi_\alpha = \partial_\alpha \phi, \;\; \phi_{\alpha\beta} = \partial_\beta \phi_\alpha, \;\; \phi^\alpha_\alpha = 0 \Rightarrow \partial^2 \phi = 0$$

- Similarly, generalized Weyl curvatures and their derivatives are also in the same field $B(X|Y)$.

- But to get the full consistent non-linear equations one needs to extend the internal space by another set of oscillators Z and an auxiliary field $S(X|Y,Z)$ in addition to $W(X|Y,Z), B(X|Y,Z)$.

- $$dW + W \star W = 0; \;\; dB + W \star B - B \star \tilde{W} = 0; \;\; dS + W \star S - S \star \tilde{W}.$$

  together with some constraints on S, B are the Vasiliev non-linear equations.

3 Free Yang-Mills Theory

Tensionless Strings in AdS

- Spectrum of gauge invariant states (operators) in free Yang-Mills theory well understood.

- They correspond to the spectrum of string theory on AdS with $\frac{R_{AdS}}{\ell_s} \sim \lambda^{\frac{1}{4}} \to 0$. Tensionless strings. (Sundborg, Witten, Sezgin-Sundell)

- However, (unlike flat space) this limit appears to be non-singular.

- Among the states are those corresponding to the twist two operators $(\Delta - s = 2)$

$$J_{(\mu_1 \ldots \mu_s)}(x) = \sum_{k=0}^{s} c_k^{(s)} \mathrm{Tr}[\partial_{(\mu_1} \ldots \partial_{\mu_k} \Phi^{\dagger}(x) \partial_{\mu_{k+1}} \ldots \partial_{\mu_s)} \Phi(x)] - (Traces).$$

- Thus dual tensionless string theory contains massless higher spin excitations.

- Analogue of $\alpha_{-1}^{\mu_1} \ldots \alpha_{-1}^{\mu_s} \tilde{\alpha}_{-1}^{\mu_1} \ldots \tilde{\alpha}_{-1}^{\mu_s} |p\rangle$ in flat space. Leading Regge trajectory.

Tensionless Strings in AdS *(contd.)*

- But there are many more states in the Yang-Mills theory (as well as the dual AdS string theory) than these twist two operators.

- Nevertheless, the sector of twist two operators in the free theory are closed amongst themselves under the OPE.

- This should therefore describe a closed subsector of the dynamics of the full theory.

- Dual of this subsector governed by the Vasiliev theory (a consistent truncation like to supergravity when $\lambda \gg 1$).

- The full free Yang-Mills (massive) also seems to be governed by the higher symmetry algebra underlying the Vasiliev theory. (Bianchi et.al.)

- What about going away from $\lambda = 0$? Indications that the higher spin symmetry is higgsed in the bulk. (Porrati et.al.)

4 Non-SUSY AdS-CFT in $d = 2, 3$

$O(N)$ Vector Models in $d = 3$

- Are there CFTs whose dual (at large N) is purely a Vasiliev-like theory (rather than being a subsector of a string theory)?

- Need a much smaller infinity of single particle operators compared to a gauge theory. Not a hagedorn density of states.

- Vector like models have far fewer degrees of freedom $\propto N$ rather than gauge theories $\propto N^2$.

- The single particle operators are *only* the bilinears

$$\sum_{k=0}^{s} c_k^{(s)} [\partial_{(\mu_1} \ldots \partial_{\mu_k} \phi_i(x) \partial_{\mu_{k+1}} \ldots \partial_{\mu_s)} \phi_i(x)] - (Traces).$$

- Therefore dual bulk fields are only the Vasiliev gauge fields (together with a scalar). (Klebanov-Polyakov, Sezgin-Sundell)

- In $d = 3$, $O(N)$ vector models have nontrivial quantum behavior.

$O(N)$ Vector Models in $d = 3$ *(contd.)*

- Can add to the free action $S_0 = \int d^3x \partial_\mu \phi_i(x) \partial_\mu \phi_i(x)$ an interaction ("double trace") term $S_1 = \lambda \int d^3x (\phi_i(x)\phi_i(x))^2$.

- There is a nontrivial fixed point of the RG in the infrared.

- The scalar bilinear $\phi_i(x)\phi_i(x)$ has dimension $\Delta = 2$ instead of the canonical $\Delta = 1$ at the free (UV) fixed point.

- The two CFTs are dual to the Vasiliev theory (with spins $s = 0, 2, 4 \ldots$) on AdS_4 but with the bulk scalar quantized in two inequivalent ways.

- Precisely agrees with general expectations about such RG flows in AdS/CFT.

- Non-trivial evidence from computation of three point functions in the CFT and matching with Vasiliev's cubic couplings. (Sezgin-Sundell, Giombi-Yin)

- Legendre transformation of correlation functions from free to interacting theory also reflected in the bulk. (Giombi-Yin)

$\mathcal{W}_N$ Minimal Models in $d = 2$

- In $d = 2$ QFTs can be *interacting* and yet have higher spin conserved currents.
- Thus the possibility of having nontrivial interacting CFTs dual to Vasiliev theories.
- From the bulk side, $D = 3$ is special too. Higher spin fields have no propagating d.o.f. (like 3d gravity).
- Also can truncate the infinite tower of spins to $s = 2, \ldots N$.
- A simple Chern-Simons action for the frame fields. (Blencowe, Bergshoeff-Blencowe-Stelle). Gauge group is $SL(N, \mathbb{R}) \times SL(N, \mathbb{R})$.
- Action is $S = S_{\rm CS}[A] - S_{\rm CS}[\tilde{A}]$ with level $k_{CS} = \frac{R_{AdS}}{4G_N}$.
- Brown-Henneaux type analysis of the asymptotic symmetry algebra (Henneaux-Rey, Campoleoni et.al.) shows that one gets a boundary $\mathcal{W}_N$ algebra.
- The central charge is *exactly* the same as Brown-Henneaux: $c = \bar{c} = \frac{3\ell}{2G_N}$.

$\mathcal{W}_N$ Minimal Models in $d = 2$ *(contd.)*

- What kind of CFTs are these dual to?

- Proposal (M.R. Gaberdiel and R.G.): $SU(N)$ coset WZW models in the large N 'tHooft limit.

$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}.$$

- Take k, N large keeping $0 \leq \lambda = \frac{N}{N+k} \leq 1$ fixed.

- A family of theories with central charge $c_N(\lambda) = N(1-\lambda^2)$ - vector like model.

- For any finite k, N these are the $\mathcal{W}_N$ minimal models since they have $\mathcal{W}_N$ symmetry.

- The $N = 2$ case corresponds to the usual Virasoro minimal models (Ising model series).

- A large nontrivial spectrum of scalar primaries labelled by two representations (Λ^+, Λ^-) of $SU(N)_k$ and $SU(N)_{k+1}$ respectively.

$\mathcal{W}_N$ Minimal Models in $d = 2$ *(contd.)*

- $h(0;\mathrm{f}) = \frac{(N-1)}{2N}\left(1 - \frac{N+1}{N+k+1}\right) \to \frac{1}{2}(1-\lambda) = h_-$.
- $h(\mathrm{f};0) = \frac{(N-1)}{2N}\left(1 + \frac{N+1}{N+k}\right) \to \frac{1}{2}(1+\lambda) = h_+$.
- Note that all the spin s currents are in the vacuum sector and have $h = (s, 0)$ etc.
- How is this reflected in the bulk theory?
- We need two additional complex scalars with $M^2 = -(1-\lambda^2)$.
- Quantised in opposite ways leading to CFT operators with $h_\pm = \frac{1}{2}(1 \pm \lambda)$.
- Then the CFT spectrum of states $(0;\Lambda), (\Lambda;0)$ organizes itself into those of multi-particle states of these scalars and their $\mathcal{W}$ descendants. (Gaberdiel - R.G.- Hartman - Raju)
- E.g. $h(0;\mathrm{adj}) = 1 - \frac{N}{N+k+1} \to (1-\lambda)$; $h(\mathrm{adj};0) = 1 + \frac{N}{N+k} \to (1+\lambda)$.
  Two particle states in the bulk - double trace operators in the CFT.

$\mathcal{W}_N$ Minimal Models in $d = 2$ *(contd.)*

- Thus bulk theory (at $N = \infty \Rightarrow G_N = 0$) has an infinite tower of higher spin fields together with two massive scalars $M^2 = -(1 - \lambda^2)$ quantized in opposite ways.
- Check of three point function CFT correlator with bulk calculation agrees (Chang-Yin, Ahn, Kraus et.al.).
- The higher spin symmetry of the theory is what is known as $hs[\lambda]$ and the boundary symmetry is $\mathcal{W}_\infty[\lambda]$ (Gaberdiel-Hartman).
- This symmetry is related to the large N 'tHooft limit of the $\mathcal{W}_N$ theory by a generalized level-rank duality.
- This makes the question of going away from $N = \infty$ quite novel.
- On the CFT side, if we go to the $\mathcal{W}_N$ coset then there are many extra light states (e.g. $h(\Lambda, \Lambda) \sim \frac{\lambda^2}{2N} C_2(\Lambda)$).
- They contribute to leading order in $\frac{1}{N}$ to four point functions (Papadodimas-Raju). What is the bulk interpretation?

5 Toy Models of Stringy Gravity

Toy Models of Stringy Gravity

- We know string theory sees the geometry of space-time differently from GTR.
- Many singularities are resolved - one has a microscopic description of black hole thermodynamics.
- These features have something to do with the large gauge symmetry and non-local nature of string theory. But how exactly?
- Vasiliev theories offer examples of relatively simpler descriptions in which to address these questions.
- This is specially so for the $D = 3$ case where gravity (despite having no propagating d.o.f.) is still non-trivial.
- BTZ black holes have played a central role in our understanding of the entropy of supersymmetric black holes.
- The duality between a Vasiliev theory and non-SUSY 2d CFTs is a window to explore these questions more generally.

Toy Models of Stringy Gravity *(contd.)*

- Recently a class of new black holes in the $SL(N,\mathbb{R}) \times SL(N,\mathbb{R})$ higher spin theories have been constructed (Gutperle-Kraus, Ammon et.al., Castro et.al.).
- They carry higher spin charges (in addition to M and J).
- Interestingly the notion of event horizon is now gauge dependent. A more invariant characterization is in terms of holonomies.
- A gauge invariant characterization of the first law of BH thermodynamics.
- Generalisation to $\mathcal{W}_\infty$ theories match with 2d CFT answers (Kraus-Perlmutter).
- Another class of geometries to study are conical defect singularities in the $SL(N)$ theories (Castro-R.G.-Gutperle-Raeymakers).
- These are actually non-singular in the higher spin theory because of the enlarged gauge invariance - $SL(N)$ rather than $SL(2)$.
- Criterion for smoothness: Holonomy of the $SL(N)$ connection must be trivial.

Toy Models of Stringy Gravity *(contd.)*

- i.e. holonomy along the spatial circle must be in centre of gauge group $SL(N)$ $P\exp\int A \in Z_N$
- For a *discrete* set of values of the conical deficit the configurations are actually smooth.
- Becomes a dense discretuum in the large N limit stretching all the way to the AdS vacuum!
- There also exist more general smooth conical surplus geometries.
- Remarkably, a certain analytic continuation of the parameters of the coset model relates this discrete spectrum to those of the light primaries (Λ, Λ).
- Exact matching of the spectrum for any finite N - not just in the large N limit.
- Gives a candidate bulk dual for the light states.
- Suggests that Vasiliev theory perhaps needs to be augmented at the quantum level with extra solitonic states.

6 Questions

Questions

☆ Understand better the role of the higher spin algebra in Yang-Mills theory for $\lambda \neq 0$.

☆ How exactly does the higgsing of the gauge invariance in the bulk take place? What constraints does it place on the theory?

☆ Develop systematic methods of expansion about $\lambda = 0$ in the bulk. What does it teach us about the string theory on AdS_5?

☆ What about $\frac{1}{N}$ corrections? How do we quantize Vasiliev's theory? How are all the vanishing $\frac{1}{N}$ corrections in a free theory seen in the bulk?

☆ Does the Vasiliev theory have to be embedded in a string theory? How do vector model dualities fit into the general class of AdS/CFT examples?

☆ Are there new qualitative features in non-SUSY AdS/CFT examples? What can vector dualities teach us about non-SUSY gauge theories in 4d?

☆ Generalizations to other 2d cosets (Ahn, Gaberdiel-Vollenweider). Other RCFTs (Kiritsis).

Questions *(contd.)*

- ☆ Can we generalize the dualities to massive theories? A large space of 2d integrable QFTs related by RG flows. (In Progress)
- ☆ Applications to real life systems? (Z_N Ising model/Parafermions, FQHE ...)
- ☆ Study other classical solutions of higher spin e.o.m. Exotic black holes? Scalar Hair?
- ☆ Can we understand microstates in non-SUSY Black Holes? (role of integrability?)
- ☆ More general characterization of BH thermodynamics. Analogue of Wald's formula?
- ☆ What kind of singularities can be resolved in higher spin theories? What role do such solutions play?
- ☆ Can we prove these vector model dualities? (Douglas-Mazzucato-Razamat) Might be the simplest examples of holography - Minimal holographic models.

The end