1st IPM school and workshop on applied

AdS/CFT

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On Holography of Julia-Zee Dyon

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Question

† What is the dual theory of the gravitational 'tHooft-Polyakov monopole or more generally gravitational Julia-Zee dyon?

Julia-Zee Dyon

$$S = \int d^4x \left\{ -\frac{1}{4}(\vec{F}_{\mu\nu})^2 - \frac{1}{2}(D_\mu \vec{\phi})^2 - \frac{\lambda}{4}(\vec{\phi}^2)^2 + \frac{1}{L^2}\vec{\phi}^2 \right\}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\,\varepsilon^{abc} A^b_\mu A^c_\nu \qquad D_\mu \phi^a = \partial_\mu \phi^a + e\,\varepsilon^{abc} A^b_\mu \phi^c$$

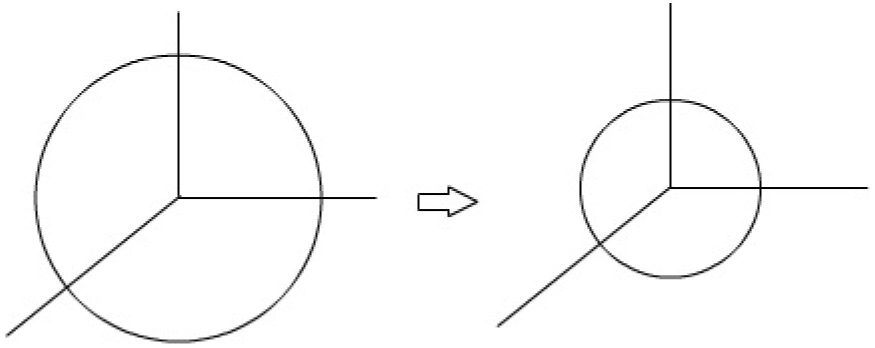
$$\phi = \vec{\tau}.\vec{\phi} = \frac{H(r)}{er}\,\tau^r \qquad A = \vec{A}.\vec{\phi} = \frac{J(r)}{er}\,\tau^r\, dt + \frac{2\left(1-K(r)\right)}{e}\left(-\tau^\varphi d\theta + \tau^\theta \sin(\theta)\, d\varphi\right)$$

$$\tau^r = \hat{r}.\vec{\tau} \quad ; \quad \tau^\theta = \hat{\theta}.\vec{\tau} \quad ; \quad \tau^\varphi = \hat{\varphi}.\vec{\tau}$$

$$F_{\mu\nu} = \hat{\phi}.\vec{F}_{\mu\nu} + \hat{\phi}.\left[D_\mu\hat{\phi} \times D_\nu \hat{\phi}\right]$$

$$\hat{\phi} = \frac{\vec{\phi}}{\sqrt{\vec{\phi}.\vec{\phi}}}$$

$$B_i = \varepsilon_{ijk} F^{jk} \rightarrow g = \oint_{S^2} B_i\, ds^i = \frac{n}{e}$$



external space   internal space

A.M.Polyakov, JTEP Lett. 20 (1974) 194
G.'tHooft, Nucl. Phys. B 79 (1974) 276
B. Julia and A.Zee , Phys. Rev. D 11 (1975) 760
J. Arafune , P.G.O.Freund and C.J.Goebel, Lect. Notes Phys. 39 (1975) 240

Julia-Zee Dyon

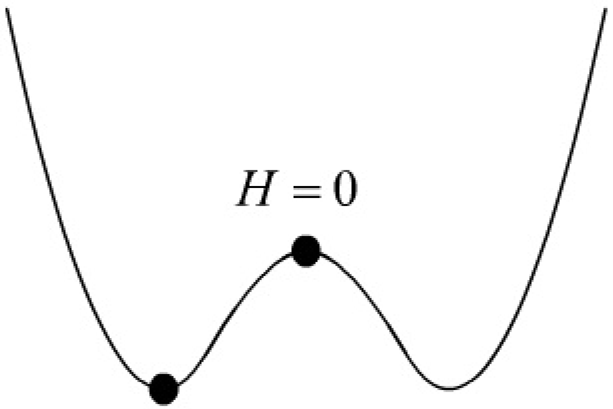
$$S=\int\sqrt{-g}\,d^4x\left\{\frac{1}{16\pi G}\left(R+\frac{6}{L^2}\right)-\frac{1}{4}(\vec{F}_{\mu\nu})^2-\frac{1}{2}(D_\mu\vec{\phi})^2-\frac{\lambda}{4}(\vec{\phi}^2)^2+\frac{1}{L^2}\vec{\phi}^2\right\}$$

$$ds^2=-e^{X(r)}dt^2+e^{Y(r)}dr^2+r^2d\Omega_2^2$$

$$H=\sqrt{\frac{2}{\lambda L^2}}\,r\,,K=0\,,J=\mu r-\rho\,,g_{\mu\nu}\rightarrow AdS-RN$$

$$H=0\,,K=0\,,J=\mu r-\rho\,,g_{\mu\nu}\rightarrow AdS-RN$$

† *In flat back ground H=0 solution is unstable but, in AdS back ground such solutions can be stable.*



M. Kasuya and M. Kamata, Nuovo Cim. B 66 (1981) 75

Julia-Zee Dyon

Dynamical stability of H=0 solution:

$$\begin{cases} T > T_C: & \text{the quasi-normal modes decay and the background is stable} \\ T < T_C: & \text{the quasi-normal modes blow up and the background is unstable} \end{cases}$$

In the space of the parameters of the theory for the H=0 solution, there is a "wall of marginal stability".

By crossing this wall a "Phase transition" occurs in the dual field theory.

$4\pi T$	ω
0.212	0.764969 + 1.66858 i
0.212	4.93743 + 3.35654 i
0.812	10.133 + 36.3126 i
0.812	0.669683 + 9.05345 i
1.212	9.9903 + 34.2773 i
2.211	0.0148391 + 35.9313 i
3.211	12.5418 + 52.0683 i
4.210	4.98854×10^{-15} - 1.30046 i
6.210	5.26857×10^{-14} - 2.09467 i
8.209	2.36149×10^{-14} - 3.01488 i
10.21	1.26155×10^{-9} - 4.13518 i
16.21	2.16507 - 7.78775 i
16.21	8.66376 - 7.8904 i
20.20	3.7928 - 9.49205 i

D. Allahbakhshi , F. Ardalan . JHEP 1010:114,2010. (arXiv:1007.4451)

Vacuum Expectation Values

- The dual field theory containes:

$$O^a \leftrightarrow \phi^a$$

$$V_\mu^a \leftrightarrow A_\mu^a$$

- Suppose the asymptotic expansion of the fiels are:

$$\begin{cases} \vec{\phi} \approx \vec{\phi}_S r^{\Delta_+} + \vec{\phi}_C r^{\Delta_-} + \dots \\ \vec{A}_i \approx \vec{A}_{Si} r^{\eta_+} + \vec{A}_{Ci} r^{\eta_-} + \dots \\ \vec{A}_t \approx \vec{A}_{St} r^{\delta_+} + \vec{A}_{Ct} r^{\delta_-} + \dots \end{cases}$$

- For Einstein-Yang-Mills-Higgs action, the 1-point functions are:

$$\begin{cases} \left\langle \vec{O} \right\rangle = \lim_{r\to\infty} \dfrac{\delta S_{o.s}}{\delta \vec{\phi}_S} \propto \left(\lim_{r\to\infty} r^{\Delta_+} \sqrt{-g}\, D^r \vec{\phi} \right)_{finite\ term} \\ \left\langle \vec{V}_i \right\rangle = \lim_{r\to\infty} \dfrac{\delta S_{o.s}}{\delta \vec{A}_{Si}} \propto \left(\lim_{r\to\infty} r^{\eta_+} \sqrt{-g}\, \vec{F}^{ri} \right)_{finite\ term} \\ \left\langle \vec{V}_t \right\rangle = \lim_{r\to\infty} \dfrac{\delta S_{o.s}}{\delta \vec{A}_{St}} \propto \left(\lim_{r\to\infty} r^{\delta_+} \sqrt{-g}\, \vec{F}^{rt} \right)_{finite\ term} \end{cases}$$

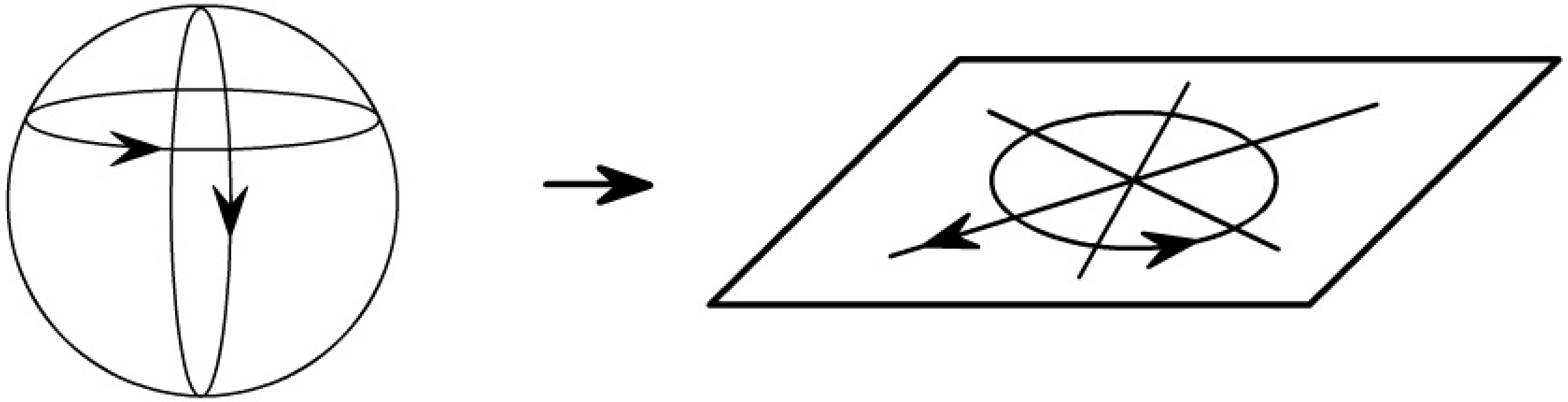
Vacuum Expectation Values

† For the Julia-Zee dyon we have:

$$
\begin{cases}
\langle O^r \rangle \propto \left(\lim\limits_{r\to\infty} r^{\Delta_+} \sqrt{-g}\,\left(D^r \phi\right)^r \right)_{finite\ term} & \to \langle O^r \rangle \\
\langle \vec{V}_i \rangle \propto \left(\lim\limits_{r\to\infty} r^{\eta_+} \sqrt{-g}\,\vec{F}^{ri} \right)_{finite\ term} & \to \langle V_\theta^{\ \varphi} \rangle, \langle V_\varphi^{\ \theta} \rangle \\
\langle \vec{V}_t \rangle \propto \left(\lim\limits_{r\to\infty} r^{\delta_+} \sqrt{-g}\,\vec{F}^{rt} \right)_{finite\ term} & \to \langle V_t^{\ r} \rangle
\end{cases}
$$

And all other v.e.vs are zero.

By stereographic projection the profile of the vectors looks like a vortex



The dual field theory has a vortex condensate.

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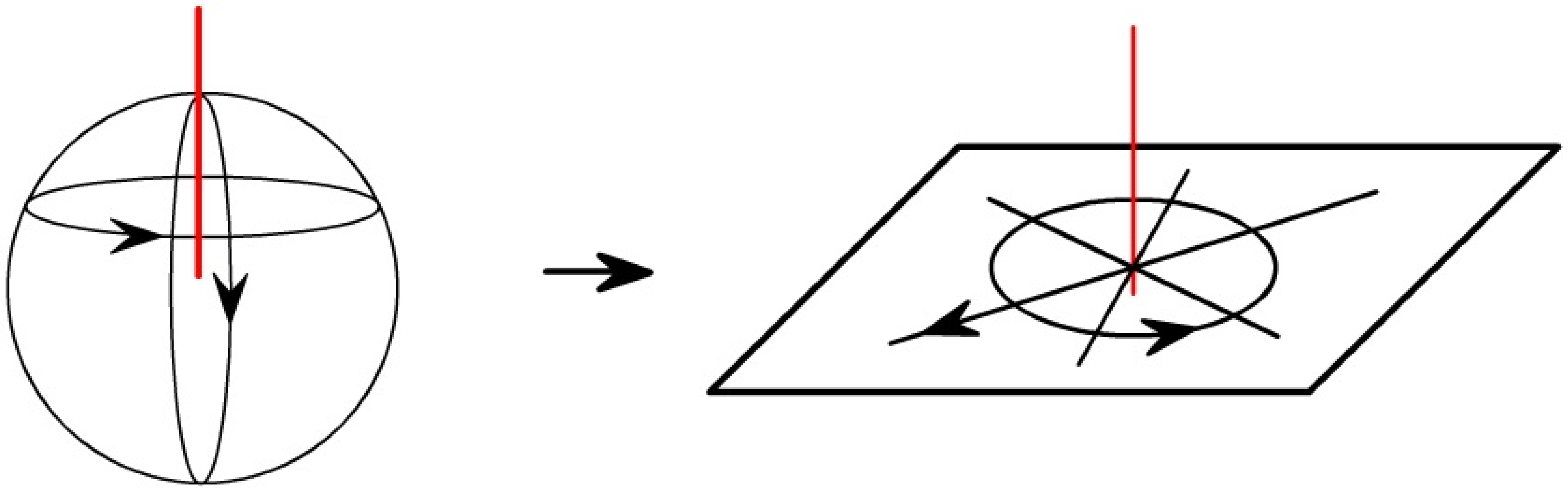
Holography of Topologies

† In the abelian gauge both scalar and vector fields are in the 3rd direction of the gauge space.
† In this gauge, there is a singularity (Dirac string) in the whole positive semi axis in the third direction of space.
† In this gauge the magnetic charge comes from the Dirac string.

$$g = \oint B^i ds_i = \oint \left(curl A^3 \right) . ds \rightarrow \oint_c A^3 . dl$$



Vacuum Expectation Values

† The phase space is a 3-dim hypersurface in the space of temperature, chemical potential, scalar and vector sources.

† Vacuum expectation value of the vector operator has different signs in different regions of the phase space, and there is a line of vanishing v.e.v.

† Vacuum expectation value of the scalar operator changes sign by lowering temperature when scalar and vector sources are fixed.