Hidden Conformal Symmetry of Warped AdS_3 Black Holes

Reza Fareghbal

Department of Physics,IPM

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Warped AdS$_3$ spacetime

- Warped AdS$_3$ spacetimes appear in the near horizon geometry of extremal Kerr black holes:
- The Kerr metric is

$$\begin{aligned} ds^2 = -\frac{\Delta}{\rho^2}\left(d\hat{t} - a\sin^2\theta d\hat{\phi}\right)^2 + \frac{\rho^2}{\Delta}d\hat{r}^2 + \rho^2 d\theta^2 \\ + \frac{\sin^2\theta}{\rho^2}\left((\hat{r}^2 + a^2)d\hat{\phi} - a d\hat{t}\right)^2 \end{aligned} \tag{1}$$

$$a \equiv \frac{J}{M}\,, \qquad \Delta \equiv \hat{r}^2 - 2Mr + a^2\,, \qquad \rho^2 \equiv \hat{r}^2 + a^2\cos^2\theta, \tag{2}$$

- Horizons are at

$$r_\pm = M \pm \sqrt{M^2 - a^2} \tag{3}$$

- For extremal cases (i.e. $r_+ = r_-$), we define new coordinates

$$t = \frac{\lambda\hat{t}}{2M^2}\,, \qquad r = \frac{\hat{r} - M}{\lambda}\,, \qquad \phi = \hat{\phi} - \frac{\hat{t}}{2M}. \tag{4}$$

- In the $\lambda \to 0$ limit, the result is the near-horizon geometry of the extremal Kerr (NHEK):

$$ds^2 = \Gamma(\theta)\left[-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2\right] + \gamma(\theta)(d\phi + r dt)^2\,, \tag{5}$$

where

$$\Gamma(\theta) = M^2(1 + \cos^2\theta)\,, \gamma(\theta) = \frac{4M^2\sin^2\theta}{1 + \cos^2(\theta)}. \tag{6}$$

- The NHEK geometry is not asymptotically flat!

- This geometry has $U(1) \times SL(2,R)$ isometry. The rotational $U(1)$ isometry is generated by the Killing vector $J_0 = -\partial_\varphi$ and the $SL(2,R)$ is generated by

$$\begin{aligned} \bar{J}_1 &= \partial_t \ , \\ \bar{J}_2 &= t\partial_t - r\partial_r , \\ \bar{J}_3 &= \left(\frac{1}{2r^2} + \frac{t^2}{2} \right) \partial_t - tr\partial_r - \frac{k}{r}\partial_\phi \ . \end{aligned} \quad (7)$$

- For fixed θ, the geometry (5) is similar to the three-dimensional warped AdS_3 spacetime,

$$ds^2 = \frac{\ell^2}{(\nu^2+3)}\left[-\cosh^2\sigma d\tau^2 + d\sigma^2 + \frac{4\nu^2}{\nu^2+3}\left(du + \sinh\sigma d\tau\right)^2\right] \tag{8}$$

which is a solution of the Topologically Massive Gravity (TMG):

$$I_{TMG} = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{-g}\left(R + 2/\ell^2\right) + \frac{\ell}{96\pi G\nu}\int_{\mathcal{M}} d^3x\sqrt{-g}\varepsilon^{\lambda\mu\nu}\Gamma^r_{\lambda\sigma}\left(\partial_\mu\Gamma^\sigma_{r\nu} + \frac{2}{3}\Gamma^\sigma_{\mu\tau}\Gamma^\tau_{\nu r}\right) \tag{9}$$

- Besides WAdS_3 geometries, the action (9) has warped AdS_3 black hole solutions. These black holes are discrete quotients of warped AdS_3 just as BTZ black holes are discrete quotients of the ordinary AdS_3.
- The metric of WAdS_3 black hole is

$$ds^2 = -N(r)^2 dt^2 + \ell^2 R(r)^2 (d\phi + N^\phi(r) dt)^2 + \frac{\ell^4 dr^2}{4R(r)^2 N(r)^2},$$

$$\begin{aligned} N(r)^2 &\equiv \frac{\ell^2(\nu^2+3)(r-r_+)(r-r_-)}{4R(r)^2}, \\ N^\phi(r) &\equiv \frac{2\nu r - \sqrt{r_+ r_-(\nu^2+3)}}{2R(r)^2}, \\ R(r)^2 &\equiv \frac{r}{4}\Big(3(\nu^2-1)r + (\nu^2+3)(r_+ + r_-) \\ &- 4\nu\sqrt{r_+ r_-(\nu^2+3)}\Big). \end{aligned} \tag{10}$$

$\mathbf{WAdS_3/CFT_2}$ conjecture

- Quantum TMG is holographically dual to a 2D boundary CFT with $c_R = \frac{(5\nu^2+3)\ell}{G\nu(\nu^2+3)}$ and $c_L = \frac{4\nu\ell}{G(\nu^2+3)}$.
  [Anninos,Li, Padi, Song and Strominger (2008)]
- The motivations for this conjecture are that application of the Cardy formula to the CFT density of states reproduces the black hole entropy.
- The close relation between WAdS_3 and NHEK geometries results in the following conjecture which is known as "Kerr/CFT correspondence":

Quantum gravity in the background of near horizon extremal Kerr black holes is holographically dual to a chiral two-dimensional conformal field theory.

[Guica, Hartman, Song, Strominger (2008)]

Hidden Conformal Symmetry

- The isometry of WAdS$_3$ and NHEK is $U(1) \times SL(2,R)$. They do not have the $SL(2,R) \times SL(2,R)$ conformal symmetry of the dual CFT.
- The attempts to enhance the $U(1) \times SL(2,R)$ isometry of WAdS$_3$ to the full conformal symmetry by making use of the Brown and Henneaux's asymptotic symmetry method, have not been unambiguously successful.
- The question which still remains unanswered is that What is the interpretation of dual conformal symmetry in terms of gravitational bulk theory?

- "Hidden conformal symmetry" may provide an answer to this question:
- In this proposal the conformal symmetry is not derived from a conformal symmetry of the spacetime geometry. It is a symmetry of solution space:
- To see this, Consider the warped AdS_3 black hole (10). We want to study the propagation of a massive scalar field in this background.
- The Klein-Gordon equation for a massive scalar with mass m is

$$\left(\frac{1}{\sqrt{-g}}\,\partial_\mu\sqrt{-g}\partial^\mu - m^2\right)\Phi = 0. \tag{11}$$

- Expanding in eigenmodes

$$\Phi(t,r,\theta,\phi) = e^{-i\omega t + ik\phi}\Phi(r,\theta)\ , \tag{12}$$

and imposing

$$\omega^2 \ll \frac{(\nu^2+3)^2}{3(\nu^2-1)}. \tag{13}$$

simplifies the wave equation (11) to the form

$$\tilde{\mathcal{H}}^2\Phi = \mathcal{H}^2\Phi = \frac{\ell^2 m^2}{\nu^2+3}\Phi \tag{14}$$

where $\tilde{\mathcal{H}}^2$ and $\mathcal{H}^2$ are the quadratic Casimir of two following $SL(2,R)$s: [R.F. (2010)]

$$
H_0 = -\frac{2i\nu}{\nu^2+3}\frac{T_L}{T_R}\partial t + \frac{i}{2\pi\ell T_R}\partial_\phi,
$$

$$
H_1 = i\, e^{-2\pi\ell T_R\phi}\Bigg[-\frac{\nu}{(\nu^2+3)\sqrt{\Delta}}\left((2r-r_+-r_-)\frac{T_L}{T_R}+r_+-r_-\right)\partial_t + \sqrt{\Delta}\partial_r + \frac{2r-r_+-r_-}{4\pi\ell T_R\sqrt{\Delta}}\partial_\phi\Bigg],
$$

$$
H_{-1} = i\, e^{2\pi\ell T_R\phi}\Bigg[-\frac{\nu}{(\nu^2+3)\sqrt{\Delta}}\left((2r-r_+-r_-)\frac{T_L}{T_R}+r_+-r_-\right)\partial_t - \sqrt{\Delta}\partial_r + \frac{2r-r_+-r_-}{4\pi\ell T_R\sqrt{\Delta}}\partial_\phi\Bigg], \tag{15}
$$

$$\bar{H}_0 = \frac{2i\nu}{\nu^2+3}\partial t,$$

$$\bar{H}_1 = i\, e^{-\frac{\nu^2+3}{2\nu}t - 2\pi\ell T_L \phi}\left[\frac{\nu\Big((\nu^2+3)(2r-r_+-r_-)+8\pi\ell T_L\Big)}{(\nu^2+3)^2\sqrt{\Delta}}\partial_t \right.$$
$$\left. + \sqrt{\Delta}\partial_r - \frac{2}{(\nu^2+3)\sqrt{\Delta}}\partial_\phi\right],$$

$$\bar{H}_{-1} = i\, e^{\frac{\nu^2+3}{2\nu}t + 2\pi\ell T_L \phi}\left[\frac{\nu\Big((\nu^2+3)(2r-r_+-r_-)+8\pi\ell T_L\Big)}{(\nu^2+3)^2\sqrt{\Delta}}\partial_t \right.$$
$$\left. - \sqrt{\Delta}\partial_r - \frac{2}{(\nu^2+3)\sqrt{\Delta}}\partial_\phi\right], \tag{16}$$

- where

$$T_R = \frac{(\nu^2+3)(r_+ - r_-)}{8\pi\ell}, \quad (17)$$

$$T_L = \frac{\nu^2+3}{8\pi\ell}\left(r_+ + r_- - \frac{\sqrt{r_+ r_-(\nu^2+3)}}{\nu}\right), \quad (18)$$

$$\Delta = (r - r_+)(r - r_-). \quad (19)$$

- It is not difficult to see that the generators (15) and (16) satisfy the $SL(2,R)\times SL(2,R)$ algebra

$$\begin{aligned} [H_n, H_m] &= i(n-m)H_{n+m} \\ [\bar{H}_n, \bar{H}_m] &= i(n-m)\bar{H}_{n+m} \qquad (n,m=-1,0,1) \\ [H_n, \bar{H}_m] &= 0 \end{aligned} \quad (20)$$

- Hence the scalar Laplacian can be written as the $SL(2,R)$ Casimir and the $SL(2,R)_L \times SL(2,R)_R$ weights of the scalar field are

$$(h_L, h_R) = (\frac{1}{2}\sqrt{1+\frac{4\ell^2 m^2}{\nu^2+3}} - \frac{1}{2}, \frac{1}{2}\sqrt{1+\frac{4\ell^2 m^2}{\nu^2+3}} - \frac{1}{2}). \quad (21)$$

- Generators of hidden $SL(2,R) \times SL(2,R)$ are not periodic under $\phi \sim \phi + 2\pi$. Periodic identification of ϕ breaks the hidden symmetry to $U(1) \times U(1)$.
- This identification is generated by the group element

$$e^{\partial_\phi} = e^{-i2\pi\ell(T_R H_0 + T_L \bar{H}_0)} \quad (22)$$

- Interestingly, the identification (22) can be used to produce the warped black holes as a quotient of the warped AdS_3 and T_R and T_L may be interpreted as the right and left temperatures of the conformal field theory dual to the warped AdS black holes.
- The Entropy calculation using the Cardy formula

$$S = \frac{\pi^2 \ell}{3}(c_L T_L + c_R T_R) \qquad (23)$$

supports this hypothesis.

Absorption cross section

- After imposing condition (13), the absorption cross section of the scalar field takes the following form

$$\sigma_{abs} \sim \sinh\left[2\pi\left(\Omega_+\omega + Uk\right)\right]\left|\Gamma\left(\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4m^2\ell^2}{\nu^2+3}} -i\left(\omega(\Omega_+ + \Omega_-) + 2Uk\right)\right)\right|^2 \times\left|\Gamma\left(\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4m^2\ell^2}{\nu^2+3}} - i\omega\left(\Omega_+ - \Omega_-\right)\right)\right|^2 \quad (24)$$

where

$$U = \frac{2}{(r_+ - r_-)(\nu^2+3)} \tag{25}$$

$$\Omega_+ = \frac{2\nu r_+ - \sqrt{r_+ r_-(\nu^2+3)}}{(r_+ - r_-)(\nu^2+3)} \tag{26}$$

$$\Omega_- = \frac{2\nu r_- - \sqrt{r_+ r_-(\nu^2+3)}}{(r_+ - r_-)(\nu^2+3)} \tag{27}$$

- By defining

$$\omega_L \equiv \delta E_L = \frac{1}{2\ell}\left(\nu(r_+ + r_-) - \sqrt{r_+ r_-(\nu^2+3)}\right)\omega$$

$$\omega_R \equiv \delta E_R = \frac{1}{2\ell}\left(\nu(r_+ + r_-) - \sqrt{r_+ r_-(\nu^2+3)}\right)\omega + \frac{k}{\ell}$$

which are the left and right conjugate charges, we can rewrite (24) as

$$\sigma_{\mathrm{abs}} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh\left(\frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R}\right) \left|\Gamma(h_L + i\frac{\omega_L}{2\pi T_L})\right|^2 \left|\Gamma(h_R + i\frac{\omega_R}{2\pi T_R})\right|^2 . \tag{28}$$

which is the well-known absorption cross section for a 2d CFT.