5D SUSY Black Ring/CFTs With Higher Derivative Terms

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■ *Plan*

- ♣ 5D SUGRA + H. D. C.
- ♣ Black Ring Solution & Macroscopic Entropy
- ♣ c-extremization Approach
- ♣ Kerr/CFT Approach
- ♣ Summary

♣ *5D SUGRA + H. D. C.*

Compactification of M-theory on a CY_3 results in $\mathcal{N}=2$ SUGRA in 5D. The bosonic action up to 4th order in superconformal formalism is

$$I = \frac{1}{16\pi G_5}\int d^5x\sqrt{|g|}\,(\mathcal{L}_0+\mathcal{L}_1)\,,$$

where

$$\begin{aligned}\mathcal{L}_0 &= \partial_a\mathcal{A}^i_\alpha\partial^a\mathcal{A}^\alpha_i + (2\nu+\mathcal{A}^2)\frac{D}{4} + (2\nu-3\mathcal{A}^2)\frac{R}{8} + (6\nu-\mathcal{A}^2)\frac{v^2}{2} + 2\nu_I F^I_{ab}v^{ab}\\ &+ \frac{1}{4}\nu_{IJ}(F^I_{ab}F^{J\,ab} + 2\partial_a X^I\partial^a X^J) + \frac{e^{-1}}{24}C_{IJK}\epsilon^{abcde}A^I_a F^J_{bc}F^K_{de},\end{aligned}$$

$$\begin{aligned}\mathcal{L}_1 &= \frac{c_{2I}}{24}\Big(\frac{1}{16e}\epsilon_{abcde}A^{Ia}R^{bcfg}R^{de}{}_{fg} + \frac{1}{8}X^I C^{abcd}C_{abcd} + \frac{1}{12}X^I D^2 + \frac{1}{6}F^{Iab}v_{ab}D\\ &+ \frac{1}{3}X^I C_{abcd}v^{ab}v^{cd} + \frac{1}{2}F^{Iab}C_{abcd}v^{cd} + \frac{8}{3}X^I v_{ab}\hat{\mathcal{D}}^b\hat{\mathcal{D}}_c v^{ac}\\ &+ \frac{4}{3}X^I\hat{\mathcal{D}}^a v^{bc}\hat{\mathcal{D}}_a v_{bc} + \frac{4}{3}X^I\hat{\mathcal{D}}^a v^{bc}\hat{\mathcal{D}}_b v_{ca} - \frac{2}{3e}X^I\epsilon_{abcde}v^{ab}v^{cd}\hat{\mathcal{D}}_f v^{ef}\\ &+ \frac{2}{3e}F^{Iab}\epsilon_{abcde}v^{cf}\hat{\mathcal{D}}_f v^{de} + e^{-1}F^{Iab}\epsilon_{abcde}v^c{}_f\hat{\mathcal{D}}^d v^{ef}\\ &- \frac{4}{3}F^{Iab}v_{ac}v^{cd}v_{db} - \frac{1}{3}F^{Iab}v_{ab}v^2 + 4X^I v_{ab}v^{bc}v_{cd}v^{da} - X^I(v^2)^2\Big)\end{aligned}$$

related to the mixed gauge-gravitational CS term by SUSY tensformations.

C_{IJK}: Intersection numbers of internal space CY_3
$C_2 I$: Second Chern class of internal space CY_3

$$\mathcal{A}^2 = \mathcal{A}^i_\alpha \mathcal{A}^\alpha_i, \qquad v^2 = v_{ab} v^{ab},$$
$$\nu = \frac{1}{6} C_{IJK} X^I X^J X^K, \;\; \nu_I = \frac{1}{2} C_{IJK} X^J X^K, \;\; \nu_{IJ} = C_{IJK} X^K.$$

Weyl multiplet→the metric, a 2-form auxiliary field, v_{ab}, a scalar auxiliary field D, a gravitino ψ^i_μ and an auxiliary Majorana spinor χ^i

Vector mutiplet→1-form gauge field A^I, a scalar auxiliary field X^I and a gaugino Ω^{Ii} (where $I = 1, \cdots, n_v$ count the number of vector multiplets) and $i = 1, 2$ is an $SU(2)$ doublet index.

Hyper multiplet→auxiliary scalar fields $\mathcal{A}^i_\alpha$ and a hyperino ζ^α $\alpha = 1, \cdots, 2r$ refers to $USP(2r)$ group.

♣ Entropy Function & Black Ring Solution

[Castro, Davis, Kraus and Larsenr(07)]
[de Wit and Katmadas(09)]

The near horizon of SUSY black ring is given by

$$ds^2 = l_{AdS^2}^2\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + l_{S^1}^2\,(d\psi + e_0\, r\, dt)^2 + l_{S^2}^2\left(d\theta^2 + \sin^2\theta d\phi^2\right),$$

$$A^I = e^I r dt - \frac{p^I}{2}\cos\theta d\phi + a^I(e_0 r\, dt + d\psi), \qquad X^I = \frac{p^I}{l_{AdS^2}}, \qquad D = \frac{12}{l_{AdS^2}^2},$$

$$Q^I = -4C_{IJK}p^J a^K, \qquad e^I + e_0 a^I = 0, \qquad v_{\theta\phi} = \frac{3}{8}l_{AdS^2}\sin\theta,$$

θ & ϕ: The coordinates of a usual 2-sphere.
ψ: The coordinate of ring, $\psi \sim \psi + 4\pi$.
The radii are given by the magnetic charges p^I,

$$l_{AdS_2} = l_{S^2} = e_0\, l_{S^1} = \frac{1}{2}\left(\frac{1}{6}C_{IJK}p^Ip^Jp^K + \frac{1}{12}c_{2I}p^I\right)^{1/3}$$

The macroscopic entropy is

$$S_{mac} = \frac{2\pi}{e_0}\left(\frac{1}{6}C_{IJK}p^Ip^Jp^K + \frac{1}{6}c_{2I}p^I\right) = 2\pi\sqrt{\frac{\hat{q}_0(C_{IJK}p^Ip^Jp^K + c_{2I}p^I)}{6}}$$

in which

$$\hat{q}_0 = \frac{1}{{e_0}^2}\left(\frac{1}{6}C_{IJK}p^Ip^Jp^K + \frac{1}{6}c_{2I}p^I\right)$$

[de Wit and Katmadas(09)]

♣ c-extremization approach

◇ Near Horizon Isometry

The isometry of the metric are $SL(2,R)\times U(1)\times SO(3)$ generated by

$$K_1 = t\partial_t - r\partial_r, \qquad K_2 = \frac{1}{2}(t^2 + r^{-2})\partial_t - rt\partial_r - \frac{e_0}{r}\partial_\psi,$$
$$K_3 = \partial_t, \qquad K_4 = \partial_\psi,$$
$$J^3 = -i\partial_\phi, \qquad J^\pm = e^{\pm i\phi}(-i\partial_\theta \pm \cot\theta\partial_\phi).$$

The first two parts of the near horizon metric is locally $\mathrm{AdS}_3$and this permit us to use the c-extremization approach to find the associated central charge. [Kraus and Larsen (05)]

The first step is choosing an appropriate ansatz,

$$ds^2 = l^2_{AdS_3} ds^2_{AdS_3} + l^2_{S^2} ds^2_{S^2},$$
$$A^I = e^I r dt - \frac{p^I}{2}\cos\theta d\phi + a^I(e_0 r\, dt + d\psi),$$

Then by extremizing thec-function

$$c = 6{l_A}^3{l_S}^2(\mathcal{L}_0 + \mathcal{L}_1),$$

with respect to the l_A and l_S we find their values in terms of the magnetic charges and the value of c-function at these radii gives the average of left and right central charges.

By doing these calculation one finds,

$$l_{AdS_3} = 2\, l_{S^2} = \left(\frac{1}{6}C_{IJK}p^I p^J p^K + \frac{1}{12}c_{2I}p^I\right)^{1/3},$$

$$A^I = e^I r dt - \frac{p^I}{2}\cos\theta d\phi + a^I(e_0 r\, dt + d\psi), \qquad X^I = \frac{p^I}{l_{AdS^2}}, \qquad D = \frac{12}{l^2_{AdS^2}},$$

$$Q^I = -4C_{IJK}p^J a^K, \qquad e^I + e_0 a^I = 0, \qquad v_{\theta\phi} = \frac{3}{8} l_{AdS^2}\sin\theta,$$

and the value of c-function at this extremum point is given by

$$c|_{ext.} = \frac{1}{2}(c_L + c_R) = C_{IJK}p^I p^J p^K + \frac{3}{4}c_{2I}p^I$$

There is a precise agreement between the above solution and the results of entropy function formalism.

For the associated dual CFT the gravitational anomaly yields to the difference between left and right central charges,

$$c_L - c_R = \frac{1}{2}c_{2I}p^I$$

Thus the left and right central charges are given by

$$c_L = C_{IJK}p^I p^J p^K + c_{2I}p^I, \qquad c_R = C_{IJK}p^I p^J p^K + \frac{1}{2}c_{2I}p^I$$

Using the Cardy formula (in extremal limit) to compute the microscopic entropy

$$S^{(c)}_{\text{mic}} = 2\pi\sqrt{\frac{c_L\,\widehat{q}_0}{6}} = \frac{2\pi}{e_0}\left(\frac{1}{6}C_{IJK}p^Ip^Jp^K + \frac{1}{6}c_{2I}p^I\right) = S_{mac}$$

Note that this entropy associated to a chiral CFT based on the $SL(2,R)$ part of the isometry of near horizon geometry of black ring such that L_0, L_1 and L_{-1} are respectively proportional to K_1, K_2 and K_3 isometries of the geometry.

Thus there is a Virasoro algebra with the above central charge

$$[L^{(c)}_m, L^{(c)}_n] = (m-n)L^{(c)}_{m+n} + \frac{c_L}{12}(m^3-m)\delta_{m+n}$$

such that

$$\left.\begin{array}{r} t\partial_t - r\partial_r \\ \partial_t \\ \frac{1}{2}(t^2+r^{-2})\partial_t - rt\partial_r - \frac{e_0}{r}\partial_\psi \end{array}\right. \Rightarrow L^{(c)}_0, L^{(c)}_{\pm 1}$$

and

$$c_L = C_{IJK}p^Ip^Jp^K + c_{2I}p^I$$

♣ Kerr/CFT approach

[Guica, Hartman, Song, Strominger(08)]

◇ Asymptotic Symmetry

Near horizon metric of 5D Black Ring :

$$ds^2 = \frac{\tilde{p}^2}{4}(-r^2dt^2 + \frac{dr^2}{r^2}) + \frac{\tilde{p}^2}{4}(d\theta^2 + \sin^2\theta d\phi^2) + \tilde{L}^2(d\psi + e_0 r dt)^2$$

$$\tilde{p}^3 \equiv \frac{1}{6}C_{IJK}p^I p^J p^K + \frac{1}{12}c_{2I}p^I \qquad \tilde{L} \equiv \frac{\tilde{p}}{2e_0}$$

Consistent boundary conditions $(t, r, \theta, \phi, \psi)$:

$$h_{\mu\nu} \sim \mathcal{O}\begin{pmatrix} r^2 & 1/r^2 & 1/r & r & 1 \\ & 1/r^3 & 1/r^2 & 1/r^2 & 1/r \\ & & 1/r & 1/r & 1/r \\ & & & 1/r & 1 \\ & & & & 1 \end{pmatrix}$$

The generators associated to these boundary conditions are given by

$$\zeta_n = -e^{-in\psi}\partial_\psi - in\ r\ e^{-in\psi}\partial_r,$$

such that satisfy a Virasoro algebra

$$i[\zeta_m, \zeta_n] = (m-n)\zeta_{m+n}.$$

◇ Computing the central charge

1st Point: To extend the Kerr/CFT approach for theory with higher derivative corrections it is useful to do the calculations in non-basis coordinates.

[Azeyanagi, Compere, Ogawa, Tachikawa, and Terashima(09)]

The vielbeins associated to near horizon geometry of black ring are

$$e^{\hat{t}} = \frac{\tilde{p}}{2} r dt, \qquad e^{\hat{r}} = \frac{\tilde{p}}{2r} dr, \qquad e^{\hat{\theta}} = \frac{\tilde{p}}{2} d\theta,$$
$$e^{\hat{\phi}} = \frac{\tilde{p}}{2} \sin\theta d\phi, \qquad e^{\hat{\psi}} = \tilde{L}(d\psi + e_0 r\, dt),$$

and the variation of the metric under Virasoro generators is given by

$$\mathcal{L}_{\zeta_n} e^{\hat{t}} = i\, n\, e^{-in\psi} e^{\hat{t}}, \qquad \mathcal{L}_{\zeta_n} e^{\hat{r}} = -e_0\, n^2\, e^{-in\psi}\left(e^{\hat{\psi}} - e^{\hat{t}}\right),$$
$$\mathcal{L}_{\zeta_n} e^{\hat{\theta}} = \mathcal{L}_{\zeta_n} e^{\hat{\phi}} = 0, \qquad \mathcal{L}_{\zeta_n} e^{\hat{\psi}} = i\, n\, e^{-in\psi}\left(e^{\hat{\psi}} - 2e^{\hat{t}}\right).$$

2nd Point: The Kerr/CFT approach was extended to the case with CS term and it was shown that for a theory with gravity and also other fields, the central charge is not affected by non-gravitational fields.

[Compere, Murata, and Nishioka(09)]

Using the BBC method the central charge is given by

[Barnich and Brandt (01), Barnich and Compere (07), Compere (07)]

$$c^{(k)} = 12\, i \int_{\partial\Sigma} \mathbf{k}^{inv}_{\zeta_n}[\mathcal{L}_{\zeta_{-n}} g; g]\Big|_{n^3}$$

$\big|_{n^3}$: Stands for the term of order n^3

$\partial\Sigma$: Spatial boundry

$$\begin{aligned}\mathbf{k}^{inv}_{\zeta_n}[\mathcal{L}_{\zeta_{-n}} g; g] = & - 2\Big[\mathbf{X}_{cd}\mathcal{L}_{\zeta_n}\nabla^c\zeta^d_{-n} + (\mathcal{L}_{\zeta_n}\mathbf{X})_{cd}\nabla^{[c}\zeta^{d]}_{-n} + \mathcal{L}_{\zeta_n}\mathbf{W}_c\zeta^c_{-n}\Big] \\ & - \mathbf{E}[\mathcal{L}_{\zeta_n} g, \mathcal{L}_{\zeta_{-n}} g; g],\end{aligned}$$

Covariant derivatives: are defined with respect to the original metric g.

X and **W** and **E**: are given by

$$(\mathbf{W}^c)_{c_3c_4c_5} = -2\nabla_d Z^{abcd}\epsilon_{abc_3c_4c_5} = 2(\nabla_d \mathbf{X}^{cd})_{c_3c_4c_5},$$

$$\mathbf{E}_{c_3c_4c_5} = \frac{1}{2}\left(-\frac{3}{2}Z^{abcd}\delta g_c{}^e \wedge \delta g_{ed} + 2Z^{acde}\delta g_{cd} \wedge \delta g^b{}_e\right)\epsilon_{abc_3c_4c_5}.$$

in which

$$Z^{abcd} = \frac{\delta^{cov} L}{\delta R_{abcd}},$$

[Azeyanagi, Compere, Ogawa, Tachikawa, and Terashima(09)]

The important points for calculating the central charge are

1- The Lie derivative of vielbeins with respect to the diffeomorphisms
2- The isometry $SL(2,R) \times U(1)$
3- The $t-\psi$ reflection symmetry of the near horizon geometry

Doing some algebric calculations the central charge associated to the Virasoro algebra is derived

$$c^{(k)} = -12 e_0 \int_\Sigma Z_{abcd} \epsilon^{ab} \epsilon^{cd} \mathrm{vol}(\Sigma) = \frac{6\, e_0}{\pi} S_{\mathrm{mac}}$$

In the last equality we used the Iyer-Wald formula for macroscopic entropy of a black hole which is generalization of Bekenstein-Hawking formula when the higher derivative terms are appeared.

The cental charge is

$$c^{(k)} = C_{IJK} p^I p^J p^K + c_{2I} p^I = c_L$$

Note that this central charge equals to the left central charge computed by c-extremization formalism.

This equality was shown for SUSY black ring without higher derivative corrections.

$$c^{(k)} = c_L = c_R = C_{IJK} p^I p^J p^K$$

and

$$S^{(k)}_{\text{mic}} = S^{(c)}_{\text{mic}} = S_{\text{mac}}$$

[Loran and Soltanpanahi (08)]

As in the other application of Kerr/CFT approach, to compute the microscopic entropy we should derive the Frolov-Thorne temperature. The temperature is an intrinsic feature of metric and its definition is not corrected by higher derivative terms.

[Azeyanagi, Compere, Ogawa, Tachikawa and Terashima (09)]

So as usual one can find the Frolov-Thorne temperature from the $t\psi$ cross term of near horizon geometry

$$T_{\mathrm{FT}} = \frac{1}{\pi e_0}.$$

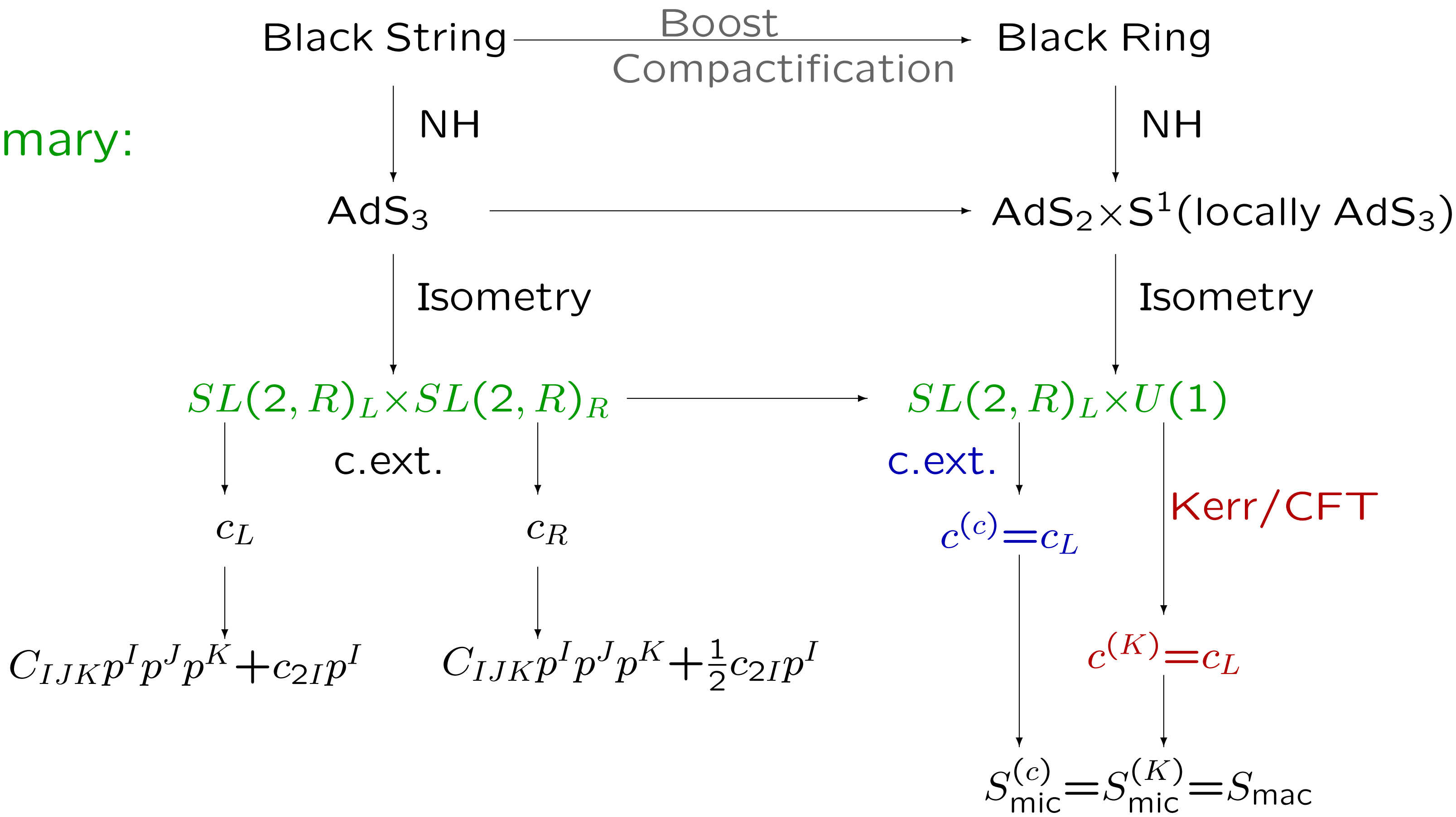
Often there is a factor **2** in the denominator but remember that the period of ψ coordinate in our notation is 4π.

The microscopic entropy of supersymmetric black ring in Kerr/CFT approach can be computed by other form of Cardy formula,

$$S^{(k)}_{\mathrm{mic}} = \frac{\pi^2}{3} c^{(k)} T_{FT} = \frac{2\pi}{e_0}\left(\frac{1}{6} C_{IJK} p^I p^J p^K + \frac{1}{6} c_{2I} p^I\right) = S^{(c)}_{\mathrm{mic}} = S_{\mathrm{mac}}$$

As we expect this microscopic entropy equals to the microscopic entropy calculated by c-extremization formalism and also equals to the macroscopic entropy.

♣ Summary:



Black String
Boost
Compactification
Black Ring
NH
NH
AdS$_3$
AdS$_2\times$S^1(locally AdS$_3$)
Isometry
Isometry
$SL(2,R)_L\times SL(2,R)_R$
$SL(2,R)_L\times U(1)$
c.ext.
c.ext.
Kerr/CFT
c_L
c_R
$c^{(c)}=c_L$
$c^{(K)}=c_L$
$C_{IJK}p^Ip^Jp^K+c_{2I}p^I$
$C_{IJK}p^Ip^Jp^K+\frac{1}{2}c_{2I}p^I$
$S^{(c)}_{\text{mic}}=S^{(K)}_{\text{mic}}=S_{\text{mac}}$

Thank you for your attention