Non-Relativistic Gauge-Gravity Duality

Drag Force in Non-Relativistic Background

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Gauge/Gravity Duality as A Technology

String Theory Gave birth to a child which is called AdS/CFT duality.[Maldacena],[Witten] and [GKP]

- The precise statement is that

  $$D = 4,\ \mathcal{N} = 4,\ SU(N) \text{ Yang-Mills} = \text{IIB string theory on } AdS$$

- $$\text{strong} \Leftrightarrow \text{weak} \qquad ; \quad \text{QFT} \Leftrightarrow \text{String theory}$$

- So we can **calculate in strong interaction theories**

- people believe that

$$\mathrm{AdS} = \mathrm{CFT}\,,$$

  is a Greatest Equation in today Physics.[Polchinski:TASI 2010]
- This duality is a tip of a large iceberg of *Gauge* $=$ *Gravity* (emergent gravity)
- Stong coupling is necessary for having gravity dual,but not sufficient

Schrödinger Group

$d+1$ dimensional Galilean group

- M_{ij}: rotation
- P_i : translation
- K_i : Galilean boost ,$[K_i, P_j] = -i\delta_{ij}M$,M: mass

Conformal extension:

- D :dilatation with (dynamical) exponent z

$$[D,\ P_i] = iP_i,\ [D,\ H] = zi\ H,$$
$$[D,\ K_i] = (1-z)iK_i,\ [D,\ M] = i(2-z)M$$

- C: special conformal transformation when $z = 2$

$$[K_i,\ C] = 0, \qquad [D,\ C] = -2iC, \qquad [H,\ C] = -iD.$$

When $z = 2$ Called Schrödinger group because of symmetry of this action (unitarity noninteracting fermions)

$$S = \int d^d x dt \left[\psi^\dagger i \partial_t \psi - \frac{1}{2m} (\partial_i \psi)(\partial_i \psi^\dagger) \right] ,$$

has this symmetry

- D: $x \rightarrow \lambda x, t \rightarrow \lambda^2 t$
- C : $x \rightarrow \frac{x}{1-\lambda t}, t \rightarrow \frac{t}{1+\lambda t}$

- Fermion at unitary in $d = 3$ [Mehen-Stewart-Wise]

$$S = \int d^3x dt \left[\psi^\dagger i\partial_t \psi - \frac{1}{2m}(\partial_i \psi)^2 + g\psi_\downarrow^\dagger \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow \right] ,$$

  when $g \to \infty$

- Experimentally realized in a system of trapped cold atoms

- Strongly coupled ,hard to solve $\to$ Might AdS/CFT help?

background with non-relativistic conformal symmetry

[D.T.Son] and [K.Balasubramanian-McGreevy] found the metric

$$ds^2 = -\beta^2\, r^{2z}(dx^+)^2 + \frac{dr^2}{r^2} + r^2(-dx^+dx^- + d\vec{x}^2)$$

which has non-relativistic conformal symmetry with dynamical exponent z

- D acts as follows

$$\vec{x} \rightarrow \lambda\vec{x},\, x^+ \rightarrow \lambda^z x^+,\, x^- \rightarrow \lambda^{2-z}x^-,\, r \rightarrow r/\lambda$$

- $x^+ \leftrightarrow H$
- Deformation of AdS_{d+2} for non-relativistic conformal system with d spatial dimention.
- Seems natural compactify x^- in $z = 2$, $x^- \leftrightarrow M$

- if one compactifies x^-,deformation is not necessary! [W.D.Goldberger],[Maldacena] the isometry of

$$ds^2 = +\frac{dr^2}{r^2} + r^2(-dx^+dx^- + d\vec{x}^2), x^- \sim x^- + r^-$$

  is exactly Schrödinger group.
- DLCQ of relativistic theory$\longrightarrow$ looks like Galilean theory. $p_+ p_- - \vec{p}^2 = 0 \longrightarrow E = \frac{\vec{p}^2}{2M}$ where $E = p_+$ and $M = p_-$
- DLCQ of relativistic conformal theory $\longrightarrow$ Galilean+Scaling+... !

- $AdS_5 \times S^5$ with $x^- \sim x^- + r^- \rightarrow$ DLCQ of $\mathcal{N} = 4$ SYM
- A theory in $d = 2 + 1$ doesn't at all look like fermions at unitarity ....!
- $AdS_7 \times S^4$ with $x^- \sim x^- + r^- \rightarrow$ DLCQ of $M5$-brane theory
- studied in non-relativistic superconformal by [Aharony-Berkooz-Seiberg]
- Theory in $4 + 1$ dimentions.It has a 4 dimension schrödinger symmetry but not cold atoms in 4-dim.

Deformed version with $z = 2$

$$ds^2 = -\beta^2\, r^{2z}(dx^+)^2 + \frac{dr^2}{r^2} + r^2(-dx^+dx^- + d\vec{x}^2)$$

times S^5 can be obtain by a solution generating technique: TsT transformation orNull Melvin Twist start from

$$ds^2 = r^2\left(-dt^2 + d\mathbf{x}^2 + dy^2\right) + \frac{dr^2}{r^2} + (d\psi + A)^2 + d\Sigma_4^2$$

where we have written the metric on the unit S^5 as a fibration over a $\mathbf{C}P^2$ base and now $\mathbf{x} = \{x_1, x_2\}$.

- Pick a translationally invariant direction (say y) and boost by amount γ along y
- T-dualize along y
- Twist some one-form σ: $\sigma \to \sigma + \alpha\, dy$
- T-dualize along y again
- Boost by $-\gamma$ along y
- Scale the boost and twist: $\gamma \to \infty$ and $\alpha \to 0$, keeping

$$\beta = \frac{1}{2}\alpha\, e^{\gamma} = \text{fixed}.$$

- Field theory side: non-commutativity ,non-local field theory=dipole theories...!
- So, this background is dual to DLCQ of non-commutative deformation of $\mathcal{N}=4$.
- It looks quite different from cold atoms(=fermion at unitarity)
  But one can study the different aspect of this theory with this caveat in mind
  Is there difference larger than that of QCD at RHIC and hot N=4SYM ?

more caveats

- Vacum solutions have x^{-} compactified $\rightarrow$ can't trust supergravity approx.
- The spacetime is conformal (with an overall conformal factor r^2) to a pp-wave spacetime, and this pp-wave spacetime is known to be non-distinguishing [Flores:2002],Hubeny:2003]. Non-distinguishing means that while the spacetime is causal there are distinct points in the spacetime which haveidentical past and future sets
- Asymtotes vacume solutions $\rightarrow$ Still bad at $r \rightarrow \infty$

BH solution

- AdS $\longrightarrow$ NMT or TsT $\longrightarrow$ Schrödinger bkg.
- Non-extremal brane solution $\longrightarrow$ NMT or TsT$\longrightarrow$ finite temperature solution [Alishahiha,Oz,D.Yamada,Herzog]

$$ds^2 = r^2\left(-\frac{\beta^2 r^2 f(r)}{k(r)}(dt+dy)^2 - \frac{f(r)}{k(r)}dt^2 + \frac{dy^2}{k(r)} + d\mathbf{x}^2\right) -$$

with

$$f(r) = 1 - \frac{r_+^4}{r^4}\,, \qquad k(r) = 1 + \frac{\beta^2 r_+^4}{r^2}$$

- Horizen at $r = r_+$

- Ratio of shear viscosity to entropy density of near-extremal D3-branes [Policastro:2001] raised the tantalizing prospect of a connection between string theory and relativistic heavy ion collisions.
- Replacing QCD by $\mathcal{N} = 4$ SYM
- Phenomenon of jet-quenching (=strong energy loss as a high-energy parton passes through the QGP) in RHI collisions
- The external quark can be prescribed to move on the boundary of AdS_5-BH
- The string is a holographic representation of the color flux from the external quark spreading out in the $3 + 1$ dimensions of the boundary theory.
- The magic of AdS/CFT $\rightarrow$ classical picture of a string in curved spacetime.

- A test string in Non-relativistic background can be described by the Nambu-Goto action:

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma\, e^{\phi/2}\sqrt{-\det g_{\alpha\beta}} \quad g_{\alpha\beta} \equiv G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$$

- Ansatz $\sigma^\alpha = (t, r)$,to describe the late-time behavior

$$x^1(t, r) = vt + x(r)$$

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$$S = -\frac{1}{2\pi\alpha'}\int dtdr\sqrt{-(g_{tt}g_{rr} + g_{tt}g_{xx}x'^2 + g_{xx}g_{rr}v^2)}$$

- Equation of motion is simply that π_ξ is a constant

$$\frac{-g_{tt}g_{xx}x'}{\sqrt{-(g_{tt}g_{rr} + g_{tt}g_{xx}x'^2 + g_{xx}g_{rr}v^2)}} = c = -2\pi\alpha'\pi_x = \text{consta}$$

- $$x'^2 = 4\pi^2\alpha'^2\pi_x^2\left(\frac{g_{rr}(-g_{tt}-g_{xx}v^2)}{g_{xx}g_{tt}(g_{xx}g_{tt}+4\pi^2\alpha'^2\pi_x^2)}\right).$$
- In terms of the constant π_x one has

$$\frac{dE}{dt} = \pi_x v, \qquad \frac{dP}{dt} = \pi_x.$$

where E and P are energy and momentum the open string gain from through its end point.

- Physically make sense if

$$\frac{1}{2}\mu^2 r_0^6 + (1-v^2)r_0^4 - \frac{1}{2}\mu^2 r_H^4 r_0^2(1+v^2) - r_H^4 = 0,$$

which can be solved for r_0.

Plugging the solution r_0 in the denominator one arrives at

$$\pi_x = -\frac{v}{2\pi\alpha'} g_{xx}|_{r_0}.$$

- Drag force becomes

$$\frac{dP}{dt} \approx \begin{cases} -\frac{v}{2\pi\alpha'} \frac{r_H^2}{R^2} \left(1 + \frac{1}{2}v^2\right) & \text{for} \quad v \ll 1, \\ -\frac{v}{2\pi\alpha'} \frac{2}{\mu^2 R^2} \left(v^2 + \mu^4 r_H^4 - 4\right) & \text{for} \quad v \gg 1. \end{cases}$$

- The first one as the non-relativistic limit of that found in [Gubser] for the relativistic field theory when $v \ll 1$.

$$\frac{dp_1}{dt} = \frac{-\pi\sqrt{g_{YM}^2 N}}{2} T^2 \frac{v}{\sqrt{1-v^2}}$$

- The second The second case is just because of the non-local nature of the dual field theory.

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