Logarithmic GCA in the context of Holography

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Galilean Conformal Algebra (GCA)

This algebra arise when we investigate relativistic conformal algebra in nonrelativistic limit

To investigate this limit we rescale space and time as

$$x \to x/c \qquad t \to t; \qquad c \to \infty$$

Galilean Conformal Algebra (GCA)

Now we can redefine operators and consider order of magnitudes of *C.* For example

$$P_0 \to P_0$$

$$J_{0i} = tc\partial_i - \frac{1}{c} x_i \partial_t$$

$$\frac{1}{c} J_{oi} \to B_i = t\partial_i$$

Galilean Conformal Algebra (GCA)

We end up with GCA :

$$P_i = \partial_i \qquad H = -\partial_t$$

$$B_i = t\partial_i \qquad J_{ij} = -(x_i\partial_j - x_j\partial_i)$$

$$D = -t\partial_t - x_i\partial_i \qquad K_i = t^2\partial_i$$

$$K = K_0 = -(2tx_i\partial_i + t^2\partial_t)$$

Note that scaling operator D, scales space and time isotropically in this nonrelativistic algebra

Galilean Conformal Algebra: *Infinite Extension*

Similar to Schrodinger algebra GCA does have an infinite extension which is called *Full GCA*

$$T^n = -(n+1)t^n x_i \partial_i - t^{n+1}\partial_t$$
$$M_i^n = t^{n+1}\partial_i$$
$$J_{ij}^n = -t^n(x_i\partial_j - x_j\partial_i)$$

which in 1+1 dimensions simplifies to :

$$[T^m, M_i^n] = (m-n)M_i^{m+n} \qquad [M_i^m, M_j^n] = 0$$
$$[T^m, T^n] = (m-n)T^{m+n}$$

Galilean Conformal Algebra:
Infinite Extension

GCA sits at the middle of Full GCA

$T^{-1} = H$ $\quad$ $T^{0} = D$ $\quad$ $T^{1} = K$

$M_i^{-1} = P_i$ $\quad$ $M_i^{0} = B_i$ $\quad$ $M_i^{1} = K_i$

Similar to the Schrodinger symmetry this symmetry can be also be realized within the AdS/CFT correspondence:

GCA From Contraction…

Now we impose contraction limit to Virasoro operators and observe

$$x \to \frac{x}{c} \qquad t \to t \qquad c \to \infty$$

$$L^n = -\frac{1}{2}(t + i\frac{x}{c})^{n+1}(\partial_t - ic\partial_x)$$
$$= -t^{n+1}(-ic\partial_x + \partial_t + (n+1)\frac{x}{t}\partial_x + O(\frac{1}{c})$$
$$\bar{L}^n = -t^{n+1}(ic\partial_x + \partial_t + (n+1)\frac{x}{t}\partial_x + O(\frac{1}{c})$$

$$T^n = L^n + \bar{L}^n + O(\frac{1}{c})$$
$$M^n = -i\frac{L^n - \bar{L}^n}{c} + O(\frac{1}{c})$$

So, in the contraction limit full GCA is obtained from CFT symmetry

GCA From Contraction …

Consider the usual eigensates of the scaling operator

$$L^0|h,\bar{h}\rangle = h|h,\bar{h}\rangle \qquad \bar{L}^0|h,\bar{h}\rangle = \bar{h}|h,\bar{h}\rangle$$

In the contraction limit we have

$$T^0|h,\bar{h}\rangle = (L^0 + \bar{L}^0)|h,\bar{h}\rangle = (h+\bar{h})|h,\bar{h}\rangle$$
$$M^0|h,\bar{h}\rangle = -\frac{i}{c}(L^0 - \bar{L}^0)|h,\bar{h}\rangle = \frac{i}{c}(\bar{h}-h)|h,\bar{h}\rangle$$

In other words

$$T^0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle \qquad \Delta = h+\bar{h}$$

$$M^0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle \qquad \xi = \frac{i}{c}(\bar{h}-h)$$

GCA From Contraction, correlation functions

Similarly for two point functions:

$$\langle\phi_1(x_1,t_1)\phi_2(x_2,t_2)\rangle_{GCA} = \lim_{c\to\infty}\langle\phi_1(x_1,t_1)\phi_2(x_2,t_2)\rangle_{CFT}$$

$$= lim_{c\to\infty}\delta_{h_1,h_2}\delta_{\bar{h}_1,\bar{h}_2}{z_{12}}^{-2h_1}{\bar{z}_{12}}^{-2\bar{h}_1}$$

$$= lim_{c\to\infty}A\delta_{h_1,h_2}\delta_{\bar{h}_1,\bar{h}_2}(t_{12}+i\,\frac{x_{12}}{c})^{-(\Delta+i\,c\,\xi)}(t_{12}-i\,\frac{x_{12}}{c})^{-(\Delta-i\,c\,\xi)}$$

$$= a\delta_{\Delta_1,\Delta_2}\delta_{\xi_1,\xi_2}{t_{12}}^{-2\Delta_1}exp\,(\frac{2\xi_1x_{12}}{t_{12}})$$

Logarithmic Conformal Field Theory

LCFT's arise when the action of scaling operators on scaling fields are not diagonal but rather Jordan:

$$L^0 \phi_h(z)|0\rangle = h\phi_h(z)|0\rangle$$

$$L^0 \psi_h(z)|0\rangle = h\psi_h(z)|0\rangle + \phi_h(z)|0\rangle$$

We can unite two equations in one equation using a nilpotent variable

$$\theta^2 = 0$$

$$L^0|h+\theta\rangle = (h+\theta)|h+\theta\rangle$$

Logarithmic GCA; Contraction Approach

Consider the most general logarithmic representation in which both left and right scaling weights have Jordan cell structure:

$$L^0|h,\bar{h},1\rangle = h|h,\bar{h},1\rangle + \acute{h}|h,\bar{h},0\rangle$$

$$\bar{L}^0|h,\bar{h},1\rangle = \bar{h}|h,\bar{h},1\rangle + \bar{\acute{h}}|h,\bar{h},0\rangle\,.$$

Logarithmic GCA; Contraction Approach

Now, follow through the contraction procedure:

$$T^0|\Delta,\xi,0\rangle = T^0\big|h,0,\bar{h},0\rangle = \Delta|\Delta,\xi,0\rangle$$

$$T^0|\Delta,\xi,1\rangle = T^0\big|h,\bar{h},1\rangle = h\big|h,\bar{h},1\rangle + \bar{h}\big|h,\bar{h},1\rangle + (\acute{h}+\acute{\bar{h}})\big|h,\bar{h},0\rangle$$

$$= \Delta|\Delta,\xi,1\rangle + \acute{\Delta}|\Delta,\xi,0\rangle$$

$$M^0|\Delta,\xi,1\rangle = M^0\big|h,\bar{h},1\rangle = -i\frac{h}{c}\big|h,\bar{h},1\rangle + i\frac{\bar{h}}{c}\big|h,\bar{h},0\rangle - \frac{i}{c}(\acute{h} - \acute{\bar{h}}\big|h,\bar{h},0\rangle$$

$$= \xi|\Delta,\xi,1\rangle + \acute{\xi}|\Delta,\xi,1\rangle$$

Logarithmic GCA; Contraction Approach

Now, follow through the contraction procedure:

$$T^0|\Delta,\xi,0\rangle = T^0\left|h,0,\bar{h},0\right\rangle = \Delta|\Delta,\xi,0\rangle$$

$$T^0|\Delta,\xi,1\rangle = T^0\left|h,\bar{h},1\right\rangle = h\left|h,\bar{h},1\right\rangle + \bar{h}\left|h,\bar{h},1\right\rangle + (\acute{h}+\acute{\bar{h}})\left|h,\bar{h},0\right\rangle$$

$$= \Delta|\Delta,\xi,1\rangle + \acute{\Delta}|\Delta,\xi,0\rangle$$

$$M^0|\Delta,\xi,1\rangle = M^0\left|h,\bar{h},1\right\rangle = -i\frac{h}{c}\left|h,\bar{h},1\right\rangle + i\frac{\bar{h}}{c}\left|h,\bar{h},0\right\rangle - \frac{i}{c}(\acute{h}-\acute{\bar{h}}\left|h,\bar{h},0\right\rangle$$

$$= \xi|\Delta,\xi,1\rangle + \acute{\xi}|\Delta,\xi,1\rangle$$

Logarithmic GCA; Contraction Approach

So, we have

$$\acute{\Delta} = \acute{h} + \acute{\bar{h}} \qquad \acute{\xi} = \frac{\acute{h} - \acute{\bar{h}}}{c}$$

Now, we can follow on and find two point function and compare them with algebraic approach

Logarithmic GCA; Contraction Approach

If we follow contraction limit for logarithmic GCA we obtain

$$\langle \psi_1(x_1, t_1)\psi_2(x_2, t_2)\rangle_{GCA} =$$

$$\delta_{\Delta_1,\Delta_2}\delta_{\xi_1,\xi_2} t^{-2\Delta_1}\, exp\left(\frac{2\xi_1 x}{t}\right)(-2a\acute{\Delta} log(t) - 2a\acute{\xi}\,\frac{x}{t} + b)$$

These correlators are exactly the same as those obtained via the algebraic approach.

Logarithmic GCA in the context of holography

Topologically massive gravity at critical point is corresponded to logarithmic CFT

$$S = \frac{1}{16\pi G}\int dx^3\sqrt{-g}\left[R + \frac{2}{l^2} + \frac{1}{\mu}\mathcal{L}_{cs}\right]$$

$$\mathcal{L}_{CS} = \frac{1}{2}\in^{\lambda\mu\nu}\Gamma^{\alpha}_{\lambda\sigma}\left[\partial_{\mu}\Gamma^{\sigma}_{\alpha\nu} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\sigma}\right]$$

$$\mu l = 1$$

Logarithmic GCA in the context of holography

$$< \bar{T}(\bar{z})\bar{T}(0) > = \frac{3l}{2G_N\bar{z}^4}$$

$$< t(z,\bar{z})T(0) > = \frac{-3l}{2G_N z^4}$$

$$< t(z,\bar{z})t(0) > = \frac{1}{2G_N}\frac{-3B_m - 11 + 6\ln\left(m^2\left|z^2\right|\right)}{z^4}$$

Logarithmic GCA in the context of holography

$$< T(z)T(0) > = \frac{3l}{2G_N}\frac{1}{(t-\frac{x}{c})^4} = \frac{3l}{2G_N}t^{-4} + \frac{6l}{G_N}t^{-4}\frac{x}{ct} + \cdots$$

$$< T(z)t(0) > = \frac{-3l}{2G_N}\frac{1}{(t-\frac{x}{c})^4} = -\frac{3l}{2G_N}t^{-4} + \frac{6l}{G_N}t^{-4}\frac{x}{ct} +$$

$$< t(z,\bar{z})t(0) > = \frac{3l}{G_N}\,ln(m^2t^2)\,t^{-4}\left(1 - 4\frac{x}{ct} + \cdots\right)$$

$$-\frac{(3B_m+11)l}{2G_N}t^{-4}\left(1 - 4\frac{x}{ct} + \cdots\right)$$

Logarithmic GCA in the context of holography: *Boundary model*

In an approach presented by Kogan et al. it is supposed that as cL goes to zero, beside energy-momentum tensor there is another operator X with conformal dimension of (2+η(cL), η(cL)) which in the limit approaches to (2, 0). C_L The non-vanishing two point-functions are

$$< \bar{T}\bar{T} > = \frac{c_R}{\bar{z}^4}$$

$$< TT > = \frac{c_L}{z^4}$$

$$< XX > = \frac{1}{c_L}\frac{\alpha(c_L)}{z^{4+2\eta(c_L)}\bar{z}^{2\eta(c_L)}}$$

I. I. Kogan, A. Nichols, arXiv:0203207v1 [hep-th].

Boundary model as a limit

Now, we have two limits in the boundary

$C_L \to 0$ $\quad\quad$ $c \to \infty$

$$T_1 = \frac{1}{\sqrt{c_L}}\bar{T} + \frac{\beta(c_L)}{c_L}X + \frac{\gamma(c_L)}{c_L}T$$

$$T_2 = \frac{1}{c}\left(\frac{1}{\sqrt{c_L}}\bar{T} - \frac{\beta(c_L)}{c_L}X - \frac{\gamma(c_L)}{c_L}T\right)$$

$$T_3 = T_z + \sqrt{c_L}T_{\bar{z}\bar{z}}$$

$$\lim_{c_L \to 0}\left[\left(\frac{\beta^2\alpha}{c^2{}_L} + \gamma^2\acute{}\right)\right] = \mathrm{C}_1 \qquad \lim_{c_L \to 0}\left[-2\acute{\alpha}\frac{\beta^2\alpha}{c^2{}_L}\right] = \mathrm{C}_3$$

$$\lim_{c_L \to 0\ \mathrm{c}\to\infty}\left[\left(\frac{2c_R}{c\ c^2{}_L}\right)\right] = \mathrm{C}_2 \qquad \lim_{c_L \to 0}\gamma + \mathrm{c_R} = C_4$$

Boundary model as a limit

Taking those limits we have free parameters in our contracted model.

$$< T_1 T_1 > = c_1 t^{-4} + 4c_2 t^{-4} \frac{x}{t} + 2c_3 t^{-4} \ln t$$

$$< T_1 T_2 > = c_2 t^{-4}$$

$$< T_1 T_3 > = c_4 t^{-4}$$

$$< T_2 T_2 > = < T_2 T_3 > = < T_3 T_3 > = 0$$

Now we need to check the bulk for these possible two-point functions

Boundary model in the limit

$$< \bar{T}\bar{T} >= \frac{3l}{2G_N}\frac{\lambda+1}{2\lambda+1}\frac{1}{\bar{z}^4} \qquad < TT >= \frac{3l}{2G_N}\frac{\lambda}{2\lambda+1}\frac{1}{z^4}$$

$$< XX >= -\frac{l}{2G_N}\frac{\lambda(\lambda+1)(2\lambda+3)}{2\lambda+1}\frac{1}{z^{2\lambda+4}\bar{z}^{2\lambda}}$$

where $\mu l = 2\lambda + 1$ or $\lambda \to 0$

$$T_1 = \frac{1}{\sqrt{\lambda}}\bar{T} - \frac{1}{\lambda}T$$

$$T_3 = T_z + \sqrt{c_L}T_{\bar{z}\bar{z}} \qquad T_2 = \frac{1}{c}(\frac{1}{\sqrt{\lambda}}\bar{T} + \frac{1}{\lambda}X)$$

Boundary model in the limit

$$\lim_{\mu l \to 1} \frac{3-(\mu l+2)t^{-2(\mu l-1)}}{(\mu l-1)} = 3\ln t^2 - 1$$

$$\lim_{\mu l \to 1}\lim_{c \to \infty} \frac{3+(\mu l+2)t^{-2(\mu l-1)}}{(\mu l-1)c} = 6$$

$$< T_1 T_1 > = t^{-4}\left(-\frac{l}{G_N} + \frac{24l}{G_N}\frac{x}{t} + \frac{6l}{G_N}\ln t\right)$$

$$< T_1 T_2 > = t^{-4}\left(\frac{6l}{G_N}\right) \qquad < T_1 T_3 > = \frac{3l}{G_N}$$

$$< T_2 T_2 > = < T_2 T_3 > = < T_3 T_3 > = 0$$

Thank you